Jurnal
Teknologi

Full Paper

THE EVALUATION OF CONTROLLER TRACKING PERFORMANCE BASED ON TAYLOR'S SERIES EXPANSION MODEL

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Article history
Received
13 March 2015
Received in revised form
14 April 2015
Accepted
15 June 2015

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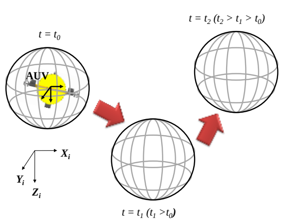
Graphical abstract



Abstract

This paper presents the controller tracking performance of Underwater Glider. The controllers are designed based on linearised model. The equations of motion are restricted to longitudinal plane. The controllers are designed and tested for the glide path moving from 45° to 30° downward and upward. The model is linearised using Taylor's series expansion linearisation method. The controller developed here is Sliding Mode Control (SMC), and Linear Quadratic Regulator (LQR). The performance of both controllers are compared and analysed. The simulations show SMC produce better performance with about over 30% faster than LQR based its convergence time.

Keywords: Underwater glider, linearisation, internal mass, longitudinal plane, dynamical model, Taylor's series expansion, LQR; SMC

Abstrak

Kertas kerja ini membentangkan prestasi kawalan penjejakan kenderaan Gelungsur Air. Pengawal direkabentuk berdasarkan model yang dilinearkan. Persamaan gerakan adalah terhad kepada satah membujur. Pengawal direka dan diuji untuk jalan meluncur bergerak daripada 45° kepada 30° ke bawah dan ke atas. Model dilinearkan menggunakan kaedah pelinearan pengembangan siri Taylor. Pengawal yang dibangunkan adalah Sliding Mode Control (SMC) dan Linear Quadratic Regulator (LQR). Prestasi kedua-dua pengawal dibandingkan dan dianalisis. Simulasi menunjukkan SMC menghasilkan prestasi lebih baik iaitu lebih kurang 30% lebih cepat daripada LQR berdasarkan masa penumpuan.

Kata kunci: Gelungsur bawah air, pelinearan, jisim dalaman, satah membujur, model dinamik, pengembangan siri Taylor, LQR, SMC

1.0 INTRODUCTION

An underwater glider is considered a special class of AUVs. The underwater glider concept was initially proposed by Stommel (1989), where later has motivated many researchers to produce several operational and laboratory scale gliders. The basic concept of underwater glider is a buoyancy-propelled and fixed-winged glider that shifts internal actuators to control its attitude.

The use of feedback control is the most commonly used method that provides a measure of robustness to uncertainty and disturbances [1], [2], [3], [4], [5], [6], [7]. PID controller is mostly used in many applications due to its simple architecture and proven algorithm used in actual platform implementation.

The optimal control approach, Linear Quadratic Regulator (LQR) is another approach has been used especially when linear system is used for designing control approach. In this approach two important tuning knobs, Q and R are varied to obtain an optimal gain that minimize the cost function and be a solution for Ricatti equation. Model Predictive Control (MPC) also recently become another candidate to be used to control underwater glider as used in [8], [9]. Francesco Tatone *et al.* in [8] using Model Predictive Control (MPC) to control the attitude of Slocum glider. They have divided the control architecture into two level controllers which is higher-level and lower-level controllers which control the internal configurations of the glider and make the actuator to execute actions for maintaining the imposed internal configuration respectively. Yuan Shan *et al.* proposed the MPC based on recurrent neural network. They formulated the control system using a time-varying constrained quadratic programming problem, which is solved by using a recurrent neural network called the simplified dual network in real time.

This paper presents tracking performance assessment of LQR and SMC based on Taylor's series expansion model. The simulation works are performed in Matlab/Simulink software and the results of the responses are presented in time domain.

The paper is organised as follows. In section 2, discusses on glider system (glider coordinates) and model linearization process. The controller designs are discussed in section 3. The results and discussion section discuss the performance of the controllers designed through simulation works. Finally, the section 5 conclude the paper which reiterates the main contributions of the work and highlights some of the possible future improvements.

2.0 GLIDER SYSTEM

The glider model include both kinematic and dynamic of rigid body, internal actuation system (ballast and movable sliding mass) and related hydrodynamics. The glider motion equations are derived by computing the kinetic energy and using kinetic energy we determine the momenta. The Newton-Euler formulation is then used to determine the forces and moments. The glider model is shown in figure 1. The model obtained is restricted to longitudinal plane. The detail working of modelling process is published in [10].

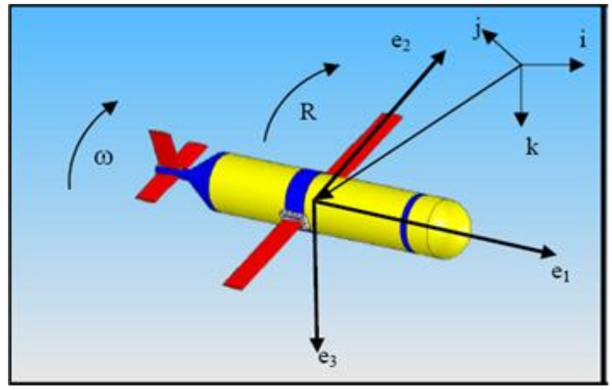


Figure 1 Coordinate systems of underwater glider

2.1 Model Linearisation

In this paper Taylor's series expansion method [11][12] is used to linearize the nonlinear model. Consider the following general nonlinear model with n state variables, m input variables and r output variables.

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1,\ldots\ldots x_n, u_1,\ldots u_m)\\
&\vdots\\
\dot{x} &= f_n(x_1,\ldots\ldots x_n, u_1,\ldots u_m)\\
y_1 &= g_1(x_1,\ldots\ldots x_n, u_1,\ldots u_m)\\
&\vdots\\
y_r &= g_n(x_1,\ldots\ldots x_n, u_1,\ldots u_m)
\end{aligned} \tag{1}$$

Therefore the elements of linearization matrices are given by:

$$A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x_d,u_d} \quad B_{ij} = \left.\frac{\partial f_i}{\partial u_j}\right|_{x_d,u_d}$$
$$C_{ij} = \left.\frac{\partial g_i}{\partial x_j}\right|_{x_d,u_d} \quad D_{ij} = \left.\frac{\partial g_i}{\partial u_j}\right|_{x_d,u_d} \tag{2}$$

With the elements in (2) define the linear system as

$$\delta\dot{x} = A\delta x + B\delta u \tag{3}$$

Where
$\delta x = x - x_d \quad \delta u = u - u_d$

The longitudinal motion equations are given by Equation 4(a)–4(k)

$$\dot{x} = v_1 \cos\theta + v_3 \sin\theta \tag{4a}$$
$$\dot{z} = -v_1 \sin\theta + v_3 \cos\theta \tag{4b}$$

$$\dot{\theta} = \omega_2 \tag{4c}$$

$$\dot{\omega}_2 = \frac{1}{J_2}((m_3 - m_1)v_1 v_3 - m_p g(r_{P1}\cos\theta + r_{P3}\sin\theta) + M_{DL2} - r_{P3}u_1 + r_{P1}u_3) \tag{4d}$$

$$\dot{v}_1 = \frac{1}{m_1}(-m_3 v_3 \omega_2 - P_{P3}\omega_2 - m_{em} g \sin\theta + L\sin\alpha - D\cos\alpha - u_1) \tag{4e}$$

$$\dot{v}_3 = \frac{1}{m_3}(m_1 v_1 \omega_2 + P_{P1}\omega_2 + m_{em} g \cos\theta - L\cos\alpha - D\sin\alpha - u_3) \tag{4f}$$

$$\dot{r}_{P1} = \frac{1}{m_p}P_{P1} - v_1 - r_{P3}\omega_2 \tag{4(g)}$$

$$\dot{r}_{P3} = \frac{1}{m_p}P_{P3} - v_3 + r_{P1}\omega_2 \tag{4(h)}$$

$$\dot{P}_{P1} = u_1 \tag{4(i)}$$

$$\dot{P}_{P3} = u_3 \tag{4(j)}$$

$$\dot{m}_b = u_4 \tag{4(k)}$$

The linearization about a steady glide path (ξ) is determined for the above motion equations. We adopt the method to calculate the equilibrium glide path from [13] . Define the state vector, x = [z', θ, ω2, v1, v3, rp1, rp3, Pp1, Pp3, mb] and input vector, u = [u1, u3, u4] where (u1, u3) are the forces acting on the internal point mass (mp) in x and z axes respectively and u4 is ballast pumping rate. Applying the Taylor's series expansion (1) and (2) to the motion Equations 4(a)–4(k) we obtain system matrix, A and input matrix, B.

$$A = \begin{bmatrix} 0 & -V & -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & -\frac{m_p\cos\theta}{J_2} & -\frac{m_p\cos\theta}{j_2} & 0 & 0 & 0 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & 0 & 0 & 0 & 0 & a_{410} \\ 0 & a_{52} & a_{53} & a_{54} & a_{55} & 0 & 0 & 0 & 0 & a_{510} \\ 0 & 0 & -r_{p3d} & -1 & 0 & 0 & 0 & \frac{1}{m_p} & 0 & 0 \\ 0 & 0 & r_{p1d} & 0 & 0 & 0 & 0 & 0 & \frac{1}{m_p} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{r_{p3d}}{J_2} & \frac{r_{p1d}}{J_2} & 0 \\ -\frac{1}{m_1} & 0 & 0 \\ 0 & \frac{1}{m_3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where

$$a_{32} = \frac{m_p g}{J_2}\left(r_{p1d}\sin\theta_d - r_{p3d}\cos\theta_d\right)$$

$$a_{33} = -m_p\left(r_{p1d}v_{1d} + r_{p3d}v_{3d}\right)$$

$$a_{34} = \frac{1}{J_2}\left((m_{f3} - m_{f1})v_{3d} + M_{v1}\right)$$

$$a_{35} = \frac{1}{J_2}\left((m_{f3} - m_{f1})v_{1d} + M_{v3}\right)$$

$$a_{42} = \frac{m_{emd}}{m_{1d}} g\cos\theta d$$

$$a_{43} = -\frac{m_{3d} + m_p}{m_{1d}} v_{3d}$$

$$a_{44} = \frac{1}{m_{1d}}\left(L_{v1}\sin\alpha_d + L\cos\alpha_d\alpha_{v1} - D_{v1}\cos\alpha d\right) + D\sin\alpha_d\alpha_{v1}$$

$$a_{45} = \frac{1}{m_{1d}}\left(L_{v3}\sin\alpha_d + L\cos\alpha_d\alpha_{v3} - D_{v3}\cos\alpha_d + D\sin\alpha_d\alpha_{v3}\right)$$

$$a_{410} = -\frac{g\sin\theta_d}{m_{1d}} + \frac{m_{emd}g\sin\theta_d}{m_{1d}^2} - \frac{1}{m_{1d}^2}\left(L\sin\alpha_d - D\cos\alpha_d\right)$$

$$a_{53} = -\frac{m_{emd}}{m_{3d}} g\sin\theta_d$$

$$a_{53} = \frac{m_{1d} + m_p}{m_{3d}} v_{1d}$$

$$a_{54} = \frac{1}{m_{3d}}\left(-L_{v1}\cos\alpha_d + L\sin\alpha_d\alpha_{v1} - D_{v1}\sin\alpha_d - D\cos\alpha_d\alpha_{v1}\right)$$

$$a_{55} = \frac{1}{m_{3d}}\left(-L_{v3}\cos\alpha_d + L\sin\alpha_d\alpha_{v3} - D_{v3}\sin\alpha_d - D\cos\alpha_d\alpha_{v3}\right)$$

$$a_{510} = \frac{g\cos\theta_d}{m_{3d}} - \frac{m_{emd}g\cos\theta_d}{m_{3d}^2} - \frac{1}{m_{3d}^2}\left(-L\cos\alpha_d - D\sin\alpha_d\right)$$

Replace all the desired parameters into matrix A and B we obtain :

Gliding Down:

$$A_d = \begin{bmatrix} 0 & -0.30 & 0 & -0.11 & 0.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.83 & 0 & 0.12 & -44.37 & -19.83 & 8.70 & 0 & 0 & 0 \\ 0 & 6.40 & -0.03 & -0.18 & 0.10 & 0 & 0 & 0 & 0 & 0.13 \\ 0 & -2.88 & 0.31 & -0.61 & -2.91 & 0 & 0 & 0 & 0 & 0.51 \\ 0 & 0 & -0.04 & -1.00 & 0 & 0 & 0 & 1.20 & 0 & 0 \\ 0 & 0 & 0.004 & 0 & -1.00 & 0 & 0 & 0 & 1.20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.11 & 0.01 & 0 \\ 0.03 & 0 & 0 \\ 0 & -0.03 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.00 & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{bmatrix}$$

Gliding Up:

$$A_u = \begin{bmatrix} 0 & -0.30 & 0 & -0.11 & 0.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7.51 & 0.05 & 8.74 & -111.86 & -179.66 & -78.85 & 0 & 0 & 0 \\ 0 & 0.25 & 0.06 & -0.81 & -0.88 & 0 & 0 & 0 & 0 & 0 -0.31 \\ 0 & 0.06 & 0.17 & 0.40 & -3.68 & 0 & 0 & 0 & 0 & 0.36 \\ 0 & 0 & -0.04 & -1.00 & 0 & 0 & 0 & 0.50 & 0 & 0 \\ 0 & 0 & -0.004 & 0 & -1.00 & 0 & 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.40 & -0.04 & 0 \\ -0.08 & 0 & 0 \\ 0 & -0.04 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.00 & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{bmatrix}$$

The controllability matrices of open loop system for both systems are 10 which indicate the open loop systems have full rank of controllability.

3.0 CONTROLLER DESIGN APPROACH

Controllers are designed to control the motion of the underwater glider. Here equations of motion are restricted to longitudinal plane. Two control techniques are designed that are Linear Quadratic Regulator (LQR) and Sliding Mode Control (SMC).

3.1 Linear Quadratic Regulator (LQR)

LQR is a method in modern control theory that uses state-space approach to analyse such a system. This the standard optimal control design which produces a stabilising control law that minimizes a cost function, J that is weighted of sum of squares of the states and input variables. Suppose we want to design state feedback control u = Kx to stabilise the system. By determines the feedback gain matrix that minimises J, we can establish the trade-off between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system. Assume that all the states are available for feedback. The cost function is to be minimised is defined as:

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t) \quad dt \tag{5}$$

Q is an n × n symmetric positive semi-definite matrix and R is an m × m symmetric positive definite matrix, (A, B) is stabilisable. Choosing Q relatively large than those of R, then deviations of x from zero will be penalised heavily relative to deviations of u from zero. On the other hand, if R is relatively large than those of Q, then control effort will be more costly and the state will not converge to zero as quickly as we wish. The Q and R matrices gliding down and gliding up are defined as:

Gliding down

Q = diag(800,500,200,200,50,50,20,10,10,50)

R = diag(1,1,1)

Gliding up

Q = diag(0.5,0.5,1,2,2,0.1,0.1,1,1,0.05)

R = diag(1,1,1)

3.2 Sliding Mode Control (SMC)

In this section, a SMC control scheme is proposed and described in detail. The design objective is to examine the performance of the controller. The controller parameters (S, ueq, and usw) of SMC controller are tuned heuristically.

The design procedure of sliding mode control comprises two steps. The first step is to design the sliding surfaces that satisfy the designer's specifications. The second is chosen n the control law such that the output trajectory is reach and stay on the sliding surfaces after a finite time. The sliding surface is given by

$$\sigma(t) = Sx(t) \tag{6}$$

where $S \in \Re^{mxn}$ is full rank. The S is chosen such that $SB \in \Re^{mxn}$ is non-singular. From the underwater glider state-space system m = 2 and n =8, hence S has 3 x 10 matrix structure.The second component of SMC design is control law. Often used structure for control law is

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{7}$$

where $u_{eq}(t)$ is the equivalent control associated with the nominal system and unique solution which satisfy $\dot{\sigma}(t) = S\dot{x}(t) = 0$ and $u_{sw}(t)$ is the switching control which satisfies the reaching condition $\sigma(t)\dot{\sigma}(t) < 0$. The equivalent control is determined using $\dot{\sigma}(t) = S\dot{x}(t) = 0$

$$u_{eq}(t) = -(SB)^{-1} SAx(t) \tag{8}$$

The switching control $u_{sw}(t)$ adopted from [15] is given by :

$$u_{sw}(t) = -(SB)^{-1} \rho \frac{\sigma(t)}{|\sigma(t) + \delta|} \quad \rho > 0 \tag{9}$$

where ρ is a design parameter that is specified by the designer and δ is the boundary layer . The following sliding surface matrices, ρ, and δ are obtained force and acceleration control inputs.

Gliding Down:

$$S_d = \begin{bmatrix} 20 & -17 & -19 & -20 & 75 & 440 & 15 & -199 & 10 & 1 \\ -17 & 7 & 10 & 5 & -50 & -2300 & 110 & -5 & 7 & -1 \\ -2 & -1 & -2 & 15 & 10 & 55 & -24 & 2 & -0.3 & 10 \end{bmatrix}$$

$\rho = 100, \delta = 0.1$

Gliding UP:

$$S_d = \begin{bmatrix} 0.2 & 2 & -2.5 & -28.4 & -36 & 440 & 15 & -199 & 10 & 1 \\ -17 & 7 & 10 & 5 & -50 & -230 & 110 & -5 & 7 & -1 \\ -2 & -1 & -2 & 15 & 10 & 55 & -24 & 2 & -0.3 & 10 \end{bmatrix}$$

$\rho = 100; \delta = 0.0$

4.0 RESULTS AND DISCUSSION

In this section, the proposed control schemes are implemented and tested within the simulation environment of the glider system and the corresponding results are presented. The output responses namely pitching angle (θ), gliding angle (ξ), horizontal (v1) and vertical velocities (v3) are observed. The controllers are evaluated using parameters adopted from [1].

Here only responses for gliding and pitching angles are depicted in figure 2 to figure 9 for downward and upward glide paths. Initial gliding angles are set to be -45o (downward) and 45o (upward). All figures are plotted for nonlinear open loop system and performance comparison between SMC and LQR.

The summarized performances of both controllers are tabulated in Table 1 for all the observed output responses. All the controller parameters are heuristically tuned until desired output are obtained. Overall results for both downward and upward glides reveal that SMC perform better than LQR with faster convergence time (at least 30% or better).

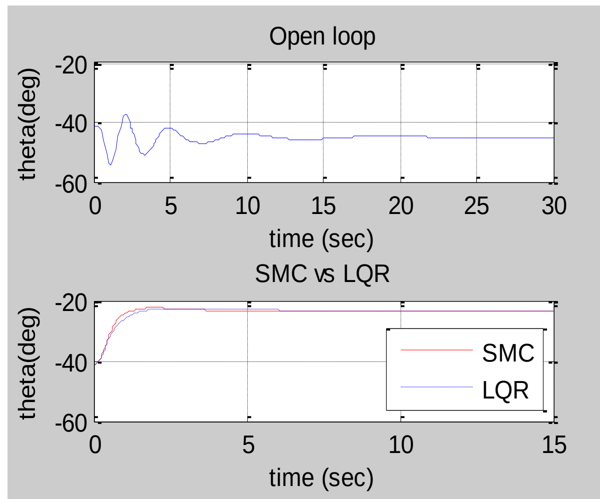


Figure 2 DOWNWARD pitching angle

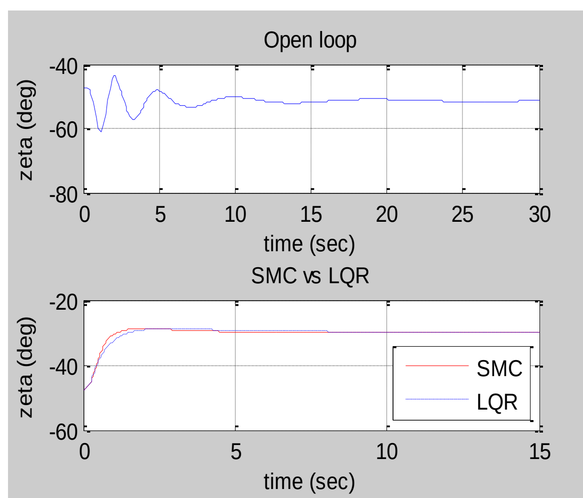


Figure 3 DOWNWARD gliding angle

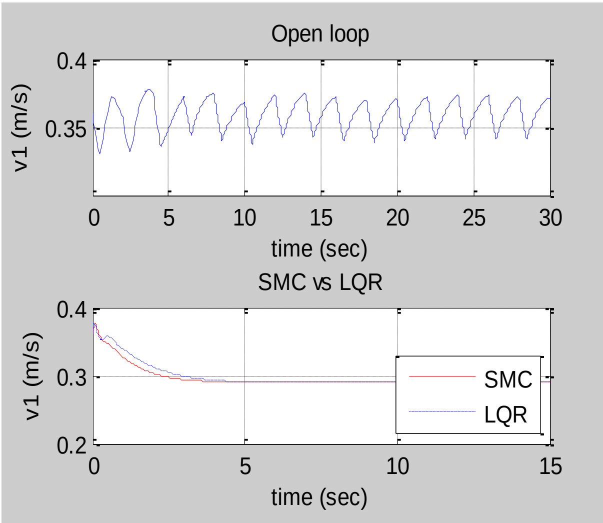


Figure 4 DOWNWARD Velocity, v_1

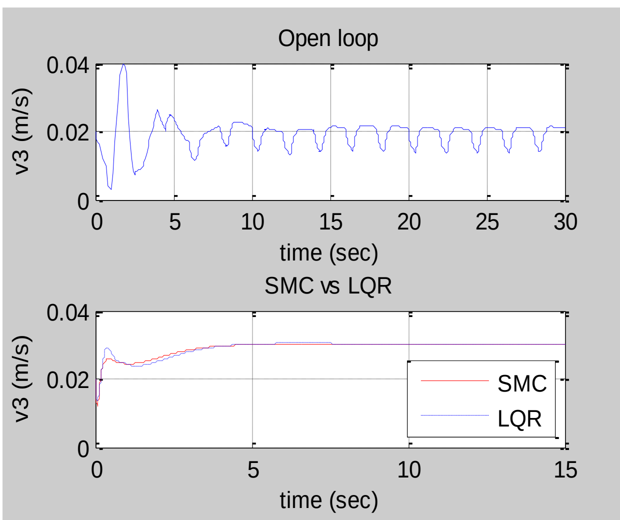


Figure 5 DOWNWARD Velocity, v_3

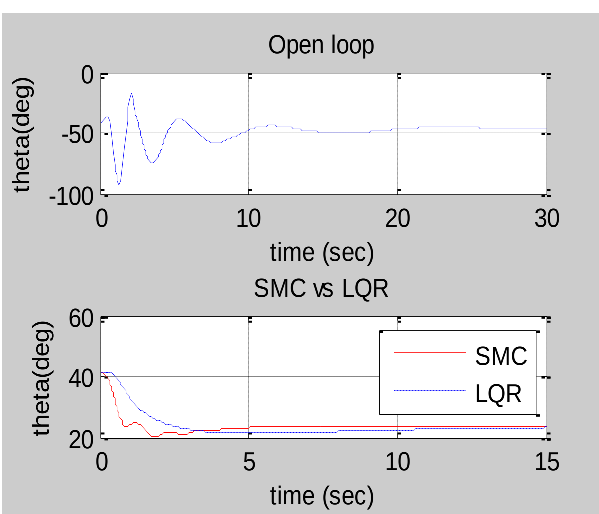


Figure 6 UPWARD pitching angle

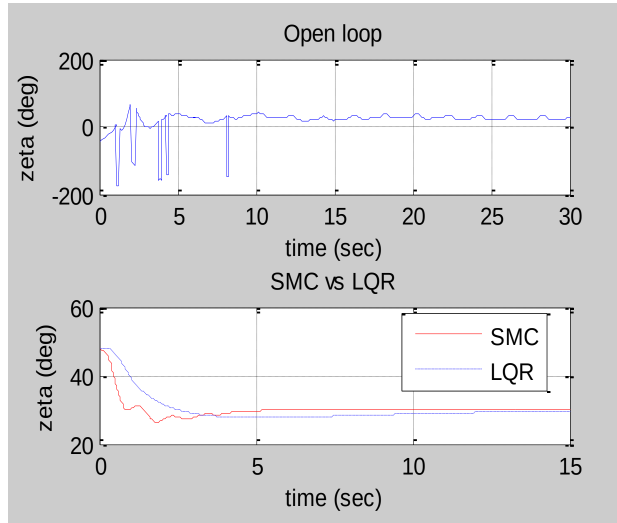


Figure 7 UPWARD gliding angle

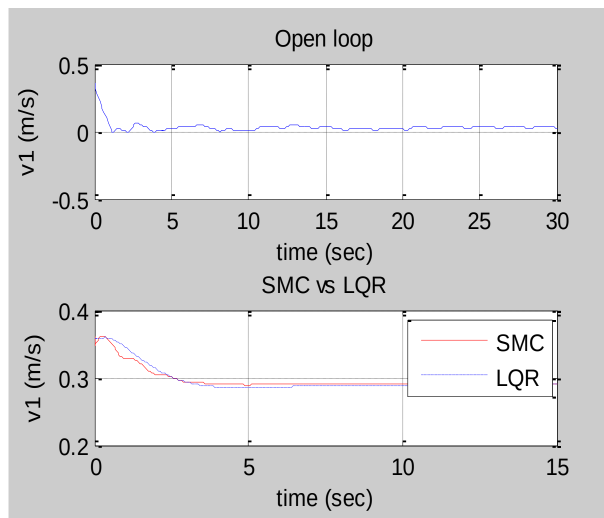


Figure 8 UPWARD velocity, v_1

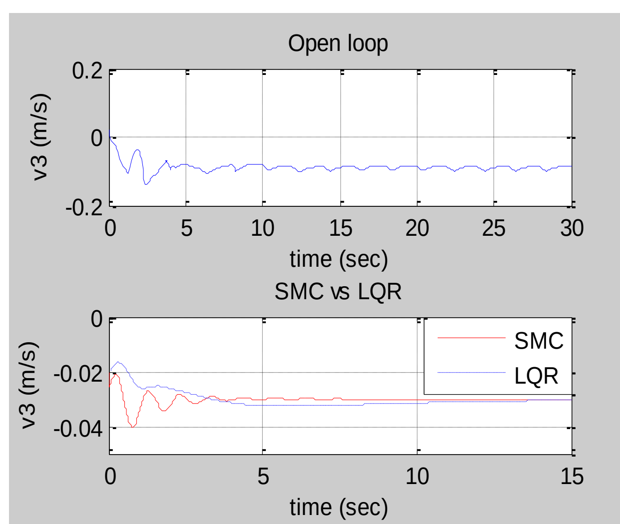


Figure 9 UPWARD velocity, v_3

Table 1 Output tracking convergence time

Output tracking	LQR (secs)		SMC (secs)	
	Down	Up	Down	Up
Gliding angle (deg.)	11.5	24	7	8
Pitching angle (deg.)	12	25	9	10
Horizontal velocity (m/s)	5	10	5.2	5
Vertical velocity (m/s)	6	20	13	8

5.0 CONCLUSION

In this paper, the LQR and SMC are successfully designed and simulated on underwater glider model. Based on the results and analysis, a conclusion has been made that SMC reveals a better results with comparing to LQR. In future, nonlinear controller algorithm will be implemented and feedback linearization approach will be used so that more results can be produced and analyzed for further improvement.

Acknowledgement

This research is supported by Universiti Malaysia Pahang (UMP) research grant Vot: RDU1303117, Development and Robust Controller Design for Underactuated Glder Platform for UMP Lake Monitoring.

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