

APPLICATION OF MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING IN SUPPLY PRODUCTION PLANNING PROBLEM

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Abstract. The objective of this paper is to establish the usefulness of modified s-curve membership function in a limited supply production planning problem with continuous variables. In this respect, fuzzy parameters of linear programming are modeled by non-linear membership functions such as s-curve function. This paper begins with an introduction and construction of the modified s-curve membership function. A numerical real life example of supply production planning problem is then presented. The computational results show the usefulness of the modified s-curve membership function with fuzzy linear programming technique in optimising individual objective functions, compared to non-fuzzy linear programming approach. Furthermore, the optimal solution helps to conclude that by incorporating fuzziness in a linear programming model through the objective function and constraints, a better level of satisfactory solution will be provided in respect to vagueness, compared to non-fuzzy linear programming.

Keywords: Vagueness, s-curve membership function, degree of satisfaction, fuzzy linear programming, multi-objectives

1.0 INTRODUCTION

In the real-world of decision-making processes in engineering and business, decision making theory has become one of the most important fields. It uses the optimization methodology connected to a single criteria, but also satisfying concepts of multiple criteria. Decision processes with multiple criteria deal with human judgment. This is really hard to be modeled. The human judgment element is in the area of preferences defined by the decision maker [1-3]. First, attempts to model decision processes with multiple criteria in business and engineering lead to concepts of multi objective fuzzy linear programming [4-6]. In this approach, the decision maker underpins each objective with a number of goals that should be satisfied [7-8]. The term “satisfying” requires finding a solution to a multi criterion problem, which is preferred, understood, and implemented with confidence. The confidence that the best solution has been found is estimated through the ideal solution. This is the solution which optimizes all criteria simultaneously. Since this is practically unattainable, a decision maker considers feasible solutions closest to the ideal solution [9-10].

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In mathematical programming, problems are expressed as optimizing some objective function given certain constraints. So, the methods of solution were directed towards single objective mathematical programs such as the simplex method for linear programming. In applying mathematical programming, the decision-makers realized that there are real-life problems that consider multiple objectives. To be able to come up with a model that can be solved by the developed solution methods for single-objective mathematical programs, these multiple objectives must be combined in some way to become one single objective.

Many problems in operations research, decision science, engineering and management have mainly been studied from optimizing points of view. As decision making is much influenced by the disturbances of a social and economical circumstances, optimization approach is not always the best. It is because under such influences, many problems are ill-structured. Therefore, a satisfaction approach may be much better than an optimization one. In this regards, it is acceptable that the aspiration level on the treated problem is resolved on the base of past experiences and current knowledge possessed by a decision maker, in the case where the aspiration level of a decision maker should be considered to solve a problem from the perspective of satisfaction strategy. Therefore, it is more natural that the vagueness in the fuzzy system is denoted by fuzzy numbers by the decision maker.

Various types of membership functions were used in fuzzy linear programming problems and some of them are linear membership function [11-12], a tangent type of a membership function [13], an interval linear membership function [14], an exponential membership function [15], inverse tangent membership function [16], logistic type of membership function [17], concave piecewise linear membership function [18] and piecewise linear membership function [19]. The tangent type, membership function, exponential membership function, and hyperbolic membership function are non-linear functions. A fuzzy mathematical programming defined with a non-linear membership function results in a non-linear programming. Usually a linear membership function is employed in order to avoid non-linearity. Nevertheless, there are some difficulties in selecting the solution of a problem written in a linear membership function. Therefore, in this paper a modified s-curve membership function is employed to overcome such deficiencies which a linear membership function has. Furthermore, s-curve membership function is more flexible enough to describe the vagueness in the fuzzy parameters for the supply production planning problems.

In this paper, the new methodology of modified s-curve membership function [20-22] using fuzzy linear programming in supply production planning and their applications to decision making is proposed. Especially, the fuzzy linear programming based on vagueness in the fuzzy parameters such as resource variables given by a decision maker is analyzed.

2.0 MODIFIED S-CURVE MEMBERSHIP FUNCTION

The modified s-curve membership function is a particular case of the logistic function with specified parameters. These parameter values are to be found out. This logistic function as given by equation (1) and depicted in Figure 1 is indicated as s-shaped membership function by Gonguen [23] and Zadeh [24].

We define here, a modified s-curve membership function as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \frac{B}{1 + Ce^{\alpha x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \quad (1)$$

where μ is the degree of membership function. The term α determines the shape of membership function $\mu(x)$, where $\alpha > 0$. The larger the value of parameter α , the lesser is the vagueness. It is necessary that parameter α , which determines the shape of membership functions, should be heuristically and experientially decided by experts. In equation (1), B and C are constants, their values' are given in equation (10).

Figure 1 shows the modified S-curve. The membership function is redefined as $0.001 \leq \mu(x) \leq 0.999$. This range is selected because in supply production, the revenue and harmful pollution need not be always 100% of the requirement. At the same time the total revenue and total harmful pollution will never be 0%. Therefore, there is a suggested range between x^a and x^b as $0.001 \leq \mu(x) \leq 0.999$. This concept of range of $\mu(x)$ is used in this research paper.

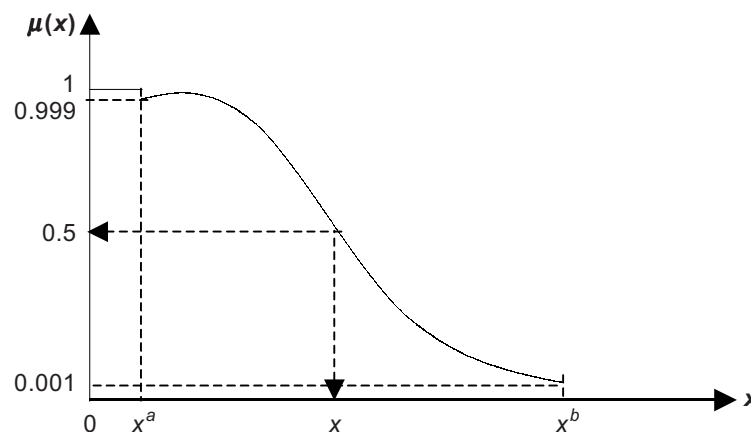


Figure 1 Modified s-Curve membership function

According to Watada [17], a triangular or trapezoidal membership function shows a lower level and upper level for μ at their grades 0 and 1, respectively. On the other hand, concerning a non-linear membership function such as a logistic function, a lower level and upper level may be approximated at the points with grades 0.001 and 0.999, respectively.

We rescale the x axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B , C , and α . Novakowska [25] has performed such a rescaling in his work of social sciences.

The values of B , C , and α are obtained from equation (1) as:

$$B = 0.999(1 + C) \quad (2)$$

$$\frac{B}{1 + Ce^\alpha} = 0.001 \quad (3)$$

By substituting equation (2) into equation (3):

$$\frac{0.999(1 + C)}{1 + Ce^\alpha} = 0.001 \quad (4)$$

Rearranging equation (4):

$$\alpha = \ln \frac{1}{0.001} \left(\frac{0.998}{C} + 0.999 \right) \quad (5)$$

Since, B and α depend on C , we require one more condition to get the values for B , C , and α .

Let, when $x_0 = \frac{x^a + x^b}{2}$, $\mu(x_0) = 0.5$. Therefore,

$$\frac{B}{1 + Ce^{\frac{\alpha}{2}}} = 0.5 \quad (6)$$

and hence,

$$\alpha = 2 \ln \left(\frac{2B - 1}{C} \right) \quad (7)$$

Substituting equation (5) and equation (6) into equation (7), we obtain,

$$2 \ln \left(\frac{2(0.999)(1 + C) - 1}{C} \right) = \ln \frac{1}{0.001} \left(\frac{0.998}{C} + 0.999 \right) \quad (8)$$

Rearrangement of equation (8) yields,

$$(0.998 + 1.998C)^2 = C(998 + 999C) \quad (9)$$

Solving equation (9):

$$C = \frac{-994.011992 \pm \sqrt{988059.8402 + 3964.127776}}{1990.015992} \quad (10)$$

Since C has to be positive, equation (10) gives $C = 0.001001001$ and from equation (2) and (5), $B = 1$ and $\alpha = 13.81$.

The modified s-curve membership function has similar shape as the logistic function [17] and that of the tangent hyperbolic function employed by Leberling [13]; but it is more easily handled than the tangent hyperbola. In addition, a trapezoidal and triangular membership functions are an approximation from a logistic function. Therefore, the s-function is considered much more appropriate to denote a vague goal level which a decision maker considers for the solution implementation. Furthermore, it is possible that the modified s-curve membership function changes its shape according to the parameter values [17]. Then, a decision maker is able to apply his strategy to a fuzzy supply production planning using these parameters. Hence, the modified s-curve membership function is much more convenient than the linear ones.

It should be noted that a triangular or trapezoidal membership function shows a lower level and upper level at their membership values of 0 and 1 respectively; on the other hand, concerning a non-linear membership function such as a modified s-curve function, a lower level and upper level may be approximated with membership values of 0.001 and 0.999, respectively. The idea of this approach is adopted from Watada [17].

3.0 FUZZY RESOURCE PARAMETER

First, we derive the equations for the fuzzy resource parameters. These equations will be used to generate fuzzy values for the respected parameters.

Fuzzy Resource Parameter \tilde{b}_i .

From equation (1), for an interval $b_i^a < b_i < b_i^b$,

$$\mu_{b_i} = \frac{B}{1 + Ce^{\alpha \left(\frac{b_i - b_i^a}{b_i^b - b_i^a} \right)}}$$

Rearranging exponential term,

$$e^{\alpha \left(\frac{b_i - b_i^a}{b_i^b - b_i^a} \right)} = \frac{1}{C} \left(\frac{B}{\mu_{b_i}} - 1 \right)$$

Taking \log_e both sides,

$$\alpha \left(\frac{b_i - b_i^a}{b_i^b - b_i^a} \right) = \ln \frac{1}{C} \left(\frac{B}{\mu_{b_i}} - 1 \right)$$

Hence,

$$b_i = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha} \right) \ln \frac{1}{C} \left(\frac{B}{\mu_{b_i}} - 1 \right) \quad (11)$$

Since b_i is the fuzzy resource variable in equation (11), it is denoted by \tilde{b}_i . Therefore,

$$\tilde{b}_i \Big|_{\mu=\mu_{b_i}} = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha} \right) \ln \frac{1}{C} \left(\frac{B}{\mu_{b_i}} - 1 \right) \quad (12)$$

The membership function for μ_{b_i} and the fuzzy interval, b_i^0 to b_i^1 for \tilde{b}_i are given in Figure 1.

4.0 NUMERICAL EXAMPLE OF FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING

The limited Supply Production Planning (SPP) problem with continuous variables are stated as:

A certain company has a factory, which produces 3 products. To produce 1 ton of product A, it needs 2 tons of material X, 3 tons of material Y, and 4 tons of material Z. To produce 1 ton of product B, it needs 8 tons of material X, and 1 ton of material Y. To produce 1 ton of product C, it needs 4 tons of material X, 4 tons of material Y, and 2 tons of material Z. At current prices, the company expects to sell product A at a rate of 5 million/ton, product B at a rate of 10 million/ton, and product C at a rate 12 million/ton. However, during production process, producing 1 ton of product A generates 1 ton of harmful pollution, producing 1 ton of product B generates 2 tons of harmful pollution, and producing 1 ton of product C produces 1 ton of harmful pollution. The company's objective is to maximize total revenue and minimize total harmful pollution produced.

This problem can be expressed as the following multi-objective linear program:

$$\text{Max } z_0 = cx \quad \text{and} \quad \text{Min } z_1 = dx$$

$$\text{s.t.} \quad Ax \leq b, \quad x \geq 0$$

$$\text{where: } x^T = (x_1, x_2, x_3), \quad c = (5, 10, 12), \quad d = (1, 2, 1)$$

$$A = [(2,8,4), (3,1,4), (4,0,2)], \quad b^T = (100, 50, 50)$$

Solving each objective separately we get:

$$\begin{aligned} \text{(a) for } \text{Max } cx \quad \text{s.t. } Ax \leq b, \quad x \geq 0. \\ z_0 = 200, \quad z_1 = 35.7144 \end{aligned}$$

- (b) for $Min cx$ s.t. $Ax \leq b, x \geq 0$.
 $z_0 = 0, z_1 = 0$

In (a) we get a maximum revenue of 200 million but this produces 35.71 ton of harmful pollution. In (b) we get the minimum 0 ton of harmful pollution but this would mean 0 million of revenue! As we can see, these two objectives are in conflict with each other. When we maximize revenue, we increase pollution. When we minimize pollution, we decrease revenue. To come up with a compromise solution in respect to vagueness and degree of satisfaction, the company defines the following rules:

Goal 1: Must retain at least 75% of maximum revenue (150 million), but would prefer 100% of maximum revenue (200 million).

Goal 2: Must not exceed 30 ton, of total pollution, but prefer that no pollution is produced.

Goal 3: The range for total revenue and total harmful pollution are to be minimum. This range is called as fuzzy band.

These first two goals can be modeled into fuzzy linear programming with modified s-curve membership function.

The fuzzy linear programming model for the above SPP problem is given as:

$$Max \sum_{j=1}^3 c_j x_j$$

Subject to:

$$\sum_{i=1}^4 a_{ij} x_j \leq b_i^a + \left[\frac{b_i^b - b_i^a}{\alpha} \right] \ln \frac{1}{C} \left[\frac{B}{\mu_{b_i}} - 1 \right] \quad (13)$$

$$\text{where } x_j \geq 0, \quad j = 1, 2, 3., \quad 0 < \mu_{b_i} < 1, \quad 0 < \alpha < \infty.$$

$C = 0.001001001, B = 1$, and $\alpha = 13.81$.

In equation (13), after trade-off between fuzzy resource parameter \tilde{b}_i and non-fuzzy parameters a_{ij} and c_j the best value for the objective function at the fixed level of μ is reached when [15]:

$$\mu = \mu_{a_{ij}} = \mu_{b_i} \quad \text{for } i = 1, 2, 3, 4.; \quad j = 1, 2, 3. \quad (14)$$

Solving equation (13) using linear programming technique, we obtain the following result. Series of iterations are carried out in order to obtain the minimum range for the total revenue and total harmful pollution. The fuzzy band for total revenue defined as $\Delta z^R = z_{\max}^R - z_{\min}^R$ and for the total harmful pollution $\Delta z^P = z_{\max}^P - z_{\min}^P \cdot z_{\max}^R$ is

maximum total revenue, z_{\min}^R is minimum total revenue and Δz^R is range for total revenue. z_{\max}^P is maximum harmful pollution, z_{\min}^P is minimum harmful pollution and Δz^P is range for harmful pollution. The input data for revenue and harmful pollution are given in Table 1 and Table 2 respectively.

Table 1 Input data for revenue

c_j	5	10	12	b_i
a_{ij}	2	8	4	100
	3	1	4	50
	4	0	2	50
	1	2	2	[0,30]

Table 2 Input data for pollution

c_j	1	2	2	b_i
a_{ij}	2	8	4	100
	3	1	4	50
	4	0	2	50
	5	10	12	[33.33,173.33]

We have to stop at iteration 3. This is because, the minimum revenue has already exceeded 30 tons even though the fuzzy band $\Delta z^P = 5.0944$ is smallest, meaning goal 2 is violated. Therefore, iteration 3 gives the good enough outcome for the maximum total revenue and minimum total harmful pollution. The result in Table 3 shows that the total maximum revenue is 194.73 and the total minimum harmful pollution is 24.48 at 99.9% degree of satisfaction with the vagueness $\alpha = 13.81$.

Table 3 Fuzzy band for total revenue

Iteration	Minimum revenue	Maximum revenue	Fuzzy band Δz^R
1	33.33	173.33	140.00
2	104.35	188.17	83.80
3	147.57	194.73	47.16

According to Zimmermann [26] and Carlsson [15], the realistic possible solution exist at 50% degree of satisfaction in a fuzzy environment. The following Tables 5 and 6 provide the fuzzy solutions for optimal revenue and optimal harmful pollution in respect to vagueness α and the the degree of satisfaction.

The following result is obtained for iteration 3.

Table 4 Fuzzy band for total pollution

Iteration	Minimum pollution	Maximum pollution	Fuzzy band Δz^P
1	15.22	33.17	17.96
2	24.47	35.58	10.10
3	30.12	35.21	5.09

Table 5 Optimal pollution and degree of satisfaction
($\alpha = 13.81$)

D.S	0.10	10.08	20.06	30.04	40.02	50.00	59.98	69.96	79.94	89.92	99.90
O.P	34.58	31.67	31.03	30.61	30.26	29.94	29.62	29.27	28.85	28.21	24.47

D.S: Degree of satisfaction (%)

O.P: Optimal pollution (ton)

Table 6 Optimal revenue and degree of satisfaction
($\alpha = 13.81$)

D.S	0.10	10.08	20.06	30.04	40.02	50.00	59.98	69.96	79.94	89.92	99.90
O.R	147.6	163.7	166.4	168.3	169.8	171.1	172.5	174.0	175.9	178.6	194.7

D.S: Degree of satisfaction (%)

O.R: Optimal revenue (ton)

From Table 5, we can see that the total harmful pollution at 50% degree of satisfaction is 29.94 ton, which satisfies goal two. The total revenue at 50% degree of satisfaction is 171.15 million, which satisfies goal one. This result explains that the optimum and satisfied solution has been obtained. It is possible to obtain the total harmful pollution less than 29.94 tons if the vagueness is less than 13.81. This can happen most likely in a less fuzzy situation. The above results are comparable to Kirsch [27].

5.0 CONCLUSION

In general, this research work has achieved its objectives in formulating a new form of membership function and investigating its applications in a limited supply production planning problem. This membership function is a modified form of the general set of s-curve membership functions. The flexibility of this membership function in applying to real world problem has been proved through an analysis.

Based on the analysis of the results of a real world supply production planning problem of maximizing revenue and minimizing harmful pollution, the following conclusions are drawn:

1. Fuzzy Linear Programming (FLP) is a simple and suitable tool for multi-objective optimization problems, compared to other methods.
2. The model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function.
3. The model can be extended to any situation not only supply production planning but any field of engineering with little modifications.
4. Analysis of results indicated that total revenue and total harmful pollution in FLP are increased by 9.164% and decreased by 20.727% respectively as compared to maximum membership values in the fuzzy decision Linear Programming (LP) model [27].

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