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The Enhanced EWMA Control Chart with Autocorrelation

Abbas Umar Farouk , Ismail Bin Mohamad*

Mathematical Sciences Department, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor Malaysia

*Corresponding author : ismailm@utm.my

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Graphical abstract

$Z_t = \lambda X_t + (1-\lambda)Z_t$

Abstract

Control charts are effective tool with regard to improving process quality and productivity, Shewhart control charts are efficiently good at detecting large shifts in a given process but very slow in detecting small and moderate shifts, such problem could be tackled through design of sensitizing rules. It has been observed that autocorrelation has an advert effect on the control charts developed under the independence assumption [1]. In this article a new EWMA control chart has been introduced with autocorrelation and some run rule schemes were introduced to enhanced the performance of the EWMA control chart when autocorrelated. The three-out-of three EWMA scheme and three-out-of- four EWMA schemes were introduced and the generated data with induced autocorrelation were used to construct the EWMA chart to sensitize the shifts presence. Simulation of autocorrelated data were carried out for the proposed schemes which detects the shifts as soon as it occurs in the given process, the performance were evaluated using the ARL (average run length) and the results were compared with the published results of Steiner (1991) and the Saccucci (1990) which were designed for large, small and moderate shift. The results indicates that the proposed schemes are more sensitive to the shifts at ARL_0=500, 300 and 200 with autocorrelation of 0.2, 0.5 and 0.9 considered in the study.

Keywords: Autocorrelation; EWMA; Run rules; Average Run Lengths, Simulation, CUSUM

Abstrak

Carta kawalan adalah alat yang berkesan dengan mengambil kira peningkatan kualiti proses dan produktiviti, carta kawalan Shewhart adalah cekap pandai mengesan anjakan besar dalam proses yang diberikan tetapi sangat perlahan dalam mengesan anjakan kecil dan sederhana, masalah tersebut boleh ditangani melalui rekabentuk kepekaan peraturan. Ia telah diperhatikan bahawa autokorelasi mempunyai kesan iklan pada carta kawalan yang dibangunkan di bawah andaian kemerdekaan [1]. Dalam artikel ini carta kawalan EWMA baru telah diperkenalkan dengan autokorelasi dan beberapa skim peraturan jangka telah diperkenalkan untuk meningkatkan prestasi carta kawalan EWMA apabila autocorrelated. Tiga keluar-tiga skim EWMA dan tiga keluar-of-empat skim EWMA diperkenalkan dan data yang dihasilkan dengan autokorelasi teraruh telah digunakan untuk membina carta EWMA untuk menyedarkan kehadiran perubahan itu. Simulasi data autocorrelated telah dijalankan bagi skim yang dicadangkan yang mengesan perubahan sebaik sahaja ia berlaku dalam proses yang diberikan, prestasi yang telah dinilai menggunakan ARL (purata panjang larian) dan keputusan dibandingkan dengan keputusan diterbitkan Steiner (1991) dan Saccucci (1990) yang telah direka untuk anjakan besar, kecil dan sederhana. Keputusan menunjukkan bahawa skim yang dicadangkan adalah lebih sensitif kepada perubahan di ARL 0 = 500, 300 dan 200 dengan autokorelasi 0.2, 0.5 dan 0.9 dipertimbangkan dalam kajian ini.

Kata Kunci: autokorelasi; EWMA; Peraturan Main; Purata Run Panjang, Simulasi, CUSUM.

1.0 INTRODUCTION

For some time, it is been assumed in statistical process control (SPC) that the observations from the fundamental procedure are independent, however this assumption was violated in practice [2]. As a result, a number of authors talked about the way the classical Shewhart, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts behave with regard, to autocorrelated processes ([3], [2, 4]), which made these schemes not good when the same control limits are used as with the situation of independent variables. Because of this it is important to use time series models to create control charts. SPC is really a significant quality control issue by which data analysis is employed to find out if the process is under control. One main purpose of SPC would be to identify immediately a process shift and adopt the required corrective action to enhance process quality. Control charts tend to be widely applied tools for monitoring processes. Harris and Ross (1991) discussed many different correlative structures and used simulations to review the impacts of those correlative structures on the traditional CUSUM as well as EWMA control charts. [5] studied Shewhart charts when process observations could be modelled as an autoregressive of order 1 AR(1) process with random error.

As it is clear that control charts are effective tool with regard to improving process quality as well as productivity. Shewhart control chart are efficiently good in detecting large shift in a given process, but it's too slow in detecting small and moderate shifts, however, such problems could be tackle through design of sensitizing rules. Autocorrelation has a significant effect on the control charts developed under the independence assumption. where some runs rules schemes are introduced.[6], and [7] presented a runs rules schemes applied to the Shewhart control charts to enhance their performance for the small and moderate shifts, keeping the false alarm rate at the pre-specified level. Also [8] presented two-runs rules schemes for the CUSUM charts and have shown that their proposed schemes perform better for small and moderate shifts while they reasonably maintain their efficiency for large shifts as well. We were inspired by their work, hence we also proposed two new schemes in this research for the design structure of EWMA control chart named as '3/3 EWMA scheme' and 'modified ¾ EWMA scheme'.

Quality improvement and statistical control charts are tools used for monitoring the quality of a given process. Shewhart – type control charts are highly effective in detecting large shifts. [9] introduced some enhancements of the EWMA chart so as to obtain the ability of Shewhart charts to detect large mean shifts and that of EWMA in detecting small/moderate shifts. Hence, the resulting control charts allows them to effectively deal with a range of shifts values of different sizes.

[10], models the classical EWMA for autocorrelated processes with model uncertainty, by developing a technique for designing residual based EWMA charts under consideration of the uncertainty within the estimated model parameters, while using method of widening the EWMA control limits in line with the worst-case design approach. However, their study was limited only to the residual-based control chart for the statistical process of autocorrelated processes.

According to [11], they investigated and discussed the performance of the combined Shewhart-EWMA control chart with estimated parameters, the results shows that estimation errors affect the in-control CSEWMA performance much more than its power detection but with the addition of the Shewhart control limits to the standard EWMA seems to offer a better protection against a higher rate of false alarms. [12], proposed schemes that yield substantially reduced ARLs of CSEWMA control charts as compared with certain established schemes.

Autocorrelated observations are common within industry, particularly when data are sampled in a high frequency from processes with inertia i.e. samples from a nonstop acceleration. The classical EWMA control chart is actually non-robust to serial correlation or autocorrelation [13]. The Modified EWMA is just capable to deal with the issue of monitoring small and large shifts with high autocorrelated observations. Hence, we are motivated to further the research to enhance the performance of the EWMA control chart with low and medium autocorrelation. [14] attempted to deal with the problem of detecting small shifts of parameter process in a small or moderate autocorrelation. However, the scheme still could not deal with the problem of detecting moderate and high shifts with large autocorrelation.

It is known that classical EWMA was established to tackle the small and moderate shifts of the process mean. As mentioned previously that Modified EWMA and Classical EWMA has serious limitations of not being able to detect moderately and large shifts with high autocorrelation and high smoothing constant (λ) at different levels of shifts (low, moderate and high). The problem at hand in this article is to construct a EWMA control chart that can cope with different levels of autocorrelation.

2.0 THE EWMA CONTROL CHART

The EWMA control chart was introduced by [15] to address the shifts of smaller and moderate magnitude. As it is a known facts that EWMA-type charts perform better than the Shewhart-type charts for small and moderate shifts in the process. The plotting statistic is defined as:

$$Z_t = \lambda X_t + (1-\lambda) Z_t \qquad (1)$$

Where X_t is the current information (for t=0,1,2....), Z_{t-1} is the past information and λ is the smoothing constant (0<λ≤ 1). The initial value for the past information is Z_0 which is equal to the target mean μ_0. The mean and variance of the EWMA statistic is

$$\text{Mean}(Z_t) = \mu_0,$$

variance

$$(Z_t) = \sigma^2 \left\{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)\right\} \qquad (2)$$

The control structure of EWMA control chart is given as:

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)},\ CL = \mu_0,$$

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \qquad (3)$$

The performance of the EWMA control charts can be further enhance using different techniques. [9] proposed the first initial response (FIR) features for the EWMA chart, which substantially improves the ARL_1 performance but at the cost of decrease in the ARL_0 of the EWMA control chart, which is not recommended by the practitioners. The application of theses extra schemes is restricted to the Shewhart-type control charts. Riaz et al proposed the use of some rules with the CUSUM and enhanced their ability. However, with this development so far on the improvement of the performance of the EWMA in detection ability, there is no available literature known to us on the enhancement of EWMA as a generalization to Shewhart performance with autocorrelated data, in view of that we proposed a two new schemes to improve the EWMA performance. The details regarding the proposed schemes for EWMA charts are provided in the next section.

3.0 RESULTS AND DISCUSSION

In this article we proposed two new schemes for the design structure of EWMA control chart named 3 -out- of 3 EWMA scheme and modified 3-out-of 4 EWMA scheme.

Proposed Scheme I (3 out of 3 EWMA scheme) : a process is said to be out-of-control if three consecutive points are plotted either below a lower signalling limit (LSL) or above an upper signalling limit (USL).

Proposed Scheme II (Modified 3 out of 4 EWMA scheme): a process is said to be out-of control if one of the following conditions is satisfied:

i. At least three out of four consecutive points fall below an *LSL* and thee point above the *LSL* (if any) falls between the *CL* and the *LSL*.

ii At least three out of four consecutive points fall above a *USL* and the point below the *USL* (if any) falls between *CL* and the *USL*

the control structure for the proposed schemes is given as follows:

$$USL = \mu_0 + A\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right.},LSL = \mu_0 - A\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right.} \quad (3)$$

Where A is the signalling limit coefficient of the proposed schemes.

3.1 Performance Evaluation Of The Proposed Schemes

In evaluating the performance of the proposed schemes we choose the popular among them as the performance measure, which is ARL, that is considered at different in-control and out-control situations. The pre-specified ARL_0 values, for this article is fixed at 200,300 and 500 with *ar* = 0.1 and the values of ARL_1 are obtained at different values of δ=0.00,0.25,...,2.00 as displayed in Tables 1- 6, for the two schemes.

Table 1 3 out of 3 run rule with ARL_0=500, ar = 0.1

3 out of 3, ARL=500				
Smoothing Constant	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
Multiplier	L=2.545	L=2.578	L=2.358	L=2.956
Shift				
0.00	486.05	501.23	503.71	499.98
0.25	101.21	189.56	233.67	245.23
0.50	29.41	56.79	76.56	89.56
0.75	14.12	20.65	30.79	45.45
1.00	8.82	12.54	15.28	20.78
1.25	6.32	7.78	9.01	12.34
1.50	4.87	5.65	6.13	7.45
1.75	3.34	2.23	4.59	3.12
2.00	1.57	2.67	3.68	2.25

Table 2 3 out of 3 run rule with ARL_0=300, ar = 0.1

3 out of 3, ARL_0=300				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.355	L=2.455	L=2.459	L=3.015
Shift				
0.00	294.32	300.87	297.54	296.46
0.25	75.32	96.76	102.87	112.45
0.50	24.04	34.89	50.67	70.65
0.75	12.36	20.45	30.68	45.78
1.00	7.90	9.67	15.87	20.54
1.25	5.72	6.89	8.45	9.45
1.50	4.46	5.65	6.23	7.32
1.75	3.71	2.67	2.65	3.45
2.00	3.22	1.56	1.67	2.04

Table 3 3 out of 3 with ARL_0=200, ar = 0.1

3out of 3, ARL_0=200				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.200	L=2.230	L=2.345	L=1.865
Shift				
0.00	197.86	200.00	200.02	196.37
0.25	59.35	70.45	90.45	120.42
0.50	20.70	30.79	45.24	53.28
0.75	10.93	17.89	20.67	25.34
1.00	7.23	8.76	10.76	13.79
1.25	5.26	6.43	7.45	8.26
1.50	4.17	5.45	6.24	5.70
1.75	2.56	3.67	4.34	4.28
2.00	1.45	1.60	1.78	3.39

Table 4 3 out of 4 run rule with ARL_0=500, ar = 0.1

3 out of 4, ARL_0=500				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.555	L=2.653	L=2.789	L=2.643
Shift				
0.00	498.17	500.00	499.99	500.01
0.25	98.98	101.45	121.23	143.23
0.50	30.54	60.97	70.97	79.97
0.75	13.91	23.89	29.92	35.56
1.00	8.76	11.78	18.76	20.67
1.25	6.21	7.45	9.45	10.43
1.50	3.34	5.11	6.34	6.78
1.75	2.25	2.34	2.54	4.89
2.00	1.54	1.67	1.90	3.45

Table 5 3 out of 4 with ARL_0=300, ar = 0.1

3 out of 4, ARL=300				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.457	L=2.595	L=2.895	L=3.105
Shift				
0.00	300.00	295.09	298.34	300.12
0.25	120.34	176.48	196.34	198.35
0.50	30.46	48.85	50.23	53.00
0.75	15.23	20.03	22.12	17.23
1.00	9.98	11.06	12.32	10.43
1.25	6.23	7.24	8.21	8.77
1.50	3.45	5.34	5.45	5.67
1.75	1.45	4.31	2.34	2.69
2.00	0.56	3.62	1.23	2.78

Table 6 3 out of 4 with ARL_0=200, ar = 0.1

3 out of 4, ARL=200				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.235	L=2.265	L=2.342	L=2.357
Shift				
0.00	199.99	200.95	200.00	200.78
0.25	68.45	85.98	105.67	124.67
0.50	19.45	29.71	45.34	67.45
0.75	8.78	13.96	20.78	34.67
1.00	6.45	8.38	15.23	21.98
1.25	5.45	5.86	8.34	10.56
1.50	3.45	4.54	5.23	8.34
1.75	1.47	3.71	3.89	4.56
2.00	0.99	3.17	2.34	2.45

The findings for the two run rules schemes are:

(i) The 3 of 3 and 3 of 4 run rules schemes' performance is satisfactorily good in detecting small and moderate shifts as well as the large shifts is also encouraging as shown in Table 1 with smoothing constant of 0.1, and in Table 2 with smoothing constant of 0.5, likewise in Table 3 with smoothing constant of 0.75 is quite good.

(ii) 3 out of 4 run rule scheme outperformed the 3 out of 3 scheme for the values of λ =0.1, 0.25, and 0.5 with ARL_0=500 as shown in Table 4

(iii) The two run rules schemes are fast at detecting shifts for small λ as shown in Tables 1,3,4 and 5

(iv) The 3 out of 4 scheme performs readily at detecting shifts for moderately large values of λ as shown in Tables 4. and 5.

3.2 Comparing The Graphical Outputs

We conduct a Markov chain simulation of 100,000 iterations using the autocorrelated data generated from standard normal distribution, using correlation coefficients of ρ= 0.2, 0.5 and 0.9, the results are compared to some existing schemes using graphical displays of ARL curves (Figure1).

Table 7 ARL values for the Classical EWMA scheme at ARL_0 =500 (cf.[15])

Classical EWMA, ARL=500				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.814	L=2.998	L=3.071	L=3.087
Shift				
0.00	500.00	500.00	500.00	500.00
0.25	106.00	170.00	255.00	321.00
0.50	31.30	48.20	88.80	140.00
0.75	15.90	20.10	35.90	62.50
1.00	10.30	11.10	17.50	30.60
1.50	6.09	5.46	6.53	9.90
2.00	4.36	3.61	3.63	4.54

Table 8 ARL values for the Proposed scheme I at ARL_0 =500

Proposed Scheme I, ARL=500				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.545	L=2.578	L=2.358	L=2.956
Shift				
0.00	486.05	501.23	503.71	499.98
0.25	101.21	189.56	233.67	245.23
0.50	29.41	56.79	76.56	89.56
0.75	14.12	20.65	30.79	45.45
1.00	8.82	12.54	15.28	20.78
1.25	6.32	7.78	9.01	12.34
1.50	4.87	5.65	6.13	7.45
1.75	3.34	2.23	4.59	3.12
2.00	1.57	2.67	3.68	2.25

Table 9 ARL values for the Proposed scheme II at ARL_0 =500

Proposed Scheme II, ARL=500				
	λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
	L=2.555	L=2.653	L=2.789	L=2.643
Shift				
0.00	498.17	500.00	499.99	500.01
0.25	98.98	101.45	121.23	143.23
0.50	30.54	60.97	70.97	79.97
0.75	13.91	23.89	29.92	35.56
1.00	8.76	11.78	18.76	20.67
1.25	6.21	7.45	9.45	10.43
1.50	3.34	5.11	6.34	6.78
1.75	2.25	2.34	2.54	4.89
2.00	1.54	1.67	1.90	3.45

Table 10 ARL values for the FIR EWMA at ARL_0 =500 (cf. [16])

ARL values for the FIR EWMA, ARL=500					
		λ = 0.1	λ = 0.25	λ = 0.5	λ = 0.75
		L=2.814	L=2.998	L=3.071	L=3.087
Shift	% HS				
0	25.00	487.00	491.00	497.00	498.00
	50.00	468.00	483.00	487.00	496.00
0.5	25.00	28.30	46.50	87.80	140.00
	50.00	24.20	43.60	86.10	139.00
1	25.00	8.75	10.10	16.90	30.20
	50.00	6.87	8.79	15.90	29.70
2	25.00	3.57	3.11	3.29	4.33
	50.00	2.72	2.50	2.87	4.09

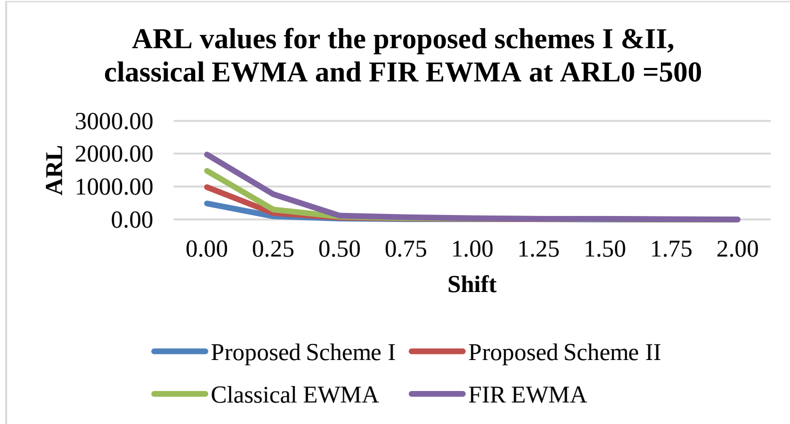

Figure 1 ARL curves for the proposed schemes I &II, classical EWMA and FIR EWMA at ARL_0 = 500

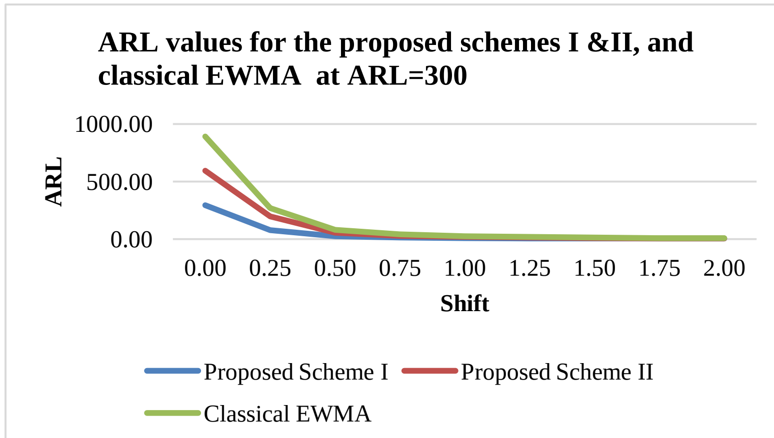

Figure 2 ARL curves for the proposed schemes I &II, and classical EWMA at ARL_0 = 300

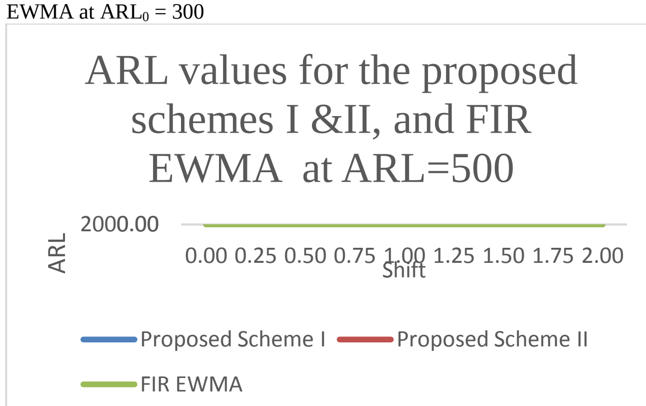

Figure 3 ARL curves for the proposed schemes 1&II, and FIR EWMA at ARL_0 = 500

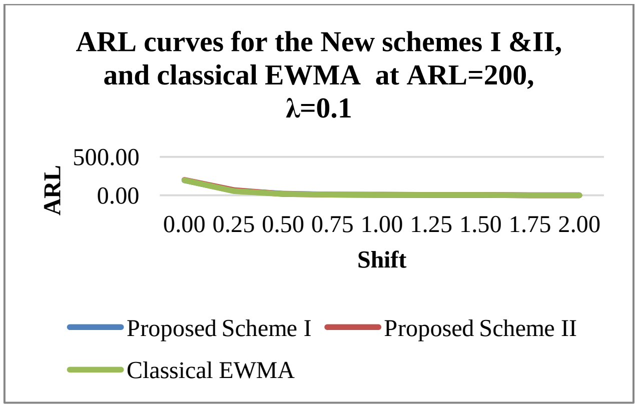

Figure 4: The ARL curves for New Schemes I &II and Classical with ARL =200

Based on these results, the following findings:

i. Proposed schemes versus the classical EWMA: The classical is defined by Roberts, whereas the ARL values of classical EWMA are displayed in Table 7, in Figures 1 and 2 at ARL_0 =500 and 300 respectively, it shows the proposed schemes I and II performs better than the classical EWMA in quick detection of shift in this study.

ii. Proposed schemes versus FIR EWMA: Lucas and Saccucci proposed the application of the FIR feature with the EWMA control chart, especially with the small values of λ, the ARL values of the FIR EWMA are provided in Table 10. Upon comparison displayed in Figure 3 shows also the proposed schemes I and II performs better than the FIR EWMA scheme.

Moreover, for an overall comparison of the proposed schemes with their existing counterparts mentioned and compared above, we have made some graphs showing ARL curves at different schemes as shown in Figure 1. Which has shown that the proposed scheme I is better than all the compared schemes in this study.

■4.0 CONCLUSION

The effectiveness of the control charts can be enhanced by using different run rule schemes along with their structural designs. This is so, if the control charts were able to detect some assignable or common causes. Hence in this research we have proposed two run rules schemes to be applied to EWMA with autocorrelated data. The proposed scheme I gave an out-of-control signal if three out of three consecutive points are plotted either below *LSL* or above *USL*, and the proposed scheme II gave an out of control signals if at least three out of four consecutive points are plotted below *LSL* or above *USL* by keeping the in-control point in between the CL and the limit. ARL was used to investigate the performance of the proposed schemes and found them better than the compared schemes published in terms of quick detection of the shift in a given process. Other control charts can be explore using the autocorrelated data is also an area for future research.

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