# MEASUREMENT OF 3-D SHAPE OF MULTI-SURFACE OBJECT IN A HOMOGENEOUS BACKGROUND USING PHASE-SHIFT FRINGE PROJECTION 

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#### Abstract

This paper presents the results of application of the phase-shift fringe projection method to the measurement of 3-D shape of a multi-surface object in a homogeneous background. The phaseshift fringes were analyzed to obtain a phase map of the object shape. An effective technique of detecting edges of the object in the homogeneous background developed in previous work was used to detect the object edges. The algorithm used to extract the 3-D data from the phase-map was developed using simulated images and later applied to a real image. Comparison of measurement results with the actual height of the object along two sections considered showed a maximum error of 0.3 mm .


Key words: Fringe projection, 3-D measurement, multi-surface


#### Abstract

Abstrak. Kertas kerja ini membentangkan keputusan mengaplikasi kaedah unjuran pinggir anjakan fasa untuk mengukur bentuk 3-D bagi objek berbilang muka dalam latar belakang berhomogen. Pinggir-pinggir anjakan fasa dianalisis untuk mendapatkan peta fasa bagi bentuk objek tersebut. Suatu kaedah untuk mengesan pinggir-pinggir bagi objek dalam latar belakang berhomogen yang dibangunkan dalam kerja dahulu digunakan untuk mengesan sempadan objek. Algoritma yang digunakan untuk menyari data 3-D daripada peta fasa dibangunkan dengan menggunakan imej-imej simulasi dan kemudian dilaksanakan ke atas imej sebenar. Bandingan keputusan pengukuran dengan tinggi sebenar menunjukkan ralat maksimum sebanyak 0.3 mm .


Kata kunci: Unjuran pinggir, pengukuran 3-D, pelbagai muka

### 1.0 INTRODUCTION

Non-contact measurement of three-dimensional (3-D) shapes of objects is of significant interest in metrological and quality control applications. Various techniques are available for such measurement including stereo vision and triangulation, active focusing, structured lighting, fringe projection, laser scanning, moiré and holographic contouring. For instance, Reich et al. [1] applied photogrammetry and fringe projection methods to measure the 3-D shape of the door of a car; Cardenas-Garcia et al. [2] used stereo imaging to reconstruct the 3-D shape of hemispherical and wavy surfaces; Hung et al. [3] presented a practical approach using computer-controlled grating projection/reflection technique to measure the 3-D shapes of several objects having dif-

[^0]fuse surfaces; Lianhua et al. used phase-shifting shadow moiré profilometry to measure the shape of a computer mouse [4]; Boxiong et al. [5] presented a new structured light encoding method for range-data acquisition and the technique was applied to measure the 3-D profile of a head model. Chen et al. [6] provided an overview of threedimensional shape measurement using optical methods and discussed the application of these methods to the measurement of panel spring back, vehicle body shape and geometry of corrugated plate.

In spite of the large amount of work worldwide on the application of non-contact optical 3-D measurement methods, most applications are for the measurement on continuous surfaces where continuity of fringe data exist throughout the surface. The measurement of multi-surface objects where discontinuity in fringes can occur has not been fully investigated, particularly if the object is located in a background of the same intensity as the object surface, i.e. in a homogeneous background. This is the focus of the current work.

This paper presents the results of measurement carried out on a multi-surface object in a homogeneous background using the phase-shift fringe projection method. The phase-shift method was used to detect the edges of the object and to increase the measurement resolution so that interpolation of measured data is unnecessary [7].

### 2.0 THEORY OF PHASE-SHIFT METHOD AND EDGE DETECTION FROM SIMULATED IMAGES

The automatic analysis of projected fringes proposed in this work is based on the phase-shift method introduced by Robinson et al. [8]. In this technique, the grayscale intensity $G(i, j)$ of any point $(i, j)$ in a sinusoidal fringe pattern can be expressed by the following equation:

$$
\begin{equation*}
G(i, j)=A(i, j)+B(i, j) \cos \phi(i, j) \tag{1}
\end{equation*}
$$

where $A(i, j)$ is the uniform background intensity, $B(i, j)$ is the amplitude of the sinusoidally varying component of the intensity and $f(i, j)$ is the phase angle at $(i, j)$. The fringe pattern is transformed into a phase-map by evaluating $f(i, j)$ from equation (1) at each position of $(i, j)$. To achieve this the fringes are shifted by $2 \mathrm{p} / 3$ radians in three stages, i.e. $0,2 p / 3$ and $4 \mathrm{p} / 3$ radians. This results in three fringe patterns having intensities given by

$$
\begin{gather*}
I_{1}=A(i, j)+B(i, j) \cos \phi(i, j)  \tag{2}\\
I_{2}=A(i, j)+B(i, j) \cos \left[\phi(i, j)+\frac{2 \pi}{3}\right]  \tag{3}\\
I_{3}=A(i, j)+B(i, j) \cos \left[\phi(i, j)+\frac{4 \pi}{3}\right] \tag{4}
\end{gather*}
$$

The three equations can be solved for $\phi(i, j)$ as follows $[7,8]$,

$$
\begin{equation*}
\phi(i, j)=\tan ^{-1}\left[\frac{\sqrt{3}\left[I_{2}(i, j)-I_{3}(i, j)\right]}{2 I_{1}(i, j)-I_{2}(i, j)-I_{3}(i, j)}\right] \tag{5}
\end{equation*}
$$

Equation (5) only gives the values of $\phi$ between $-\pi$ to $+\pi$. However, by taking into account the signs of the numerator and denominator, the value of $\phi$ between 0 to $2 \pi$ can be determined. Equation (5) was coded into a C/C++ program for an image of size $250 \times 250$ pixels. The result of applying the above algorithm to simulated fringe patterns, shown in Figures 1(a)-(c), is shown in Figure 1(d). The phase-map in Figure 1(d) has a saw-tooth intensity profile as shown in Figure 2(a), whereas the original fringe pattern in Figure 1(a) has a sinusoidal profile as shown in Figure 2(b). In the presence of noise in the image, the analysis carried out on the phase-map to detect the edges of the object improves the accuracy of the detection when compared to analysis performed on any of the original fringe patterns. The detailed results of the edge detection work has been published separately $[9,10]$. A brief outline of the edge detection technique is given here.


Figure 1 (a) Original fringe pattern, (b) fringe pattern after $2 \pi / 3$ phase shift, (c) fringe pattern after $4 \pi / 3$ phase shift and (d) the phase-map


Figure 2 Gray scale intensity profiles on the image background for (a) phase-map and (b) original fringe pattern (along the path shown in the inset images)

In most existing applications of edge detection, the variation in pixel intensity between the object surface and the background is used to locate the edges. This is often achieved using gradient-based edge detectors such as Sobel, Prewitt, Roberts, Canny etc. [11]. However, in the absence of a clear demarcation of intensities at the edge pixels, such as when the object is in a background of the same intensity as the object surface, i.e. a homogeneous background, intensity-based operators will not be able to detect the edges successfully. This is shown by our results in Figures 3(a)-(c) in which two intensity-based operators, i.e the Sobel and Canny edge detectors, were applied to detect the edges of a real object in a homogeneous background. Figure 3(b) shows that the Sobel operator is able to detect only edges where an intensity gradient is present and Figure 3(c) shows that Canny operator is sensitive to noise and picks up false edges. Other intensity based edge detectors such as Prewitt, Laplacian of Gaussian and zero-crossing will not work successfully in such application either because a clear intensity variation does not exist between the object surface and the background at all the edges. However, it may be possible to detect the edges if the depth of the object is sufficiently large to cause intensity difference between the object surface and the background due to the different distances travelled by the reflected light from the two surfaces before reaching the camera. This, however, may require a sensitive edge detector to detect the edges due to the low contast between the object surface and the background.


Figure 3 (a) Object in a homogeneous background, (b) results of edge detection using the Sobel operator and (c) results of edge detection using the Canny operator

In the proposed method of edge detection, a sinusoidally-coded structured light is projected onto the object surface and the object is viewed from an angle different from the projection angle. Due to the difference in the projection and viewing directions and the height of the object, the intensity of the fringes will break at the edges of the object. By locating the intensity break points it is possible to detect the edges. The grayscale intensity $G b(x, y)$ at any point $(x, y)$ in the background of the object due to the sinusoidally-coded light can be expressed as a sine wave function given by,

$$
\begin{equation*}
G_{b}(x, y)=A+B \cos [\phi(x, y)] \tag{6}
\end{equation*}
$$

where $A$ is due to the uniform background intensity, $B$ is the amplitude of the intensity fluctuation of the fringes and $\phi(x, y)$ is the phase value at $(x, y)$. If the depth of the object is $h$ and a fringe is displaced by $\Delta L$ between the object surface and the background, then this displacement is equivalent to a phase-shift of $\Delta \phi$ given by,

$$
\begin{equation*}
\Delta \phi=2 \pi\left(\frac{\Delta L}{L}\right) \tag{7}
\end{equation*}
$$

where $L$ is the distance between adjacent fringes, known as the fringe pitch. Hence, the light intensity on the object surface will be given by,

$$
\begin{equation*}
G_{o}(x, y)=A+B \cos [\phi(x, y)+\Delta \phi] \tag{8}
\end{equation*}
$$

If the fringes are projected vertically (parallel to the $y$-axis), the intensity will vary only along the $x$-axis and equations (6) and (8) will be reduced, respectively, as follows:

$$
\begin{gather*}
G_{b}(x)=A+B \cos \phi(x)  \tag{9}\\
G_{o}(x)=A+B \cos [\phi(x)+\Delta \phi] \tag{10}
\end{gather*}
$$

For a fixed value of $x$ an edge will, therefore, exist if

$$
\begin{equation*}
\left|G_{b}(x)-G_{o}(x)\right|>t \tag{11}
\end{equation*}
$$

where $t$ is the gray level threshold whose value depends largely on the standard deviation of noise in the image. Equation (11), however, assumes that the fringes are exactly vertical. This condition must be satisfied in the experimental setup.

When the analysis is carried out using the raw fringe pattern images, the accuracy of the detected edge from equation (11) drops when the standard deviation of noise increases [10]. However, the accuracy can be improved significantly if the analysis is performed on a phase map obtained by applying equation (5). The results of edge detection using the proposed method is shown in Figures 4(a)-(b). Comparison of


Figure 4 (a) Phase-map for the object in Figure 3(a), and (b) results of edge detection from the phase-map

Figures 3 (b) and 4 (b) shows that edge detection from the phase map produces accurate results compared to intensity-based edge detector. The average error was found to be 2.53 pixels in the column direction and 3.75 pixels in the row direction. This error is considered not significant for pattern recognition applications and approximate measurement of 2-D dimensions of 3-D objects. Although the results illustrated in Figure 4(b) is for an object having a single surface, the same algorithm is applicable to objects having multi ple surfaces provided a break in the fringe continuity can be detected. Having detected the edges successfully, it is then possible to determine the height variations to extract the 3-D information.

### 3.0 3-D MEASUREMENT ON SIMULATED IMAGE

Figure 5 shows an isometric view of an object that is used in the simulation study to develop the algorithms used in the extraction of 3-D data from the 2-D images. The object is assumed to be 'illuminated' using sinusoidally structured light and the image is 'recorded' from a normal position to the background. The average value of the angle q between the illumination and recording directions is assumed to be $45^{\circ}$. This value was assumed because in the actual experiment the angle can vary between $40^{\circ}$ to $50^{\circ}$. Figures $6(\mathrm{a})$-(c) are the phase-shifted fringe patterns and Figure $6(\mathrm{~d})$ is the phase-map obtained by applying the program developed in this work.


Figure 5 Isometric view of the 'object' used in the simulation study


Figure 6(a)-(c) Simulated fringe patterns phase-shifted by $0,2 \pi / 3$ dan $4 \pi / 3$ respectively, (d) phasemap image

The height $\mathrm{D} z$ of the object surface measured from the background can be determined by comparing the gray values of each pixel $G(i, j)$ with the average gray value $G_{\mathrm{av}}(i)$ of 20 pixels on the background along the same column measured from the edge of the image. Since 20 pixels from the edge of the image is used to compute the average gray value, it is necessary to ensure in the experimental setup that the object is at least 20 pixels from the edge of the image. This can be achieved by adjusting the camera position so that the object is not too close to the edge of the image being captured. The value of $\mathrm{D} z$ can then be determined from the following triangulation equation:

$$
\begin{equation*}
\Delta z=\frac{1}{\tan \theta}\left\{\frac{\left|G(i, j)-G_{a v}(i)\right|}{255} \times L \times S\right\} \tag{12}
\end{equation*}
$$

where $L$ is the fringe pitch and $S$ is the scaling factor for converting image pixel distance to actual spatial distance. Equation (12) was coded into a $\mathrm{C} / \mathrm{C}++$ program for use in the 3-D measurement.

The results obtained from the analysis carried out on the phase-map is shown in Figure 7. The dimensions of the object in Figure 7 do not have any physical meaning and are merely arbitrary because the analysis was carried out on a simulated image. The results, however, show the validity of the algorithm and program developed for the measurement.


Figure 7 3-D shape reconstructed from the analysis of the phase-map for the simulate image

### 4.0 EXPERIMENTAL WORK AND ANALYSIS ON A REAL IMAGE

Figure 8 shows a schematic view of the experimental layout used to record the fringe patterns on a multi-surface object. A slide projector was used to project the sinusoidal grating onto the object surface. A mirror, mounted on a rotatable stage, was placed in the path of the illumination so that phase shifting can be carried out by rotating the mirror. Three fringe patterns on the object were recorded using a Kodak DC120 digital camera corresponding to phase-shift angles of $0,2 \pi / 3$ and $4 \pi / 3$ radians. These angles correspond to fringe movements of $0,1 / 3$ and $2 / 3$ of a fringe spacing. Four reference points were located on the background of the object so that the scaling factor can be determined. The recorded images were transferred to a personal computer for the analysis. The experiment was carried out in a dark room where the lighting is controlled to exclude external light. An isometric view of the object used in the experiment is shown in Figure 9. The fringe patterns and the phase-map after the analysis are shown in Figures 10(a)-(d). The 3-D profile of the object surface obtained by applying equation (6) at every pixel on the phase-map is shown in Figure 11.


Figure 8 Schematic diagram of the experimental layout


Figure 9 Isometric view of the object used in the experiment


Figure 10 (a)-(c) Fringe pattern images on object for phase-shifts of $0,2 \pi / 3$ and $4 \pi / 3$, (d) phase map image


Figure 11 3-D profile of the object from analysis

The cross-sectional shape of the object from the analysis along $y=30 \mathrm{~mm}$ and $y=50$ mm shown in Figures 12(a)-(b) indicate that the object surface is not uniform, that is, a maximum variation in height of about $\pm 0.5 \mathrm{~mm}$ occurs. This can be attributed to noise in the original images that causes uneven pixel intensities across the object surface. However, by fitting the surface data using a first order polynomial approximation, uniform surface profile can be obtained as shown in Figures 13(a)-(b). From Figures 13(a)-(b) the maximum difference in height between measurement using phaseshift method and actual height was found to be approximately 0.3 mm .


Figure 12 Surface profile at sections along (a) $y=30 \mathrm{~mm}$ and (b) $y=50 \mathrm{~mm}$


Figure 13 Surface profile after polynomial fitting at sections along (a) $y=30 \mathrm{~mm}$ dan (b) $y=50 \mathrm{~mm}$

### 5.0 CONCLUSION

The phase-shift fringe projection method was applied to measure the three-dimensional shape of a multi-surface object in a homogeneous background and the results are presented. An algorithm developed in previous work that can effectively detect the edges of the object has been applied in the 3-D measurement. In this manner, the analysis on the 2-D image of the phase-map can be confined within the object region. The comparison between measurement and actual height showed a maximum error of 0.3 mm along the section studied. Although this error is too large for precision measurement, the technique is applicable in less stringent applications, for instance in rapid 3-D measurement for more accurate probing using the coordinate measuring machine.

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