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HYDROLOGICAL ANALYSIS OF A DRAINED PEAT BASIN USING TIME SERIES CORRELATION AND CROSS-CORRELATION FUNCTIONS

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Abstract. A simple algebraic correlation and cross-spectral approach was applied to study the stream flow hydrograph of peat basin. The analysis involved the transformations of input variables (rainfall) and output variables (stream flow and water table). Analogous to the input-output signal in electronics, the peat aquifer is considered as a filtering system. The filter is able to transform, retain or eliminate the input variables (input signal) before the output variables (output signal) are created. The behaviour of flow through basin aquifer system is deduced from the degree of the transformation of the input signal. Several important aquifer parameters were deduced from the transformation process. They are the response time, the distinction between flows (quick-flows, intermediate flow or base-flow) and the delay. These hydrological parameters are required as design parameters for a water resource project within a basin-underlain peat soil aquifer.

Keywords: Peat hydrology, cross-correlation function, spectral correlation function, complex basin

Abstrak. Satu pendekatan algebra mudah korelasi dan korelasi silang telah digunakan untuk mengkaji hidrograf aliran sungai di lembangan tanah gambut. Analisis melibatkan pemindahan pemboleh ubah input (hujan) and pemboleh ubah output (aliran sungai atau paras air tanah). Analogi kepada sistem signal input-output dalam bidang elektronik, akuifer tanah gambut dianggap sebagai suatu sistem penapis. Sistem penapis tersebut mampu untuk memindah, menahan atau menghapus pemboleh ubah input (signal input) sebelum pemboleh ubah output dihasilkan. Sifat aliran yang melalui sistem akuifer boleh diterjemahkan daripada darjah pindahan signal input. Beberapa parameter akuifer penting telah dikenal pasti, iaitu tempoh masa gerak balas, perbezaan antara jenis aliran (aliran cepat, sederhana atau aliran dasar) dan masa tunda. Kesemua parameter hidrologi tersebut diperlukan sebagai parameter reka bentuk bagi suatu projek sumber air dalam kawasan yang dilitupi oleh akuifer bertanah gambut.

Kata kunci: Hidrologi gambut, fungsi korelasi silang, fungsi korelasi spektra, lembangan kompleks

1.0 INTRODUCTION

Having more than 2.8 million hectares of peatlands (Mutalib *et al*., 1992), Malaysia is one of the premier peat countries in the world. Being a wetland and hydrophobic in nature, peat land is perishable and easy to destroy (Page and Rieley, 1998). Upon drainage, these soils tend to decompose, consolidate and subside (Wosten *et al*.,

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1997). Peat basin is believed to have a complex aquifer system. The property of the aquifer materials is difficult to characterise and very uncertain. This type of land is naturally hydrophobic. The water table is shallow and their groundwater flow can be multi-dimensional. The hydraulic properties of the peat material are heterogeneous and non-isotropy (Chason and Siegel,1986; Hemond and Fifield, 1982). Their hydraulics properties vary in both spatial and temporal domain. The peat aquifer system can be unconfined, multi-layer, dual porosity and discontinuous, depending on the degree of humification of the peat material (Hill and Siegel, 1991). More seriously, the water flow within peat soil can be deviated from the most popular principle of water flow in porous media, that is, Darcy's law (Hemond and Goldman, 1985). Due to this complex aquifer system, research effort, particularly for the tropical peatland are minimal. Consequently, many water resources projects situated on or within the vicinity of a peat basin is justified based on ad-hoc, trial and error basis (Ramli, 1999).

Understanding the hydraulic properties of peat material contained in a peat aquifer is of primary importance in water resources development. However, the complexity of the system does not allow research workers to quantify every single property of the materials, before the potential aquifer yield can be estimated. To simplify the job, researchers usually make measurements at point scale. Unfortunately in many cases, point scale measurement does not represent the spatial and temporal aspect of the watershed behaviour. Thus, an indirect method to investigate the general behavior of a basin underlaid by peat aquifer should be worked out. Hydrologists believed that the information contained in a stream flow hydrograph and water table regimes of a basin could provide some indication on the hydro-geological behaviour of the aquifer system underneath (Shedlock *et al*., 1993). Stream flow dynamic of a wetland system such as for peat basin for instance, can provide an indicator to the aquifer performance of the basin. Stream flow hydrographs are easy and inexpensive to collect and easily available from the relevant agencies such the Department of Irrigation and Drainage of Malaysia. The purpose of this research is to study the behaviour of peat aquifer by means of hydrograph properties using an indirect statistical mathematics method. This paper demonstrates that correlation and spectral analyses can be valuable tools in understanding the hydrologic characteristics of a complex peat basin.

2.0 MATERIALS AND METHODS

2.1 General Approach

The knowledge in mathematical multivariate time series and hydrological processes was applied in this study. Two most important time series analyses at frequency domain and their associated functions were employed. They are Cross-Correlation Function (CCF) and Cross-Spectral Correlation Function (SDF). The knowledge in rainfall-runoff hydrologic transfer function was integrated. The finding from these analyses was interpreted according to the physical meaning of the study aquifer system.

Brief definition

A brief overview of the theory of mathematical functions used in this paper is presented. The function includes Auto-Correlation Function (ACF), Spectral Density Function (SDF), Cross-Correlation Function (CCF) and Cross-Amplitude Function (CAF).

Time series variables

Let x_t (x_1, x_2, x_3, . . ., x_n) be a time series variable of a hydrologic system. Let y_t (y_1, y_2, y_3, . . .,y_n) be another time series variable of the same hydrologic system. Let x_t (x_1, x_2, x_3, . . ., x_n) be an input time series data and y_t (y_1, y_2,y_3, . . .,y_n) be an output time series data. Let x_t causes y_t. Let x_t represents rainfall data and y_t represents River flow or Water table data.

2.2 Time Domain Analysis

Auto-Correlation Function (ρ_{xx}, ρ_{yy})

In a univariate time series modelling approach, the ACF quantifies the linear dependency of successive values over a period. It is a time domain function. It is used to demonstrate the correlation between time lags. Mathematically, ACF for a stationary process can be written as (Jenkins and Watts, 1968; Larocque *et al.*, 1998; Padilla and Pulido-Bosch, 1995):

$$\rho_{xx(k)} = \frac{\gamma_{xx(k)}}{\gamma_{xx(0)}} \tag{1}$$

$$\gamma_{xx(k)} = \frac{1}{n}\sum_{t=1}^{n-k}(x_t - \bar{x})(x_{t+k} - \bar{x}) \tag{2}$$

Where $\rho_{xx(k)}$ is the auto-correlation coefficient at lag k, $\gamma_{xx(k)}$ and $\gamma_{xx(0)}$ is the auto-covariance and lag k and 0 respectively.

Spectral Density Function ($S_{xx}(f)$) and Regulation time (T_{reg})

Spectral Density Function of a time series variable is a Fourier transformation of its ACF. Thus, it is a frequency domain function. In time series modelling approach, it is used to quantify periodical characteristics of time series variables. Mathematically, $S(f)$ can be represented as:

$$S_{xx}(f) = 2\left[1 + 2\sum_{k=1}^{m} D(k)\,\rho_{xx}(k)\cos(2\pi fk)\right] \tag{3}$$

where,

$$D(k) = \frac{\left(1 + \cos\pi \frac{k}{m}\right)}{2} \tag{4}$$

Where $f = j/2\text{m}$, $j = 1$ to m and f is the frequency.

From spectral density function analyses, the regulation time, T_{reg}, can be computed as (Laroque *et al.*, 1998):

$$T_{reg} = \frac{S_{xx}(f = 0)}{2} \tag{5}$$

Analogous to electronics, regulation time is similar to the passing band in a signal device. It defines the duration of the influence of the input signal and it gives an indication of the length of the impulse response of the system.

Cross-Correlation Function (ρ_{xy})

In general terms the CCF, analysis is used to study the interactions between two or more time series variables with possibly having different variances (Jenkins and Watt 1968). In this study, a CCF analysis is required to investigate the interaction between input variables (Rainfall) and output variables (Streamflow) or water tables.

Consider a single input-output discrete time series modeling hydrologic system. Let x_t be an input series, y_t be an output series data, and x_t causes y_t. Let x_t represents rainfall data and y_t represents River flow or Water table data. The cross-correlation function obtained from these two series at a time lag k is defined as:

$$\rho_{xy}(k) = \frac{C_{xy}(k)}{\sigma_x \sigma_y} \tag{6}$$

$$\rho_{yx}(k) = \frac{C_{yx}(k)}{\sigma_x \sigma_y} \tag{7}$$

Where

$$C_{xy}(k) = \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{x})(y_{t+k} - \bar{y}) \tag{8}$$

$$C_{yx}(k) = \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{y})(y_{t+k} - \bar{x}) \tag{9}$$

Where $C_{xy}(k)$ and $C_{yx}(k)$ are the cross-correlogram, $\bar{x}$ and are $\bar{y}$ the means of the series x_t and y_t respectively and σ_x and σ_y are the standard deviations of the time series x_t and y_t respectively.

Unlike in autocorrelation function (ACF) of a univariate time series analysis, $\rho_{xy}(k)$ is asymmetric at time lags (truncation span), that is $\rho_{xy}(k) \neq \rho_{yx}(k)$. From the standpoint of the rainfall-runoff transfer function model, $\rho_{xy}(k)$ represents the impulse response of the aquifer system if the rainfall series is uncorrelated (white noise) (Mangin and Pulido-Bosch, 1983).

Cross Spectral Density Function ($S_{xy}(f)$)

The basic idea for having $S_{xy}(f)$ of a multivariate time series is to obtain the information about the dependencies between the two variables. $S_{xy}(f)$ is the Fourier transform of $\rho_{xy}(k)$ function and can be expressed using a complex number as (Padilla and Pulido-Bosch, 1995):

$$S_{xy}(f) = |\alpha_{xy}(f)| \exp\left[-i\phi_{xy}(f)\right] \tag{10}$$

Where i represents $\sqrt{-1}$, $\alpha_{xy}(f)$ and $\phi_{xy}(f)$ are values have cross-amplitude and phase functions for frequency f. The function $\alpha_{xy}(f)$ can be associated with the duration of impulse response function and $\phi_{xy}(f)$ shows the delay for different frequencies. The more detailed definition of these functions can be referred from articles by Jenkins and Watts (1968) and Padilla and Pulido-Bosch (1995).

Coherence and Gain Function

Having and of the time series, coherence function,and gain function for the series can be established and given as follows:

$$K_{xy}(f) = \frac{\alpha_{xy}(f)}{\sqrt{S_{xx}(f)S_{yy}(f)}} \tag{11}$$

$$G_{xy}(f) = \frac{\alpha_{xy}(f)}{\sqrt{S_{xx}(f)}} \tag{12}$$

The coherence function relates the periodic variation of y_t with that of x_t and gain function express the attenuation of x_t attributable to the impulse system.

2.3 Time series data and the study basin

The long-term time series rainfall, stream flow and water tables data taken from an experimental peat catchment located in Benut, Johor was used as a basis of this study. The site was located within the Phase I of the Johor Barat Reclamation project. The site represented a small, drained peat basin in Malaysia. The area was typically dome-shaped and completely covered with peat material of different peat depths spatially.

As shown in Figure 1, the general degree of peat humification according to USDA soil classification standard varies from sapric in the top, hemic in the middle and fibric at the bottom of the soil (Ayob and Ahmad Khairi, 2003). The thickness of the sapric, hemic and fibric layers varies from point to another point within the study catchment, making basin classification more complex (Ayob and Ahmad Khairi, 1999). Analogous to the input-output signal processing in electronics, the peat aquifer (profile) is considered as a filtering system.

The filter is able to transform, retain or eliminate the input variables (input signal) before the output variables (output signal) are created. The behaviour of flow through basin aquifer system is deduced from the degree of the transformation of the input signal. The observed time series data consisted of daily rainfall, mean daily stream flow and water table depths. Data is available from 1981 to 1995 with some missing values.

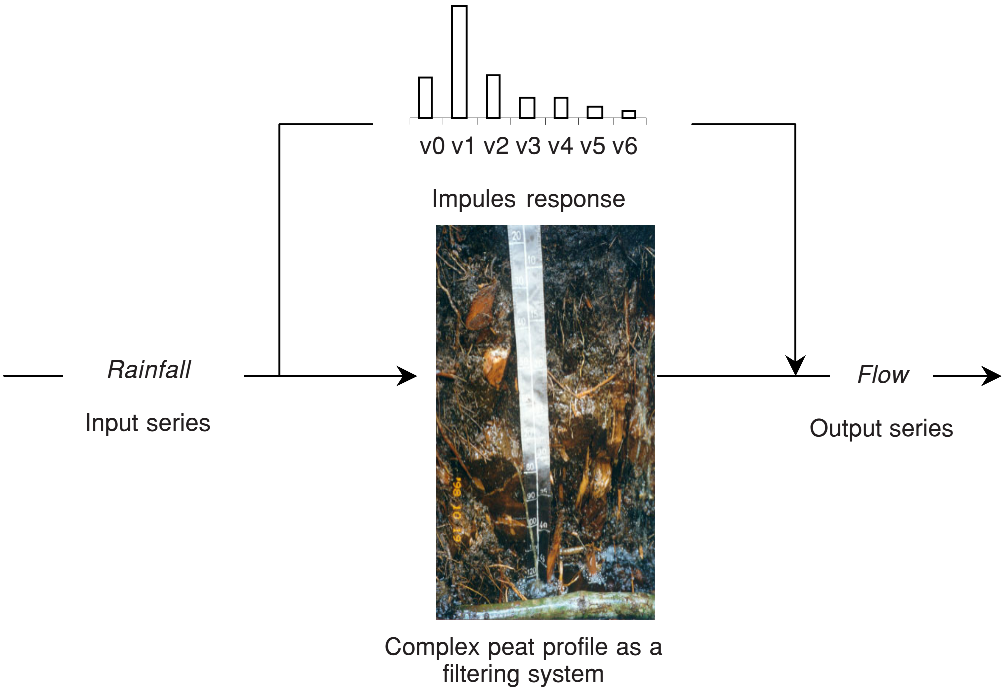

Figure 1 The typical peat profile of the study basin showing a complex hydrologic system

Only data for 1983-1984 hydrological years (Figures 2, 3 and 4) was used as the basis in this paper. The data series for these years were excellent in the sense that no missing data was observed

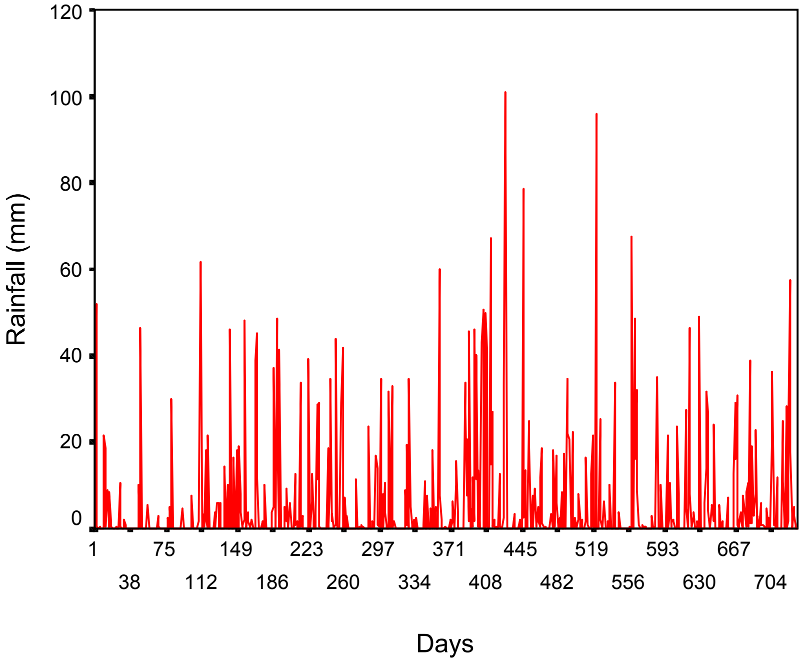

Figure 2 The rainfall series

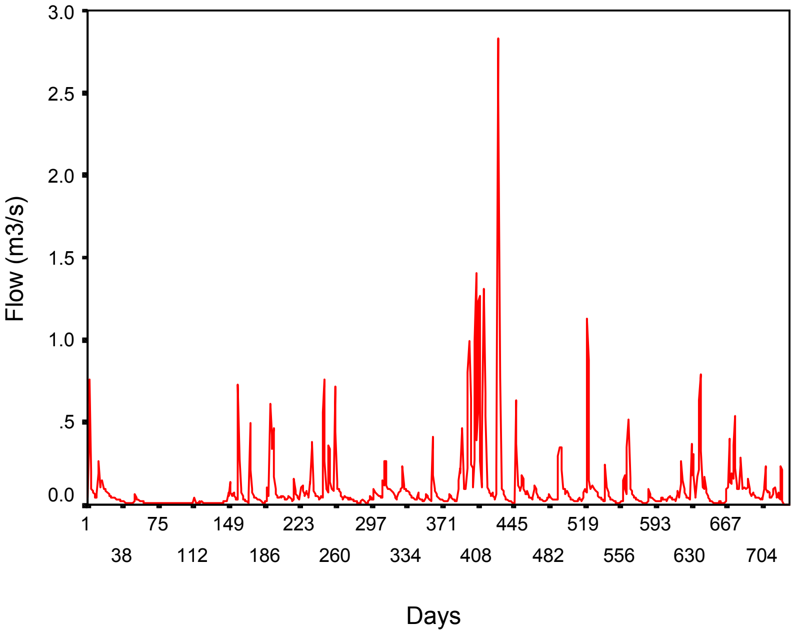

Figure 3 The flow series

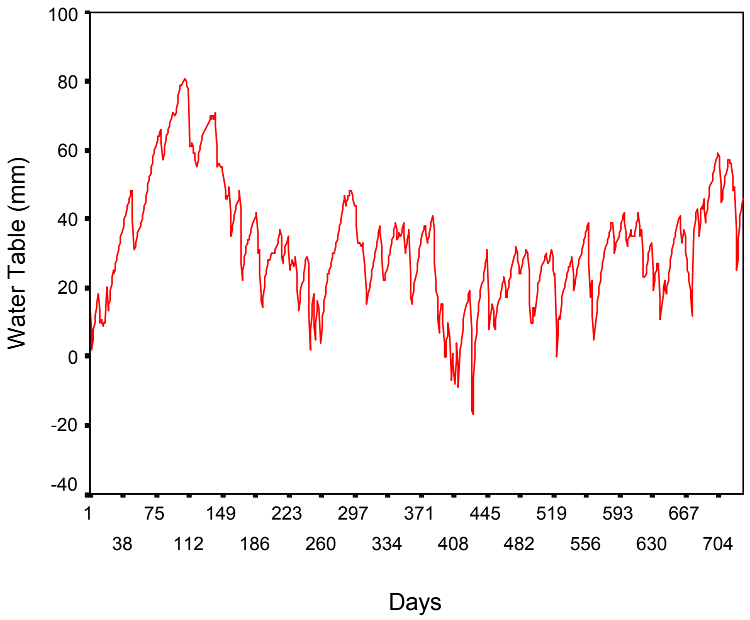

Figure 4 The water table series

3.0 RESULTS AND DISCUSSION

The summary of statistical outputs from various functional analyses is given in Table 1 and 2. Table 1 is on univariate analysis while Table 2 illustrates the functional analysis using multivariate approach. Based on the values of the functional coefficient and the univariate analysis, it was found that the regulation time for flow and water table series was about 12 and 38 days respectively, indicating that the water table series have longer memory effect. Figures 5 to 9 present examples of the functional plots used in this study. The results from the cross-correlation and spectral analysis between rainfall as input and stream flow as output have provided a better understanding on the hydrological behaviour of peat basin. Quick flow was occurring during the first 3 days after rainfall and peat basin can be categorised as a quick-emptying aquifer system.

Table 1 Summary of results of univariate analysis

Type of data	**Functional analysis**	**Functional coefficient behavior**	**Physical meaning**
Rainfall	$\rho_{xx}(k)$	Diminish quickly	No clear serial correlation
Flow	$\rho_{yy}(k)$	Diminish moderately	Moderate serial correlation. $T_{reg} = 12$ days, impulse response is short
Water table	$\rho_{yy}(k)$	Diminish slowly	Strong serial correlation. $T_{reg} = 38$ days, impulse response is longer

Table 2 Summary of results of multivariate analysis between rainfall and stream flow

Type of data	**Functional analysis**	**Functional coefficient behavior**	**Physical meaning**
Rainfall with Flow	$\rho_{xy}(k)$	Peak coefficient at 0.6 after 3 days	Short time delay with high coefficient, i.e. input signal from rainfall is significantly increased during its passage, a quick emptying aquifer system
	$\alpha_{xy}(f)$	Null value at $f > 0.3$ (periods of less than 3 days)	Quick flow occurring during less than 3 days after rainfall
	$G_{xy}(f)$	No clear result	–

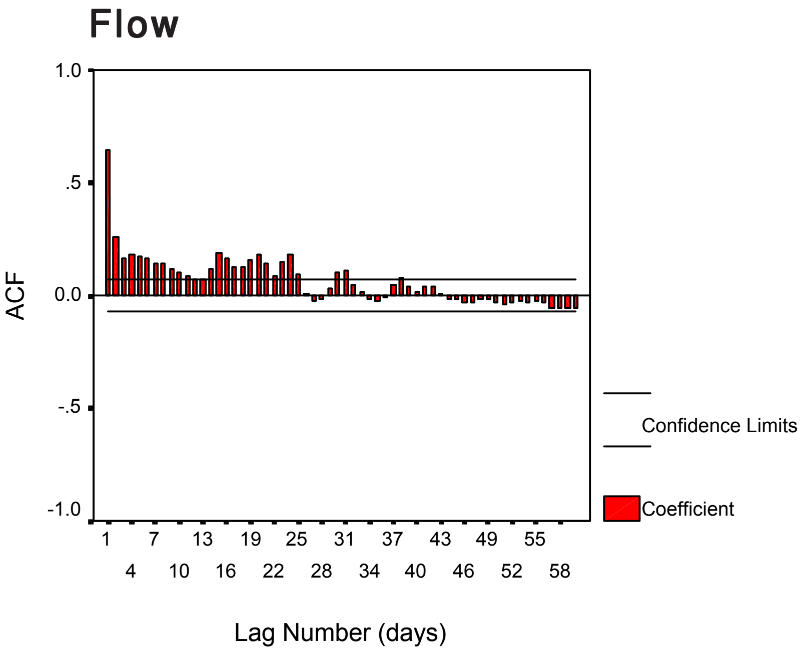

Figure 5 Auto-correlation function of daily flow series

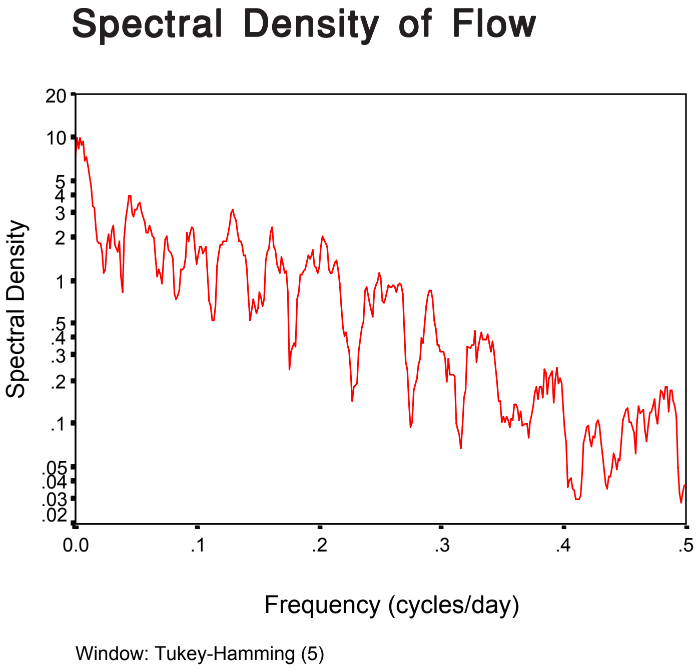

Figure 6 Spectral density function of daily flow data

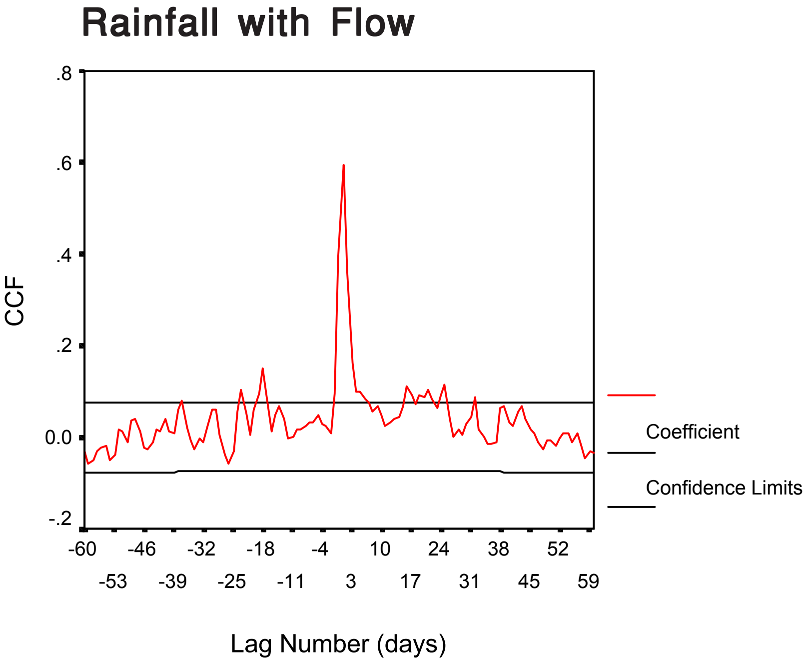

Figure 7 Cross-correlation function of daily rainfall and daily flow

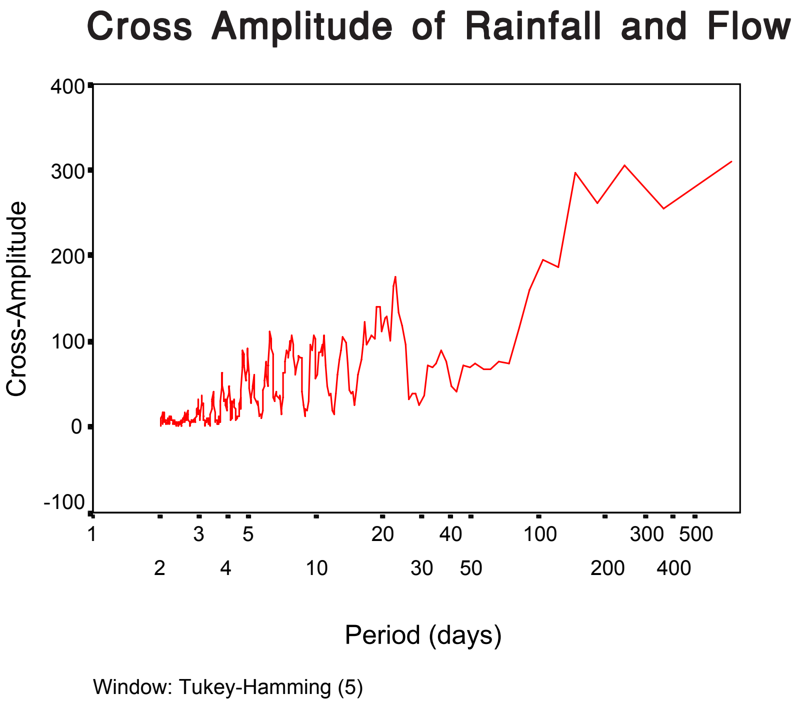

Figure 8 Cross-amplitude function of daily rainfall and flow

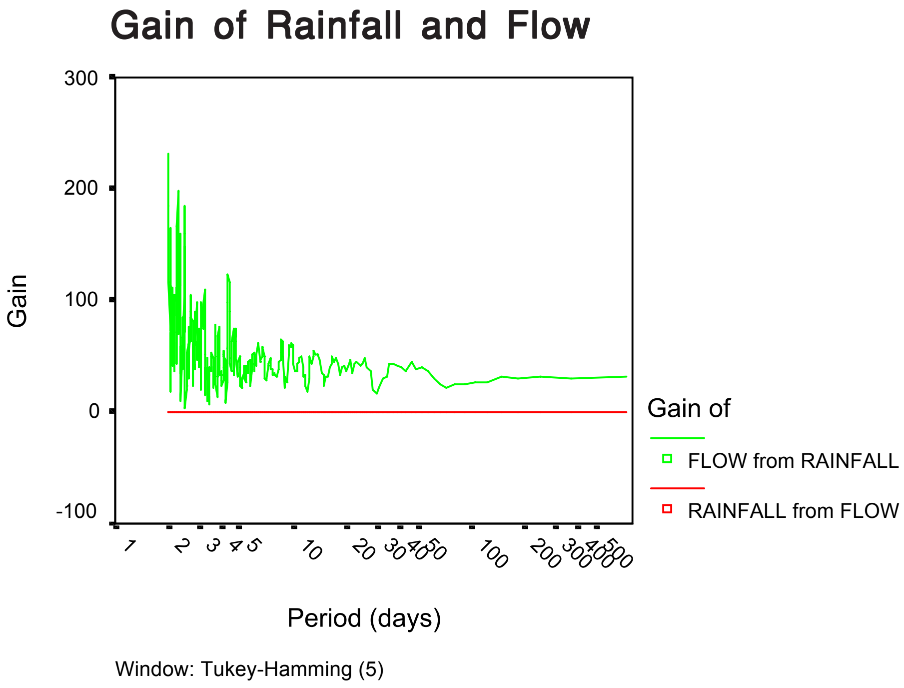

Figure 9 Gain function of rainfall and flow

4.0 CONCLUSION

The stream flow hydrograph provides an integrated representation of the mechanism of storing for delivering water to basin outflow point. Cross-correlation and cross-spectral correlation analysis of hydrological time series data can provide an understanding of the behaviour of a complex hydrological system such as peat basin. Using rainfalls, streamflows and water tables time series data obtained from a small drained peat basin in the analysis has concluded that drained peat basin is a fast-emptying type of aquifer with short delays. This could provide an additional knowledge that of practical importance to water resource planner in a similar vicinity.

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