

# An Artificial Neural Network Approach for Prediction of Bearing Capacity of Spread Foundations in Sand

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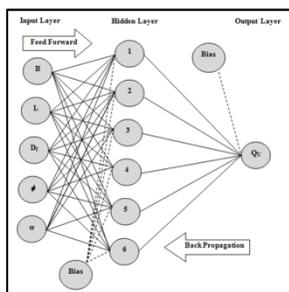
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## Graphical abstract



ANN model architecture

## Abstract

This study highlights the application of Back-Propagation (BP) feed forward Artificial Neural Network (ANN) as a tool for predicting bearing capacity of spread foundations in cohesionless soils. For network construction, a database of 75 recorded cases of full-scale axial compression load test on spread foundations in cohesionless soils was compiled from literatures. The database presents information about footing length (L), footing width (B), embedded depth of the footing ( $D_f$ ), average vertical effective stress of the soil at  $B/2$  below footing ( $\sigma'$ ), friction angle of the soil ( $\phi$ ) and the ultimate axial bearing capacity ( $Q_u$ ). The last parameter was set as the desired output in the ANN model, while the rest were used as input of the ANN predictive model of bearing capacity. The prediction performance of ANN model was compared to that of Multi-Linear Regression analysis. Findings show that the proposed ANN model is a suitable tool for predicting bearing capacity of spread foundations. Coefficient of determination  $R^2$  equals to 0.98, strongly indicates that the ANN model exhibits a high degree of accuracy in predicting the axial bearing capacity of spread foundation. Using sensitivity analysis, it is concluded that the geometrical properties of the spread foundations (B and L) are the most influential parameters in the proposed predictive model of  $Q_u$ .

**Keywords:** Bearing capacity, spread foundations, artificial neural network, sensitivity analysis, multi-linear regression analysis.

## Abstrak

Kajian ini menekankan penggunaan Back- Propagation (BP) feedforward ANN sebagai alat untuk meramalkan keupayaan galas asas tersebar di tanah berpasir. Untuk pembinaan rangkaian, pangkalan data 75 kes yang direkodkan daripada skala penuh ujian beban mampatan paksi pada asas tersebar di tanah berpasir dikumpulkan dari kajian literatur. Pangkalan data ini mengandungi maklumat tentang panjang asas (L), lebar asas (B), kedalaman terbenam asas tersebut ( $D_f$ ), purata tegasan berkesan tanah pada  $B/2$  di bawah kedudukan ( $\sigma'$ ), sudut geseran tanah ( $\phi$ ) dan keupayaan galas paksi muktamad ( $Q_u$ ). Parameter terakhir ditetapkan sebagai output yang dikehendaki dalam model ANN, manakala yang lain telah digunakan sebagai input model ramalan ANN bagi keupayaan galas. Prestasi ramalan model ANN telah dibandingkan dengan Multi- Linear Regression Analisis. Hasil kajian menunjukkan bahawa model ANN adalah lebih baik dalam meramalkan keupayaan galas asas tersebar. Pekali penentuan,  $R^2$ , bersamaan dengan 0.98 menunjukkan bahawa model ANN memberikan darjah tahap kejituan keupayaan yang tinggi dalam meramal galas paksi asas penyebaran. Kesimpulannya, menggunakan analisis sensitiviti, didapati bahawa sifat-sifat geometri asas penyebaran (B dan L) adalah parameter yang paling berpengaruh dalam model ramalan cadangan  $Q_u$ .

**Kata kunci:** Keupayaan galas, asas tersebar, ANN, analisis sensitiviti, analisis regresi multi linier

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## 1.0 INTRODUCTION

Spread or shallow foundations are often used to transfer the column-load of the low to moderate rise structures to the underlying soils. Das [1] reported that a foundation can be considered shallow if its embedded depth to width ratio is equal to or less than four. In geotechnical designing of spread foundations, two essential criteria should be controlled: settlement of the foundations under structural loads, and ultimate bearing capacity.

The latter is defined as the maximum load which the soil under footing can bear before shearing failure. Many researchers have developed semi-empirical solutions for bearing capacity of spread foundations (e.g. [2];[3];[4]).

Nevertheless, most of these methods incorporate several assumptions to simplify the problem. Moreover, the aforementioned methods mostly rely on empiricism and are site specific [5]. Hence, they need to be validated with more reliable methods. The most reliable method to determine the ultimate

bearing capacity of foundations is axial compression test of foundation in the field. However, experimental loading of foundations is time-consuming and costly. The use of Artificial Neural Network (ANN) as a quick, accurate and feasible tool in solving the bearing capacity problems has been highlighted in literature recently (e.g. [6]). The main objective of this study is to propose an ANN-based predictive model of ultimate bearing capacity of spread foundations in cohesionless soils.

## ■2.0 BACKGROUND

In the recent past, the soft computation techniques, especially ANNs, have been applied successfully for solving bearing capacity problems in both deep and shallow foundations (e.g. [7];[8]). Shahin et al. [9] demonstrated the feasibility of ANN in predicting the settlement of spread foundations. They compiled a database of 189 individual cases for network construction. It was suggested in their study that footing width (B), footing net applied pressure, and average standard penetration test blow count ( $N$ ), footing embedded ratio and footing geometry are the important factors in settlement analysis and foundation design. According to their conclusion, ANN method works reasonably well in predicting the spread foundation settlement.

Soleimanbegi and Hataf [10] utilized feedforward BP-ANN for predicting the bearing capacity of shallow foundations. They have compiled a database including 351 records of laboratory and field measurement of spread foundations bearing capacity on reinforced cohesionless soils. The architecture of their proposed predictive model comprises 10 input layers, one hidden layer and one output layer. They used footing geometrical and soil engineering properties to train ANN models. According to their conclusion, ANN-based predictive model outperforms the conventional methods of bearing capacity estimation.

In another study, Adrash et al. [11] discussed the application of soft computation techniques in predicting the bearing capacity of cohesionless soils. The length of the footing,  $L$ , length to width ratio,  $L/B$ , density of the soil and internal friction angle of the soil,  $\phi$ , were used as the inputs of their proposed predictive models of ultimate bearing capacity. It was highlighted in their study that predicted bearing capacity using ANN is in good agreement with the experimental results.

Ornek et al. [12] focused on the application of ANN in estimating the bearing capacity of circular footing on soft clay stabilized with granular soil. They have conducted several field tests on seven footings with various diameters in multilayer granular soil with different thicknesses. In their study, diameter of the footing, thickness of the granular fill layer and the settlement of the footing were used as inputs of the network while the output was set to be the predicted bearing capacity of spread foundations in natural clay deposits. According to their conclusion, the correlation coefficients values equal to 0.99 and 0.95 for training and testing datasets respectively suggest that ANN models serve as a simple and reliable tool for predicting the bearing capacity of foundations.

## ■3.0 ARTIFICIAL NEURAL NETWORK

Artificial Neural Network (ANN) is an information processing technique which is based on our understanding of human-brain information process. It comprises several layers of many interconnected processing elements (nodes). A specific ANN network is identified by three important components: transfer function, network architecture and learning rule [13].

Nevertheless, determination of these components depends on the type of the problem [14].

There are two major types of ANN: recurrent and feedforward ANNs. The latter can be implemented if there is no time-dependent parameter in defining ANNs [9]. One of the most popular feedforward ANNs is Multi-Layer Perceptron (MLP) neural network [15]. This type of ANN consists of a number of nodes in various layers (input, hidden(s) and output(s) layers) connected to each other via different weights.

Utilization of MLP-ANN is of interest due to its high efficiency in approximating different functions in high dimensional spaces ([16];[17]). After selecting the architecture of the network and prior to interpreting net information, the ANN model should be trained. Back Propagation (BP) algorithm is the most widely used technique for training the MLP feedforward neural networks ([18];[19]).

As illustrated by Kuo et al. [20] in BP-ANN, the propagation of information is initiated from input layer where the real input data are fed in. In the next step, the input from each node in previous layer ( $X_i$ ) is multiplied by an adjustable connection or weight,  $W_{ij}$ . At each node, the sum of weighted input signals is obtained and subsequently this value is added to a threshold value or bias ( $\theta_j$ ) as shown in equation 1.

This combined input ( $I_j$ ) is then passed through a nonlinear transfer function,  $f(I_j)$ , such as sigmoidal function, to produce the output of the node (see equation 2). Details regarding transfer functions are beyond the scope of this paper and can be found elsewhere [21]. However, the output of each node provides the input to the next layer node. This procedure is continued until the output is generated.

The generated output then is compared to the desired output (presented output to the network) and the error is calculated. The purpose of BP training is to change iteratively the weights between the nodes in a direction that minimizes the mean square error (MSE) of the system where MSE is defined as the squared difference between the desired and the actual outputs [8]. Detail of the BP algorithm is out of the scope of this study and can be found in many literatures (e.g. [22]).

$$I_j = \sum (w_{ij} \cdot x_i) + \theta_j \quad (1)$$

$$y(i) = f(I_j) \quad (2)$$

## ■4.0 DATABASE

A database of 75 recorded cases of axial compression tests on spread foundations was compiled from literature. The recorded database include the geometric properties of the tested footings including width (B) and length (L) of the foundation, embedded depth of the foundation ( $D_f$ ) in addition to some of the soil engineering properties of the project site *i.e.* soil internal friction angle ( $\phi$ ), average vertical effective stress at B/2 under the footing ( $\sigma'$ ) and ultimate axial bearing capacity ( $Q_u$ ). Apart from the last parameter, the other parameters were set as inputs of the ANN model due to the fact that they are influential parameters in designing spread foundations ([9];[10]). All tests were conducted in different type of cohesionless soils ranging from fine sand to coarse gravel. Table 1 presents the range of the feeding data used in this study. More detail regarding the database is reported in the study by Akbas and Kulhawiy [23].

**Table 1** Input and output parameters used in the predictive models

Variable category	Symbol	Unit	Minimum	Maximum	Average
Input	B	(m)	0.25	3.02	0.75
	L	(m)	0.25	3.02	0.80
	D	(m)	0	1.04	0.25
	$\phi$	Degree	28	50	38.59
	$\sigma'$	kN/m <sup>2</sup>	2.3	124.1	14.98
Output	Qu	(kN)	15	10300	709.76

### 5.0 PERSPECTIVE MODEL ARCHITECTURE

The performance of ANN stoutly relies on its network architecture. This is due to the fact that the network topology directly affects its computational complexity and generalization capability [24].

However, the network architecture design depends on the training algorithm as well as the number of nodes in input, hidden, and output layers. Although many scholars have defined the structural design of the ANNs (e.g. [25];[26]), there is no straight forward approach for selecting the optimum architecture of the network. It is often determined through a trial-and-error method. This is to say, several networks with different architectures *i.e.* various numbers of nodes and hidden layers are trained, tested and validated. Consequently the network that performs best is selected as the optimum network. Thus, the database needs to be randomly divided into three subsets: training, testing and validation. There is no clear guidance for selecting the size of the subsets. It is often determined based on a particular problem [12]. In this study, 70 percent of the data was used for training purpose, and the other 30 percent was distributed equally for testing and validation of the ANN model. The problem was then imposed on the ANN models via five input parameters:  $B$ ,  $D$ ,  $L$ ,  $\phi$ ,  $\sigma'$ . As mentioned earlier, they were selected due to their importance in the design of spread foundations.

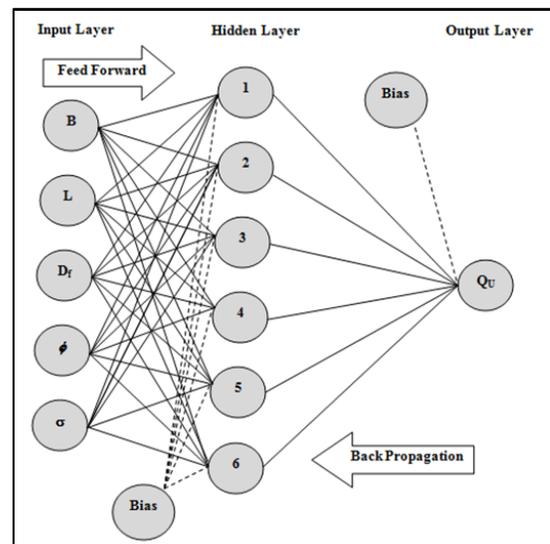
According to Hornik et al. [27], a network with one hidden layer can approximate any continuous function. Moreover, Cybenko [28] mentioned that, for practical problems, networks with maximum two hidden layers perform well enough. Nevertheless, in designing ANN architectures, increasing the number of the hidden layers should be the last options. In essence, focus should be on adding the number of nodes rather than the number of hidden layers [26]. Study by Swingler [29] suggests that the number of hidden layers should be smaller than that of the input layers. Based on previous studies, to develop the optimum network of the problem in hand, nine different ANN models with six, nine, and 11 hidden nodes in one, two, and three hidden layers were trained, tested and validated.

Table 2 shows the architecture and performance of different ANN models used in trial-and-error method. The coefficients of determination ( $R^2$ ) values were used to assess the performance of the network in this stage. It was found that the second model which comprises six nodes in one hidden layer performs best (see Figure 2). As displayed in this figure, the  $R^2$  values of the second

model suggest that the prediction performance of this model is better than that of other models. Hence, the second model was selected as the optimum network. The architecture of this model is shown schematically in Figure 1.

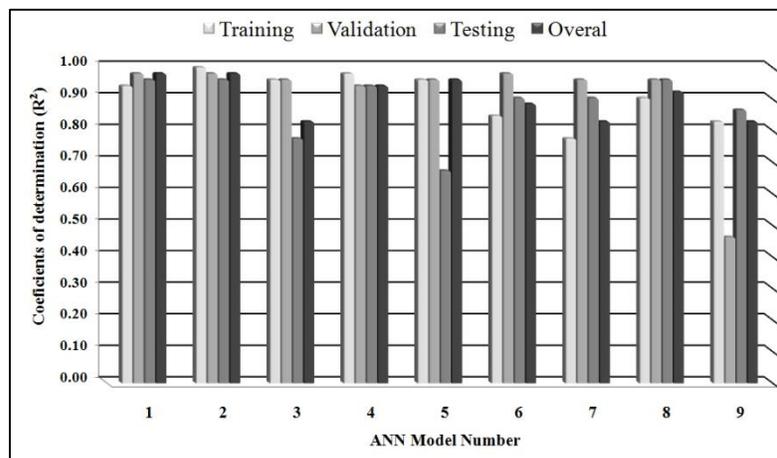
Nevertheless, it is worthy to note that in this study, the ANN models were trained using Levenberg–Marquardt (LM) learning algorithm due to its efficiency for training networks which have up to a few hundred weights. Moreover, it is well established that LM algorithm works faster than the conventional gradient descent technique (e.g. [30]).

According to Hagan and Menhaj [30], there are many reported case where Marquardt algorithm converges while other back-propagation techniques diverge. In fact, LM algorithm is an approximation of Newton's method. More details of this training algorithm are reported elsewhere [30].

**Figure 1** ANN model architecture

**Table 2** The architecture and performance of ANN models

ANN Model	Network Architecture		Coefficient of determination, R <sup>2</sup>			
	Node numbers	Hidden Layers	Training	Validation	Testing	Overall
1	9	1	0.94	0.98	0.96	0.98
2	6	1	1.00	0.98	0.96	0.98
3	11	1	0.96	0.96	0.77	0.83
4	6	2	0.98	0.94	0.94	0.94
5	9	2	0.96	0.96	0.67	0.96
6	11	2	0.85	0.98	0.90	0.88
7	6	3	0.77	0.96	0.90	0.83
8	9	3	0.90	0.96	0.96	0.92
9	11	3	0.83	0.46	0.86	0.83

**Figure 2** Prediction performance of different ANN models

## 6.0 MULTI-LINEAR REGRESSION ANALYSIS

Multi-Linear Regression (MLR) analysis is employed to establish a relationship between more than one Independent Variables (IV) and one Dependent Variable (DV). In this study, to have a better understanding of prediction performance of ANN model, MLR analysis was utilized using statistical software Microsoft Excel. The ANN input layer nodes were selected as the IVs in MLR analysis; and the ultimate bearing capacity was set to be the DV.

However, it is found that the following multivariate correlation suggests the best fitting statistical measure for the database given in Table 1. Details of the statistical information of the conducted MLR analysis are given in Table 3. The result of MLR analysis is discussed later.

$$Q_u = 3988.26B - 1046.1L + 302.74D + 27.73\phi + 3.37\sigma - 2621.70 \quad (3)$$

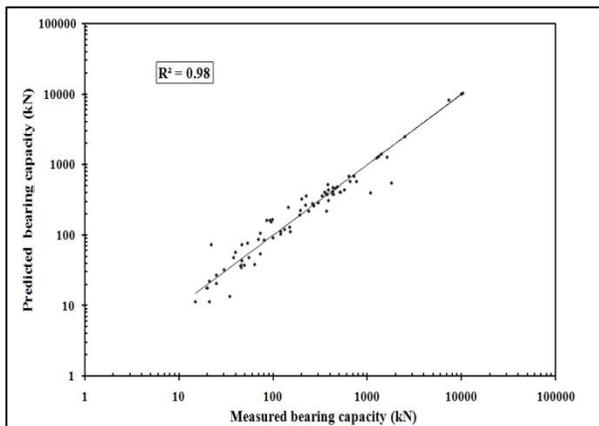
## 7.0 RESULT AND DISCUSSION

The prediction performance of the proposed ANN-model is shown in Figure 3. The coefficient of determination (R<sup>2</sup>) value equals to 0.98 reveals a strong regression and robust relationship between the predicted and measured  $Q_u$ . It is shown in Figure 3. Due to high variation of ultimate bearing capacities of the reported cases in the database, the measured and predicted  $Q_u$  are plotted against each other in logarithmic scale.

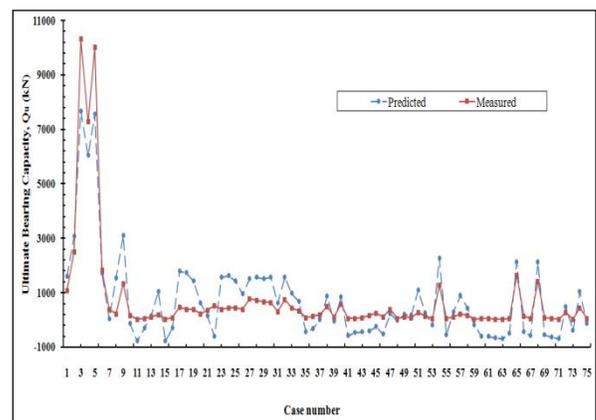
Figure 4 displays the performance of the proposed ANN-based predictive model of  $Q_u$  for all reported cases. As shown in this figure, in overall, the agreement between the predicted  $Q_u$  using ANN and the measured  $Q_u$  is indeed good.

**Table 3** Details of the statistical information of MLR predictive model

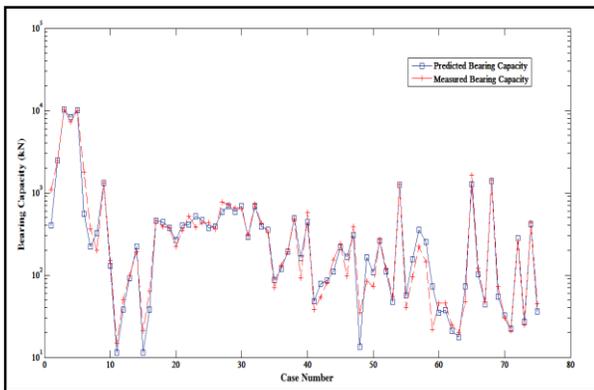
Independent variables (inputs)	Coefficient	Standard Error	T Stat	P-value
Intercept	-2621.69	933.16	-2.81	0.0064
B : Footing width	3988.26	499.24	7.99	2.097E-11
L : Footing Length	-1046.1	446.07	-2.35	0.0219
D : Footing embedded depth	302.74	303.49	0.99	0.3219
$\phi$ : Soil internal friction angle	27.73	23.039	1.20	0.2328
$\sigma'$ : Vertical effective stress at $B/2$	3.37	5.57	0.60	0.55



**Figure 3** Predicted  $Q_u$  using ANN against measured  $Q_u$



**Figure 5** The performance of the MLR based predictive model in predictive  $Q_u$



**Figure 4** The performance of the proposed ANN based predictive model in predicting  $Q_u$

To have a better understanding towards the prediction power of the ANN model, the performance of ANN is compared with that of MLR analysis. Figure 5 depicts the performance of MLR method in predicting the bearing capacity of foundations.

It can be seen in this figure that MLR method performs poorly in predicting the  $Q_u$ . In fact, in many cases the predicted  $Q_u$  is negative which is not acceptable. A simple comparison reveals the superiority of the proposed ANN model in predicting  $Q_u$ .

### 8.0 SENSITIVITY ANALYSIS

Sensitivity analysis is used to determine the relative importance of the input variables to the output value. In essence, sensitivity analysis is a method for extracting the cause-and-effect relationship between the inputs and outputs of a system. Cosine Amplitude Method (CAM) is one of the sensitivity methods which is used to identify the significance of each input parameter ([31];[32]). Utilization of this method in geotechnical engineering has been reported in literature (e.g. [14]). Hence, this method was used for the problem in hand.

In CAM method, the strength of ratio ( $r_{ij}$ ) can be determined by using the following formula. This ratio represents the percentage of influence of a particular input parameter on the output of the system. The presented  $U_{ik}$  and  $U_{jk}$  in the following equation represent the value of desired input parameter and the value of the desired output parameter respectively.

$$r_{ij} = \frac{\sum_{k=1}^n u_{ik} u_{jk}}{\sqrt{\sum_{k=1}^n u_{ik}^2 u_{jk}^2}} \quad (4)$$

The value of strength ratio close to zero suggests the less impact of the input parameter on the model output while the value close to one indicates that the input parameter is an influential parameter. However, using CAM method, it was found that the effect of footing geometry *i.e.* width and length of

the foundation on its ultimate bearing capacity is more pronounced as shown in Figure 6.

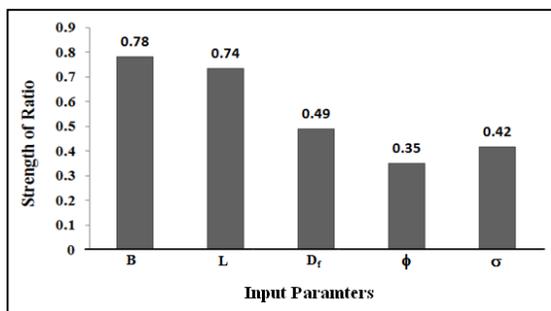


Figure 6 Strength of ratio between  $Q_u$  and input parameters

## 9.0 CONCLUSION

In this study, a feedforward back-propagation neural network with the Levenberg–Marquardt training technique was used to predict the bearing capacity of spread foundations in cohesionless soils. The database used to develop the ANN-based predictive model of  $Q_u$ , was based on 75 recorded cases compiled from literature.

Different network architecture were trained, tested and validated in order to find the best network architecture. For this reason, the database was randomly divided into three subsets: training, testing and validation. 70 percent of the data was considered for training purpose, 15 percent was used for testing the ability of model in generalization, and the 15 percent was assigned for validation. It was concluded that a network with six nodes in one hidden layer performs best; hence, this model was selected as the optimum ANN model. The prediction performance of the network was assessed based on of coefficient of determination ( $R^2$ ).

Findings show that ANN is a powerful and reliable tool in predicting the bearing capacity of spread foundations. The coefficient of determination ( $R^2$ ) equals to 0.98 suggests that the predicted bearing capacity by ANN model is in close agreement to the measured bearing capacity. Comparison between the prediction performance of ANN and MLR reveals the superiority of ANN in solving bearing capacity problems. However, to find the relative importance of input parameters in the predictive model, a sensitivity analysis was conducted using CAM method. Results showed that the width and length of the spread foundations are more significant parameters.

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