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# THE CONJUGACY CLASSES OF METABELIAN GROUPS OF ORDER AT MOST 24

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## Graphical abstract

$\mathrm{cl}(a) = \{ g \in G: \text{there exists } x \in G: g = xax^{-1} \}.$

## Abstract

In this paper, $G$ denotes a non-abelian metabelian group and $\mathrm{cl}(x)$ denotes conjugacy class of the element $x$ in $G$. Conjugacy class is an equivalence relation and it partitions the group into disjoint equivalence classes or sets. Meanwhile, a group is called metabelian if it has an abelian normal subgroup in which the factor group is also abelian. It has been proven by an earlier researcher that there are 25 non-abelian metabelian groups of order less than 24 which are considered in this paper. In this study, the number of conjugacy classes of non-abelian metabelian groups of order less than 24 is computed.

*Keywords*: Conjugacy class, metabelian groups

## Abstrak

Dalam artikel ini, $G$ menandakan kumpulan metabelan tak abelan manakala $\mathrm{cl}(x)$ menandakan kelas konjugat bagi unsur $x$ di dalam $G$. Kelas konjugat merupakan suatu hubungan kesetaraan dan ia memetakkan kumpulan tersebut kepada kelas atau set kesetaraan yang tak berhubung. Sementara itu, satu kumpulan dipanggil metabelan sekiranya ia mempunyai satu subkumpulan normal abelan yang mana kumpulan faktornya juga adalah abelan. Penyelidik terdahulu telah membuktikan bahawa terdapat 25 kumpulan metabelan tak abelan berperingkat kurang daripada 24 yang dipertimbangkan dalam artikel ini. Dalam kajian ini, bilangan kelas konjugat bagi kumpulan metabelan tak abelan peringkat kurang daripada 24 dihitung.

*Kata kunci*: Kelas konjugat, kumpulan metabelan



## 1.0 INTRODUCTION

The concept and term of metabelian groups was used initially to prove theorems pertaining to algebraic number theory, knot theory, and the foundations of geometry[1]. Metabelian groups are groups that are close to being abelian, in the sense that every abelian group is metabelian, but not every metabelian group is abelian[2]. This closeness is reflected in the particular structure of their commutator subgroups. In the Russian mathematical literature, by a metabelian group one sometimes means a nilpotent group of nilpotency class two[2]. Groups that are close to being abelian are termed as metabelian groups. The following is the definition of a metabelian group.

**Definition 1.1**[3] A group $G$ is called a metabelian if it has an abelian normal subgroup $H$ such that the quotient group $G/H$ is also abelian.

In 2010, Abdul Rahman[4] classified all metabelian groups up to order 24 into 25 groups. The groups are stated in the following:

1. $D_3 \cong S_3 \cong \langle a,b : a^3 = b^2 = 1, bab = a^{-1} \rangle$,
2. $D_4 \cong \langle a,b : a^4 = b^2 = 1, bab = a^{-1} \rangle$,
3. $Q_3 \cong \langle a,b : a^4 = 1, b^2 = a^2, aba = b \rangle$,
4. $D_5 \cong \langle a,b : a^5 = b^2 = 1, bab = a^{-1} \rangle$,
5. $Z_3 \rtimes Z_4 \cong \langle a,b : a^4 = b^3 = 1, aba = a \rangle$,
6. $A_4 \cong \langle a,b,c : a^2 = b^2 = c^3 = 1, ba = ab, ca = abc, cb = ac \rangle$,
7. $D_6 \equiv \langle a,b : a^6 = b^2 = 1, bab = a^{-1} \rangle$,
8. $D_7 \cong \langle a,b : a^7 = b^2 = 1, bab = a^{-} \rangle$
9. $D_8 \cong \langle a,b : a^8 = b^2 = 1, bab = a^{-1} \rangle$,
10. $G \cong \langle a,b : a^8 = b^2 = 1, bab = a^3 \rangle$,
11. $Q_8 \cong \langle a,b : a^8 = 1, a^4 = b^2, aba = b \rangle$,
12. $D_4 \times Z_2 \cong \langle a,b,c : a^4 = b^2 = c^2 = 1, ac = ca, bc = cb, bab = a^{-1} \rangle$,
13. $Q_3 \times Z_2 \cong \langle a,b,c : a^4 = b^4 = c^2 = 1, b^2 = a^2, ba = a^3 b,$ $ac = ca, bc = cb \rangle$,
14. Modular-16 $\cong \langle a,b : a^8 = b^2 = 1, ab = ba^5 \rangle$,
15. $B \cong \langle a,b : a^4 = b^4 = 1, ab = ba^3 \rangle$,
16. $K \cong \langle a,b,c : a^4 = b^2 = c^2 = 1, bab = a, ac = ca \rangle$,
17. $G_{4,4} \cong \langle a,b : a^4 = b^4 = abab = 1, ab^3 = ba^3 \rangle$,
18. $D_9 \cong \langle a,b : a^9 = b^2 = 1, bab = a^{-1} \rangle$,
19. $S_3 \times Z_3 \cong \langle a,b,c : a^3 = b^2 = c^3 = 1, b = a^{-1}, ac = ca, bc = cb \rangle$,
20. $(Z_3 \times Z_3) \rtimes Z_2 \cong \langle a,b,c : a^2 = b^3 = c^3 = 1, ba = cb,$ $bab = a, cac = a \rangle$,
21. $D_{10} \cong \langle a,b : a^{10} = b^2 = 1, bab = a^{-1} \rangle$,
22. $Fr_{20} \cong Z_5 \rtimes Z_4 \cong \langle a,b : a^4 = b^5 = 1, ba = ab^2 \rangle$,
23. $Z_5 \rtimes Z_4 \cong \langle a,b : a^4 = b^5 = 1, bab = a \rangle$,
24. $Fr_{21} \cong Z_7 \rtimes Z_3 \cong \langle a,b : a^3 = b^7 = 1, ba = ab^2 \rangle$,
25. $D_{11} \cong \langle a,b : a^{11} = b^2 = 1, bab = a^{-1} \rangle$.

Throughout this research, we will refer to the groups in this classification as groups of type (1)- (25).

A conjugacy class is an equivalence relation; therefore it partitions the group into disjoint equivalence classes or sets. The number of the elements in the classes together with those at the centre as well as the identity must collectively match with the order of the group. The following is the definition of conjugacy class and the class number.

**Definition 1.2**[5] Let G be a finite group. Then the conjugacy class of the element $a$ in $G$ is given as:

$$\mathrm{cl}(a) = \{ g \in G : \text{there exists } x \in G, \; g = xax^{-1} \}.$$

The conjugacy classes of G, denoted as $K(G)$, is the number of distinct (non-equivalent) conjugacy classes. All elements belonging to the same conjugacy class have the same order.

Moreover, those conjugacy class in the centre of a group is called The central conjugacy class i.e.

$$\mathrm{cl}(x) = \{x \in G : gx = xg, \text{for all } g \in G\} \subseteq Z(G).$$

Otherwise, the conjugacy class is called non-central.

The propositions below explain the equivalence conjugacy classes.

**Proposition 1.1**[5] Let $G$ be finite group and let $a$ and $b$ be elements of $G$. The elements $a$ and $b$ are conjugate if they belong to precisely one conjugacy class, that is $\mathrm{cl}(a)$ and $\mathrm{cl}(b)$ are equal.

This paper is divided into three sections. The first section includes the classification of all metabelian groups of order 24, and some background topics in group, while the second section provides some earlier and recent research that are related to the metabelian groups and conjugacy class. In the third section, we present our main results which include the list of conjugacy classes of metabelian groups of order less than 24.

## 2.0 PRELIMINARIES

In this section, we provide some previous works related to the metabelian groups and conjugacy classes.

In 2010, Abdul Rahman[3] gave the classifications of metabelian groups of order less than 24. All groups of order at most 24 are proved as metabelian groups using their group presentations and some of them are helped by Groups, Algorithms and Programming (GAP) software[3].

However, the elements of any group will be partitioned into conjugacy classes; members of the same conjugacy class share many properties. The study of conjugacy classes of non-abelian groups reveals many important features of their structure. The conjugacy classes have been used in the concept of the commutativity degree[6] and it generalizations[7-9]. In addition, the conjugacy classes has widely been used in graph theory in which different kinds of graphs are introduced including conjugacy class graph[10], generalized conjugacy class graph[11] and others.

## 3.0 MAIN RESULTS

In this section, the conjugacy classes and the of conjugacy classes of all non-abelian metabelian groups of order less than 24 are determined, starting with the first type, namely type (1).

**Lemma 3.1** Let G be a metabelian group of type (1), $D_3 \cong S_3 \cong \langle a,\ b\colon a^3 = b^2 = 1, bab = a^{-1}\rangle$. Then $K(D_3) = 3$.

**Proof** Using Definition 1.2, the conjugacy classes of elements of $D_3$ are determined as follows: Let $x = \mathrm{e}$. Then, $\mathrm{cl}(\mathrm{e}) = \{\mathrm{e}\}$. Next, let $x = a$, $\mathrm{cl}(a) = gag^{-1}$, where $g \in D_3$.

If $g = e, g^{-1} = e$ thus, $(e)a(e) = a$.
If $g = a, g^{-1} = a^2$ thus, $(a)a(a^2) = (a^2)(a^2) = a$.
If $g = a^2, g^{-1} = a$ thus, $(a^2)a(a) = (a^2)(a^2) = a$.
If $\mathrm{g} = b, \mathrm{g}^{-1} = \mathrm{b}$ thus, $(b)a(b) = (b)(ab) = a^2$.
If $g = ab, g^{-1} = ab$ thus, $(ab)a(ab) = (ab)(a^2b) = a^2$.

Hence, $\mathrm{cl}(a) = \{a, a^2\}$. By Proposition 1.1, $\mathrm{cl}(a) = \mathrm{cl}(a^2)$. Now, let $x = b$, then $\mathrm{cl}(b) = gbg^{-1}$, where $g \in D_3$. The conjugate elements with $b$ are determined as below.

If $g = e, g^{-1} = e$ thus, $(\mathrm{e})\mathrm{b}(\mathrm{e}) = \mathrm{b}$.
If $g = a, g^{-1} = a^2$ thus, $(a)b(a^2) = (ab)(a^2) = a^2b$.
If $g = a^2, g^{-1} = a$ thus, $(a^2)b(a) = (a^2)(a^2b) = ab$.
If $g = b, g^{-1} = b$ thus, $(b)b(b) = (e)(b) = b$.
If $g = ab, g^{-1} = ab$ thus, $(ab)b(ab) = (a)(ab) = a^2b$.
If $g = a^2b, g^{-1} = a^2b$ thus, $(a^2b)b(a^2b) = (a^2)(a^2b) = ab$.

Hence, $\mathrm{cl}(b) = \{b, ab, a^2b\}$. By Proposition 1.1, $\mathrm{cl}(b) = \mathrm{cl}(ab) = \mathrm{cl}(a^2b)$. It follows that, the number of conjugacy classes in $D_3$ is equal to three listed as follows: $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(a) = \{a, a^2\} = \mathrm{cl}(a^2)$, $\mathrm{cl}(b) = \{b, ab, a^2b\} = \mathrm{cl}(ab) = \mathrm{cl}(a^2b)$. That is, $K(D_3) = 3$. ■

**Lemma 3.2** Since $S_3 \cong D_3$, the number of conjugacy classes in $S_3$ is the same as the number conjugacy classes in Theorem 3.1.

Using the same computation as in Lemma 3.1, we list the conjugacy classes for the rest of the groups in the following theorems.

**Lemma 3.3** Let G be a metabelian group of type (2), $D_4 \cong \langle a, b : a^4 = b^2 = 1, bab = a^{-1}\rangle$, Then, $K(D_4) = 5$.

**Proof** Using Definition 1.2, the conjugacy classes of $D_4$ are described as follows:

$\mathrm{cl}(e) = \{e\}, \mathrm{cl}(a) = \{a, a^3\} = \mathrm{cl}(a^3), \mathrm{cl}(b) = \{b, a^2b\} = \mathrm{cl}(a^2b)$, $\mathrm{cl}(a^2) = \{a^2\}$, and $\mathrm{cl}(a^3b) = \{ab, a^3b\} = \mathrm{cl}(ab)$. It follows that, $K(D_4) = 5$. ■

**Lemma 3.4** The number of conjugacy class of metabelian group of type (3), $Q = < a, b : a^4 = 1, a^2 = b^2, aba = b >$ is the same as the number of conjugacy class in Lemma 3.3.

**Lemma 3.5** Let G be a metabelian group of type (4), $D_5 \cong \langle a, b : a^5 = b^2 = 1, bab = a^{-1}\rangle$. Then, $K(D_5) = 4$.

**Proof** Using Definition 1.2, the conjugate elements in $D_5$ are listed as follows: $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(a) = \{a, a^4\} = cl(a^4)$, $\mathrm{cl}(b) = \{b, ab, a^2b, a^3b, a^4b\} = \mathrm{cl}(ab) = \mathrm{cl}(a^2b) = \mathrm{cl}(a^3b) = \mathrm{cl}(a^4b)$, and $\mathrm{cl}(a^2) = \{a^2, a^3\} = \mathrm{cl}(a^3)$. Therefore, $K(D_5) = 4$. ■

**Lemmma 3.6** The number of conjugacy classes of metabelian group of type (6), $A_4 = < a, b : a^2 = b^2 = c^3 = 1, ba = ab, ca = abc, cb = ac >$ is the same as the number of conjugacy classes in Lemma 3.5.

**Lemma 3.7** Let G be a metabelian group of type (5), $\mathbb{Z}_3 \rtimes \mathbb{Z}_4 \cong \langle a, b : a^4 = b^3 = 1, aba = a\rangle$. Then $K(\mathbb{Z}_3 \rtimes \mathbb{Z}_4) = 6$.

**Proof** Using Definition 1.2, the conjugate elements in $\mathbb{Z}_3 \rtimes \mathbb{Z}_4$ are given as follows: $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(b) = \{b, a^3ba\} = \mathrm{cl}(a^3ba)$, $\mathrm{cl}(a) = \{a, ab, ba\} = \mathrm{cl}(ab) = \mathrm{cl}(ba)$, $\mathrm{cl}(a^2) = \{a^2\}$, $\mathrm{cl}(a^2b) = \{a^2b, aba\} = \mathrm{cl}(aba)$, and $\mathrm{cl}(a^2ba) = \{a^2ba, a^3, a^3b\} = \mathrm{cl}(a^3) = \mathrm{cl}(a^3b)$. Therefore, $K(\mathbb{Z}_3 \rtimes \mathbb{Z}_4) = 6$. ■

**Lemma 3.8** The metabelian group of type (7), $D_6 = < a, b : a^6 = b^2 = 1, bab = a^{-1} >$ and also the metabelian group of type (24), $Fr_{21} \cong \mathbb{Z}_7 \rtimes \mathbb{Z}_3 =< a, b : a^3 = b^7 = 1, ba = a\, b^2 >$ have the same number of non-central conjugacy classes as in Lemma 3.7.

**Lemma 3.9** Let G be a metabelian group of type (8), $D_7 =< a, b : a^7 = b^2 = 1, bab = a^{-1} >$. Then, $K(D_7) = 5$.

**Proof** Based on Definition 1.2, the conjugacy classes of $D_7$ are described as follows: $cl(e) = \{e\}$, $cl(a) = \{a, a^6\} = cl(a^6)$, $cl(b) = \{b, ab, a^2b, a^3b, a^4b, a^5b, a^6b\} = cl(ab) = cl(a^2b) = cl(a^3b) = cl(a^4b) = cl(a^5b) = cl(a^6b)$, $cl(a^2) = \{a^2, a^5\} = cl(a^5)$, and $cl(a^3) = \{a^3, a^4\} = cl(a^4)$. From which follows that, $K(D_7) = 5$. ■

**Lemma 3.10** The number of conjugacy classes of metabelian group of type (22) is the same as that of the conjugacy classes of Lemma 3.9.

**Lemma 3.11** Let G be a metabelian group of type (9), $D_8 \cong \langle a, b : a^8 = b^2 = 1, bab = a^{-1}\rangle$. Then, $K(D_8) = 7$.

**Proof** Using Definition 1.2, the conjugacy classes of $D_8$ are determined as follows: $cl(e) = \{e\}$, $cl(a) = \{a, a^7\} = cl(a^7)$, $cl(b) = \{b, a^2b, a^4b, a^6b\} = cl(a^2b) = cl(a^4b) = cl(a^6b)$, $cl(ab) = \{ab, a^3b, a^5b, a^7b\} = cl(a^3b) = cl(a^5b) = cl(a^7b)$, $cl(a^2) = \{a^2, a^6\} = cl(a^6)$, $cl(a^3) = \{a^3, a^5\} =$

$cl(a^5)$, and $cl(a^4) = \{a^4\}$. From which follows that, $K(D_8) = 7$. ∎

**Lemma 3.12** The number of non-central conjugacy classes of metabelian group of type (10) and type 12) are the same the number of non-central conjugacy classes in Lemma 3.11.

**Lemma 3.13** Let *G* be a metabelian group of type (13), $Q_3 \times Z_2 \cong \langle a,b,c : a^4 = b^4 = c^2 = 1, b^2 = a^2, ba = a^3b, ac = ca, bc = cb\rangle$. Then, $K(Q_3 \times Z_2) = 10$.

**Proof** According to Definition 1.2, the conjugacy classes of $Q \times \mathbb{Z}_2$ are determined as follows: $cl(e) = \{e\}$, $cl(a) = \{a, a^3\} = cl(a^3)$, $cl(b) = \{b,\ a^2b\} = cl(a^2b)$, $cl(c) = \{c\}$, $cl(ab) = \{ab,\ a^3b\} = cl(a^3b)$, $cl(ac) = \{ac, a^3c\} = cl(a^3c)$, $cl(bc) = \{bc, a^2bc\} = cl(a^2bc)$, $cl(abc) = \{abc, a^3bc\} = cl(a^3bc)$, $cl(a^2) = \{a^2\}$, and $cl(a^2c) = \{a^2c\}$. It follows that $K(Q_3 \times Z_2) = 10$. ∎

**Lemma 3.14** The number of non-central conjugacy classes of metabelian groups of type (15), (16) and (17), namely $B$, $K$, and $G_{4,4}$ all of order 16 are the same with the number of non-central conjugacy classes of the metabelian group of Lemma 3.13.

**Lemma 3.15** Let *G* be a metabelian group of type (18), $D_9 \cong \langle a,b : a^9 = b^2 = 1, bab = a^{-1}\rangle$. Then, $K(D_9) = 6$.

**Proof** Based on Definition 1.2, the conjugacy classes of $D_9$ are determined as follows: $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(a) = \{a, a^8\} = \mathrm{cl}(a^8)$, $\mathrm{cl}(b) = \{b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b\} = cl(a^2b) = \mathrm{cl}(a^3b) = \mathrm{cl}(a^4b) = \mathrm{cl}(a^5b) = \mathrm{cl}(a^6b) = \mathrm{cl}(a^7b) = \mathrm{cl}(a^8b)$, $\mathrm{cl}(a^2) = \{a^2, a^7\} = \mathrm{cl}(a^7)$, $\mathrm{cl}(a^3) = \{a^3, a^6\} = \mathrm{cl}(a^6)$, and $\mathrm{cl}(a^4) = \{a^4, a^5\} = \mathrm{cl}(a^5)$. Therefore, $K(D_9) = 6$. ∎

**Lemma 3.16** The number of non-central conjugacy classes of the metabelian groups of type (14), (20) and (23) Modular−16, $(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_2$ and $\mathbb{Z}_4 \rtimes \mathbb{Z}_5$ of order 16, 18 and 20, respectively are the same as the number of conjugacy classes of the metabelian group in Lemma 3.15.

**Lemma 3.17** Let *G* be a metabelian group of type (19), $S_3 \times Z_3 \cong \langle a,b,c : a^3 = b^2 = c^3 = 1, b = a^{-1},\ ac = ca, bc = cb\rangle$. Then, $K(S_3 \times Z_3) = 9$.

**Proof** Using Definition 1.2, the conjugacy classes of $S_3 \times \mathbb{Z}_3$ are determined as follows. $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(a) = \{a, a^2\} = \mathrm{cl}(a^2)$, $\mathrm{cl}(b) = \{b,\ ab, a^2b\} = \mathrm{cl}(ab) = \mathrm{cl}(a^2b)$, $\mathrm{cl}(c) = \{c\}$, $\mathrm{cl}(ac) = \{ac, a^2c\} = \mathrm{cl}(a^2c)$, $\mathrm{cl}(bc) = \{bc, abc, a^2bc\} = \mathrm{cl}(abc) = \mathrm{cl}(a^2bc)$, $\mathrm{cl}(c^2) = \{c^2\}$, $\mathrm{cl}(a^2c) = \{a^2c, a^2c^2\} = \mathrm{cl}(a^2c^2)$, and $\mathrm{cl}(bc^2) = \{bc^2, abc^2, a^2bc^2\} = \mathrm{cl}(a^2bc^2) = \mathrm{cl}(abc^2)$. Hence, $K(S_3 \times \mathbb{Z}_3) = 9$ . ∎

**Lemma 3.18** Let *G* be a metabelian group of type (21), $D_{10} \cong \langle a,b : a^{10} = b^2 = 1, bab = a^{-1}\rangle$. Then, $K(D_{10}) = 8$.

**Proof** Using Definition 1.2, the conjugacy classes of $D_{10}$ are given as follows: $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(a) = \{a, a^9\} = \mathrm{cl}(a^9)$, $\mathrm{cl}(b) = \{b,\ a^2b, a^4b, a^6b, a^8b,\} = \mathrm{cl}(a^2b) = \mathrm{cl}(a^4b) = \mathrm{cl}(a^6b) = cl(a^8b)$, $\mathrm{cl}(ab) = \{ab, a^3b, a^5b, a^7b, a^9b\} = \mathrm{cl}(a^3b) = \mathrm{cl}(a^5b) = \mathrm{cl}(a^7b) = \mathrm{cl}(a^9b)$, $cl(a^2) = \{a^2, a^8\} = cl(a^8)$, $\mathrm{cl}(a^3) = \{a^3, a^7\} = \mathrm{cl}(a^7)$, $\mathrm{cl}(a^4) = \{a^4, a^6\} = \mathrm{cl}(a^6)$, and $\mathrm{cl}(a^5) = \{a^5\}$. Hence, $K(D_{10}) = 8$. ∎

**Lemma 3.19** Let *G* be a metabelian group of type (25), $D_{11} \cong \langle a,b : a^{11} = b^2 = 1, bab = a^{-1}\rangle$. Then, $K(D_{11}) = 7$.

**Proof** Using the definition of conjugacy classes, the conjugacy classes of $D_{11}$ are determined as follows: $\mathrm{cl}(e) = \{e\}$, $\mathrm{cl}(a) = \{a, a^{10}\} = cl(a^{10})$, $\mathrm{cl}(b) = \{b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b, a^8b, a^9b, a^{10}b\} = \mathrm{cl}(ab) = \mathrm{cl}(a^2b) = \mathrm{cl}(a^3b) = \mathrm{cl}(a^4b) = \mathrm{cl}(a^5b) = \mathrm{cl}(a^6b) = \mathrm{cl}(a^7b) = \mathrm{cl}(a^8b) = \mathrm{cl}(a^9b) = \mathrm{cl}(a^{10}b)$, $\mathrm{cl}(a^2) = \{a^2, a^9\} = \mathrm{cl}(a^9)$, $\mathrm{cl}(a^3) = \{a^3, a^8\} = \mathrm{cl}(a^8)$, $\mathrm{cl}(a^4) = \{a^4, a^7\} = \mathrm{cl}(a^7)$, and $\mathrm{cl}(a^5) = \{a^5, a^6\} = \mathrm{cl}(a^6)$. Thus, $K(D_{11}) = 7$. ∎

## 4.0 CONCLUSION

In this paper, the conjugacy classes of metabelian groups of order less than 24 are computed. It is proven that the number of conjugacy classes of $D_3$ and $S_3$ is three with two non-central classes. Similarly, the number of conjugacy classes of $\mathrm{D}_5$ is 4 which is equal to the number of conjugacy classes of metabelian group of type (6). In addition, the metabelian groups $\mathrm{D}_6$, $\mathbb{Z}_3 \rtimes \mathbb{Z}_4$ and $\mathrm{Fr}_{21}$ have the same number of non-central conjugacy classes, namely 4. Meanwhile, the metabelian groups $\mathrm{D}_7$ and $\mathrm{Fr}_{20}$ both have 5 conjugacy classes as well as non-central conjugacy classes. Moreover, the metabelian groups of types (9), (10) and 12) have the same number of non-central conjugacy classes of order greater than one, namely 5, and the metabelian groups types (13), (15), (16) and (17) also have the same number of conjugacy classes which is 10 as well as non-central conjugacy classes of 6 of the same sizes. However, the metabelian groups types (14), (20), (23) have the same number of non-central conjugacy classes of 5. Whereas, the metabelian groups of types (19), (21) and (25) each has a unique number of conjugacy classes of 9, 8 and 7 respectively.

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