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THE GROWTH OF MIXING ZONE IN HETEROGENEOUS POROUS MEDIA

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Abstract. Utilizing currently available analytical solutions that incorporate fractal exponent, the growth of mixing length of injected solvent was determined for a one-dimensional model. Mixing zone size was found to increase as porous medium becomes increasingly heterogeneous. In a heterogeneous porous media, mixing zone size increases as dispersion coefficient increases particularly at relatively short duration of flow. There are three important factors influencing the size of the dispersive mixing zone, Δx_D. Of particular importance in this study is reservoir heterogeneity, which is represented by a fractal exponent, β. It was discovered that as β becomes smaller (porous medium becomes increasingly heterogeneous), the size of the mixing zone increases. Another factor affecting Δx_D is time dependent dispersion coefficient, $K(t_D)$. In a heterogeneous reservoir, mixing zone increases with increasing value of dispersion coefficient at relatively short duration of flow. For relatively long period of flow, however? Δx_D continues to increase even though $K(t_D)$ remains constant. The third factor is average fluid velocity, ν. Mixing zones have inverse relationship with fluid velocity in that Δx_D increases as v decreases.

Key words: Homogeneity, heterogeneity, dispersion coefficient, fractal exponent, mixing zone, dimensionless concentration, porous media

Abstrak. Dengan menggunakan penyelesaian analitikal yang merangkumi fraktal eksponen, pembesaran jarak pencampuran telah dapat ditentukan bagi model satu dimensi. Saiz zon pencampuran didapati meningkat apabila media berliang menjadi semakin heterogen. Dalam media berliang yang heterogen, saiz zon pencampuran meningkat apabila pemalar penyerakan meningkat terutama sekali pada aliran jangkamasa singkat relatif. Terdapat tiga faktor penting mempengaruhi saiz zon pencampuran penyerakan, Δx_D. Perkara terpenting dalam kajian ini ialah keheterogenan takungan, yang dipersembahkan oleh eksponen fraktal, β. Hasil kajian mendapati bahawa apabila β menjadi kecil (media berliang menjadi semakin heterogen), saiz zon pencampuran meningkat. Satu lagi faktor yang mempengaruhi Δx_D ialah pekali penyerakan bersandar masa, $K(t_D)$. Di dalam takungan heterogen, zon pencampuran meningkat terhadap peningkatan nilai pekali penyerakan pada aliran jangkamasa singkat relatif. Bagaimanapun, bagi aliran jangkamasa panjang relatif, Δx_D terus meningkat walaupun $K(t_D)$ malar. Faktor ketiga ialah purata kelajuan bendalir, ν. Zon pencampuran mempunyai perkaitan songsang terhadap kelajuan bendalir di mana Δx_D meningkat apabila v berkurangan.

Kata kunci: Kehomogenan, keheterogenan, pekali penyerakan, eksponen fraktal, zon pencampuran, media berliang

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1.0 INTRODUCTION

The success of a miscible oil recovery process depends on the length and integrity of the mixing zone within which dispersion works to cause mixing and dissipation of the injected solvent. Subsurface mixing behavior can be determined from inter-well tracer tests in which the tracer concentration at a producing well is monitored. The concentration profile is a function of the evolution of the mixing zone with time.

For a gravity stable one dimensional, miscible flood in a dipping homogeneous porous media (reservoir), the less dense solvent displaces oil down-dip at a rate below a critical displacement rate such that gravity acts to keep the solvent segregated from the oil and prevents protrusions of solvent fingers into the oil. Assuming homogeneity, such a system has been modeled as an infinitely long one dimensional flow system containing no solvent initially, but into which a constant solvent concentration is continuously injected beginning at time zero. For one dimensional longitudinal dispersion, the relevant convection-dispersion equation in a semi-infinite homogeneous medium having a plane source at $x = 0$, is given as

$$K\frac{\delta^2 C}{\delta x^2} - v\frac{\delta C}{\delta x} = \frac{\delta C}{\delta t} \qquad (1)$$

The exact analytical solution for constant K in (1) employing the initial and boundary conditions for one dimensional first type boundary where,

$$C(x, 0) = 0 \quad ; x^3 \geq 0$$
$$C(0, t) = Co \quad ; t^3 \geq 0$$
$$C(\infty, t) = 0 \quad ; t^3 \geq 0$$

results in (Lake, 1989 and Marle, 1981),

$$\frac{C}{Co} = \frac{1}{2} erfc\left(\frac{x - vt}{2\sqrt{Kt}}\right) + \frac{e^{\frac{xv}{K}}}{2} erfc\left(\frac{x - vt}{2\sqrt{Kt}}\right) \qquad (2)$$

rewriting the equation (2) above in a dimensionless form gives,

$$C_D = \frac{1}{2} erfc\left(\frac{x_D - t_D}{2\sqrt{\frac{Kt_D}{vL}}}\right) + \frac{e^{\frac{x_D vL}{K}}}{2} erfc\left(\frac{x_D + t_D}{2\sqrt{\frac{Kt_D}{vL}}}\right) \qquad (3)$$

For continuous-constant injection into a flow field, the initial and third conditions of the second boundary are given as,

$$C(x,0)=0 \qquad ;-\infty < x < +\infty$$

$$\int_{-\infty}^{\infty} \Phi_e C(x,t)dx = C_o \Phi_e v_x t \qquad ;t>0$$

$$C(\infty,t)=0 \qquad ;t \geq 0$$

The boundary states that the injected mass of solute over the domain from $-\infty < x < +\infty$ is proportional to the length of time of the injection. ϕ_e is defined as the effective porosity and v_x is average linear flow velocity in a longitudinal direction. Following the solution by Sauty (1980), the exact solution is in the form,

$$\frac{C}{Co}=\frac{1}{2}erfc\left(\frac{x-vt}{2\sqrt{Kt}}\right)-\frac{e^{\frac{xv}{K}}}{2}erfc\left(\frac{x+vt}{2\sqrt{Kt}}\right) \tag{4}$$

In a dimensionless configuration it is represented by,

$$C_D=\frac{1}{2}erfc\left(\frac{x_D-t_D}{2\sqrt{\frac{Kt_D}{vL}}}\right)-\frac{e^{\frac{x_D vL}{K}}}{2}erfc\left(\frac{x_D+t_D}{2\sqrt{\frac{Kt_D}{vL}}}\right) \tag{5}$$

Defining time dependent dispersion coefficient, K(t) (Erkal,1997),

$$K(t)=\frac{d}{dt}(f(t).t) \tag{6}$$

where,

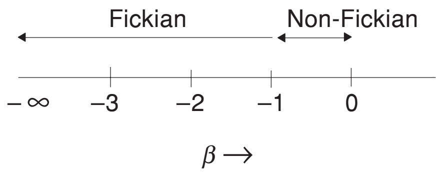
$$f(t)=\frac{1}{f}\left[-\frac{1}{(1+\beta)(2+\beta)}-\frac{1}{(1+\beta)}+\frac{(1+t)^{\beta}}{(1+\beta)(2+\beta)}+\frac{2t(1+t)^{\beta}}{(1+\beta)(2+\beta)}+\frac{t^2(1+t)^{\beta}}{(1+\beta)(2+\beta)}\right]$$

and multiply $f(t)$ by time and then differentiate with respect to t, the following time dependent dispersion coefficient is obtained,

$$K(t)=-\frac{t}{(1+\beta)}+-\frac{\left[\beta(1+t)^{\beta-1}+2\beta t(1+t)^{\beta-1}+2(1+t)^{\beta}+\beta t^2(1+t)^{\beta-1}+2t(1+t)^{\beta}\right]}{(1+\beta)(2+\beta)}$$

for $\beta \neq -1, -2$ (7)

β is called a fractal exponent. It normally takes values from less than zero to negative infinity. For $\beta < -1$, normal dispersion prevails which characterizes a Fickian model of dispersion. For fractal exponent in the range of $0 > \beta \geq -1$, anomalous dispersion or non-Fickian dispersion takes place in the reservoir of interest. Positive values of β are not applicable to a physical system (Zhang, 1991). Therefore, in this work, the discussion is restricted only to Fickian model of dispersion.



The approximate PDE solution employing the first type boundary condition can be obtained by introducing the time dependent dispersion coefficient to yield,

$$\frac{C}{Co} = \frac{1}{2} erfc\left(\frac{x - vt}{2\sqrt{K(t)t}}\right) - \frac{e^{\frac{xv}{K}}}{2} erfc\left(\frac{x + vt}{2\sqrt{K(t)t}}\right) \tag{8}$$

The exact solution, if known, should satisfy equation (1) and the first boundary conditions simultaneously. The error in the approximate solution can be evaluated by substituting equation (8) into equation (1) in the following form where y denotes the error from the true solution,

$$y = K(t)\frac{\delta^2 C}{\delta x^2} - v\frac{\delta C}{\delta x} - \frac{\delta C}{\delta t} \tag{9}$$

For the approximate solution to be the true solution of the convection-dispersion equation, the value of y should be zero. For the second type boundary condition, the solution can be approximated from,

$$\frac{C}{Co} = \frac{1}{2} erfc\left(\frac{x - vt}{2\sqrt{K(t)t}}\right) - \frac{e^{\frac{xv}{K}}}{2} erfc\left(\frac{x + vt}{2\sqrt{K(t)t}}\right) \tag{10}$$

2.0 DISCUSSION

Figure 1 depicts combined illustration of first and second type boundary solution for dimensionless concentration against dimensionless time at various fractal exponents. "S" shape profile is observed from the figure. When longitudinal mixing takes place in a miscible displacement process, where a first contact miscible solvent is injected into a reservoir to displace oil that has the same density and viscosity as the solvent,

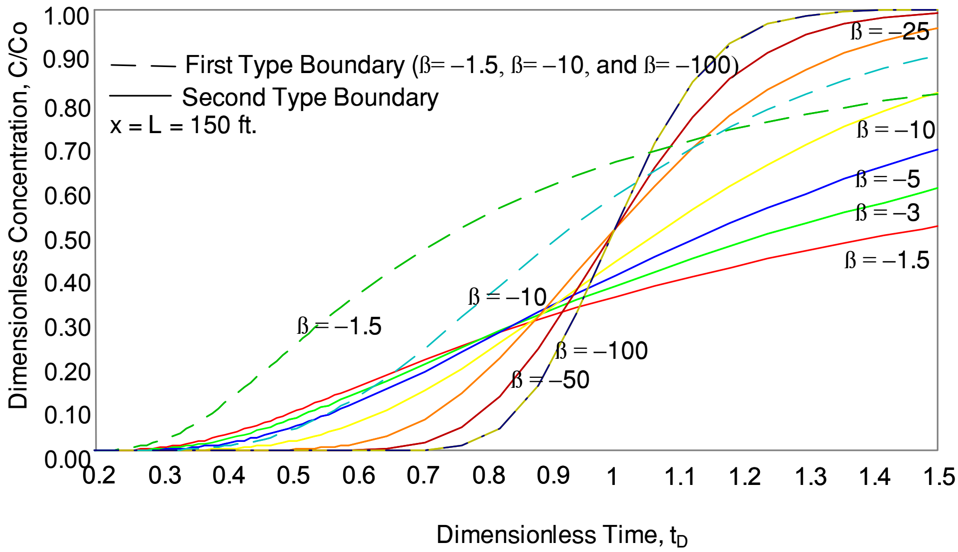


Figure 1 Dimensionless concentration against dimensionless time for various values of the fractal exponent-First and Second Type Boundary

the effluent solvent concentration initially is produced at low concentration. Then, it is followed by a period of rising concentration and finally a period where effluent concentration gradually approaches injected concentration. Thus, the 'S' shaped concentration profile is observed from the figure. For perfectly homogeneous reservoir, represented by $\beta = -100$, both first and second type boundary solutions become similar.

The dispersive mixing zones for a reservoir can be determined by plotting dimensionless concentration against distance. The mixing zone, Δx_D, is defined (Lake 1989)

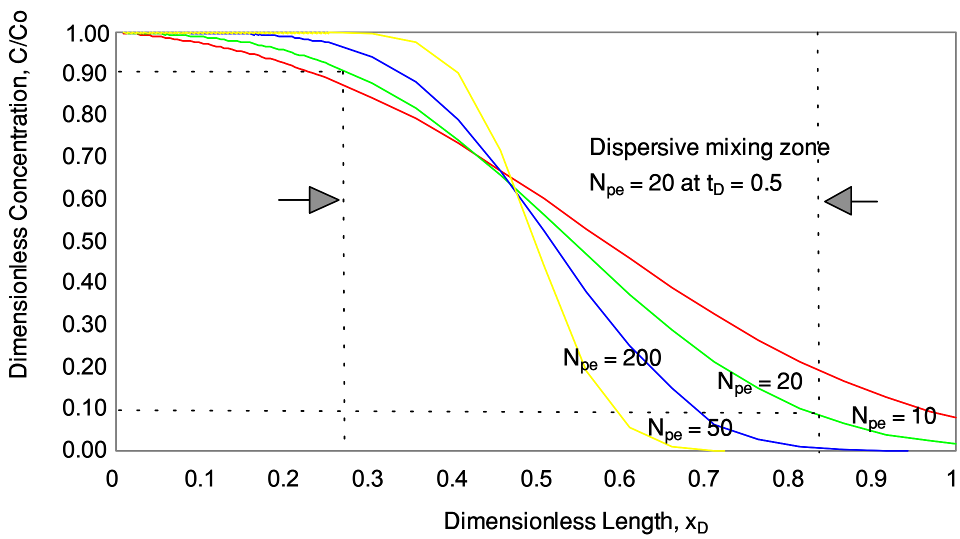


Figure 2 Dispersive mixing zone

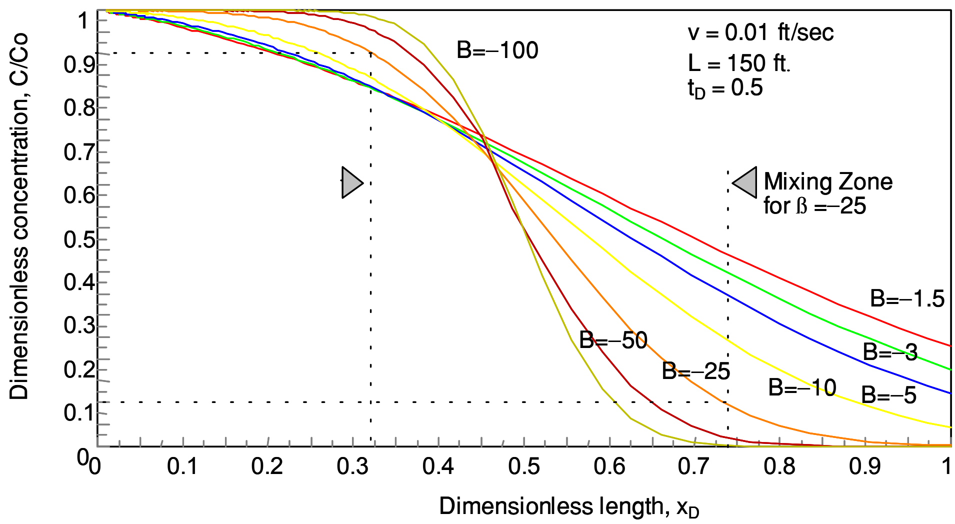


Figure 3 Dimensionless concentration against dimensionless length at different fractal exponent - First Type Boundary

as the distance between the distances where $C_D = 0.1$ and $C_D = 0.9$. This is shown in Figure 2 with inverted "S" shape profiles. The figure illustrates change in concentration profiles at a fixed dimensionless time of 0.5 for different values of Peclet number. The figure shows that the mixing zones grow with decrease in N_{pe}. The mixing zone becomes almost non-existent as N_{pe} approaches infinity.

Figure 3 is another illustration of concentration distribution against dimensionless distance at a fixed dimensionless time for different values of fractal exponent. At a given time, Δx_D increases as β increases. Δx_D appears to grow with increase in dispersion coefficient (as β is proportionally related to K). The mixing zone becomes almost non-existent as β approaches negative infinity. This indicates that as a reservoir becomes less heterogeneous, the dispersive mixing zone becomes smaller, eventually approaching a point where no mixing takes place. The results also demonstrate that the same value of dimensionless concentration and mixing zone can be obtained when extremely small fractal exponents or large Peclet numbers are reached.

Figure 4 shows the growth of mixing zone with increase in dimensionless time and dispersion coefficient at a fixed value of $\beta = -3$. The mixing zone size increases with K. The curve (thinnest line) represents K at $t_D = 0.15$ used to predict the mixing zone size at $t_D = 0.50$. The solid curves represent K that varies with time. The graph indicates that K can never be constant since the use of the same value of K can lead to under-estimation of the mixing zone size.

Figure 5 illustrates the relationship between mixing zone size and the average fluid velocity, ν, at fixed β, L, and t_D of –10, 150 ft, and 0.5 respectively. As n decreases, the size of mixing zone appears to increase. When the velocity is relatively slow, the time that the fluid takes to travel increases, giving more time for the fluid to interact and

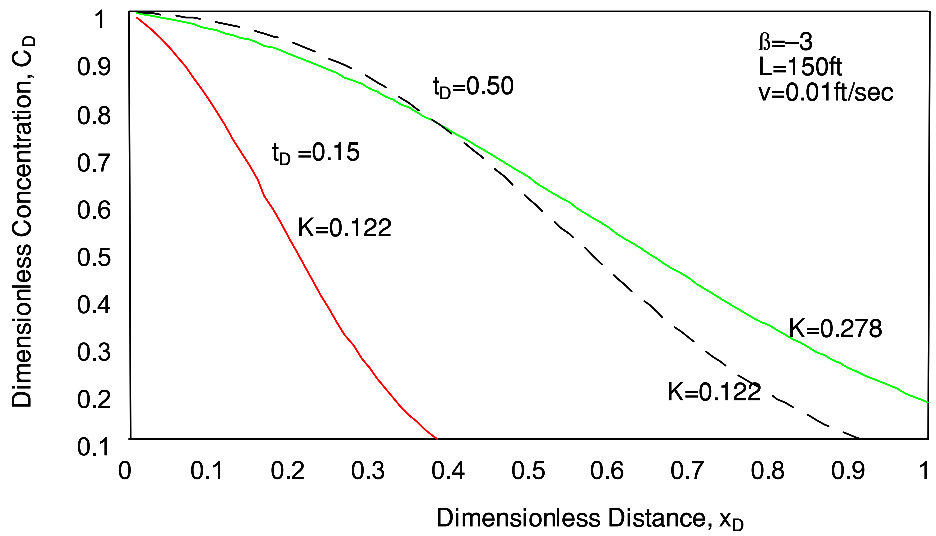


Figure 4 Dimensionless concentration as a function of dimensionless distance-First Type Boundary

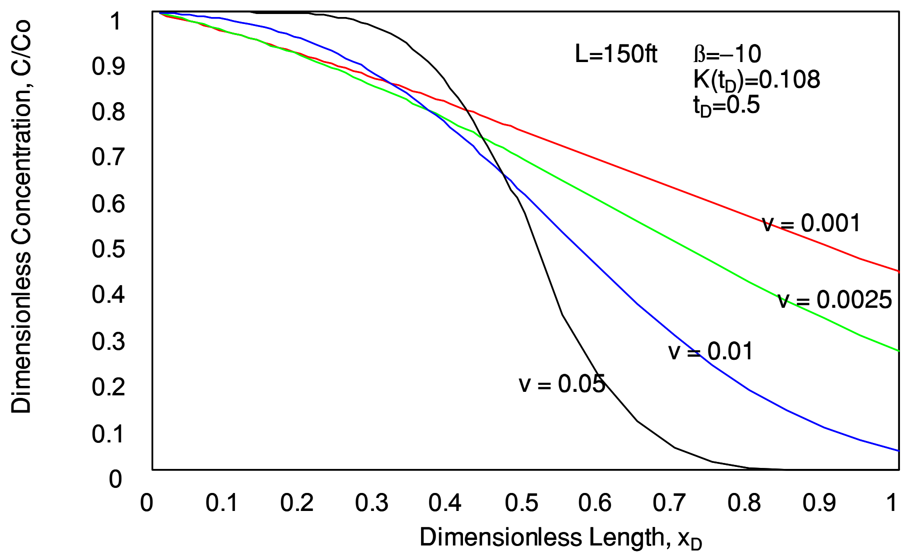


Figure 5 Dimensionless concentration as a function of dimensionless length at different average velocity - First Type Boundary

mix with the reservoir fluid. The fluid that flows relatively slow however, takes much longer time to reach its initial concentration at the producer as depicted in Figure 6.

When continuous injection is applied, the concentration profile for a reservoir at t_D = 0.5 is given in Figure 7. The plot for continuous injection (second boundary condition) differs from that for the first boundary condition in that the curves at relatively

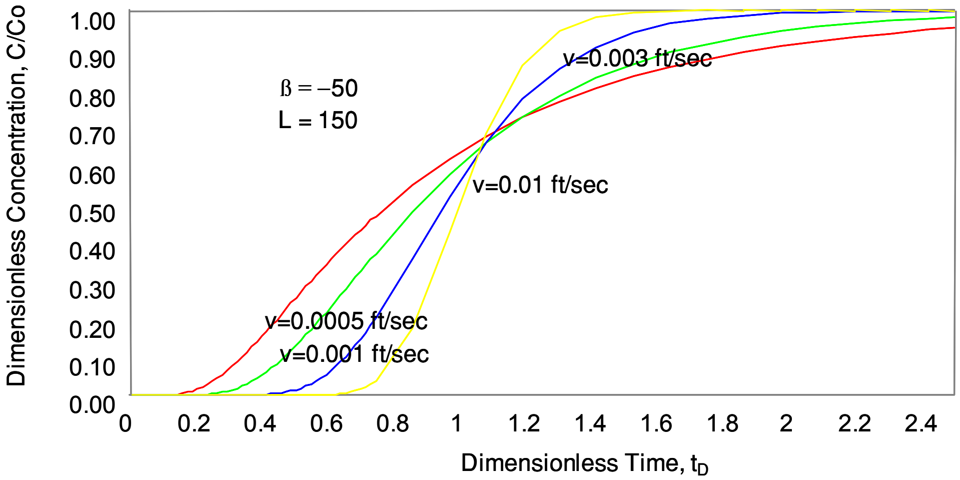


Figure 6 Dimensionless concentration against dimensionless time at different average velocity - First Type Boundary

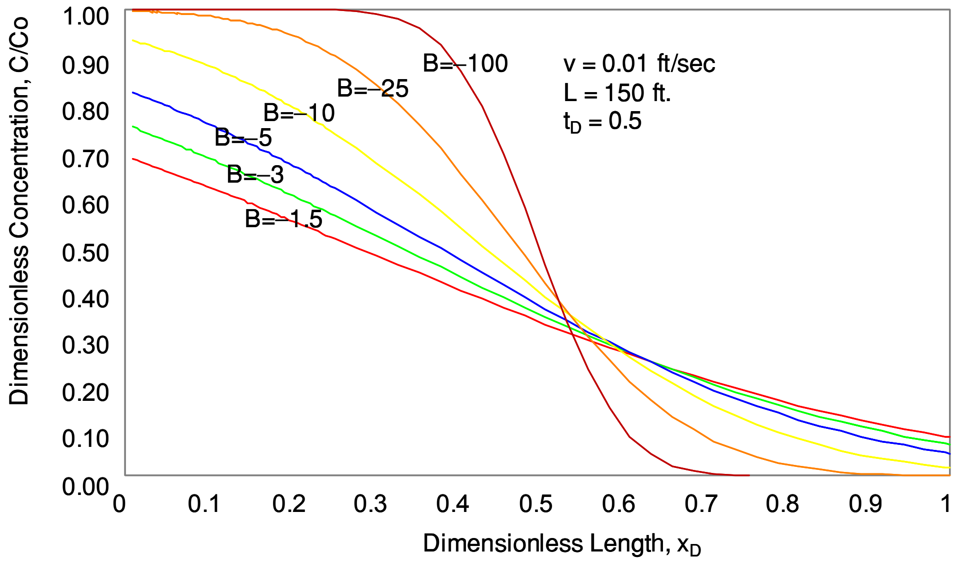


Figure 7 Dimensionless concentration against dimensionless length at different fractal exponent - Second Type Boundary

higher values of β do not start at the same dimensionless concentration. This is due to the mass of the injected solute that is equal to $Co.\phi_e.\nu.t$, represented by the area under the curves. The area under the curve for $\beta = -100$ should be the same as the area under the curve for $\beta = -1.5$. In order for this relationship to occur, the difference in curve exists in such a way that curve for $\beta = -100$ is higher but shorter than the curve for $\beta = -1.5$. Hence the difference in the starting points. This difference however is eliminated when b becomes relatively small.

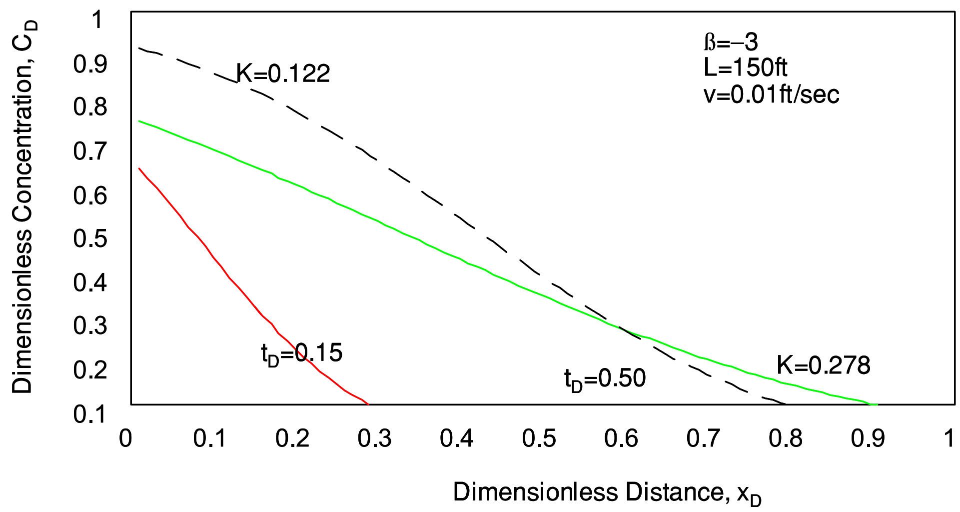


Figure 8 Dimensionless concentration as a function of dimensionless distance - Second Type Boundary

Figure 8 is attempted to explain the effect on mixing zone size in a heterogeneous reservoir during short flow when constant dispersion coefficient time is used instead. The graph depicts discrepancy that exists between constant dispersion coefficient and time dependent dispersion coefficient for the second boundary case. Similar to the first boundary solution, the use of constant dispersion coefficient leads to under-estimation of the mixing zone size.

Figure 9 illustrates the effect of fluid velocity on the size of dispersive mixing zone.

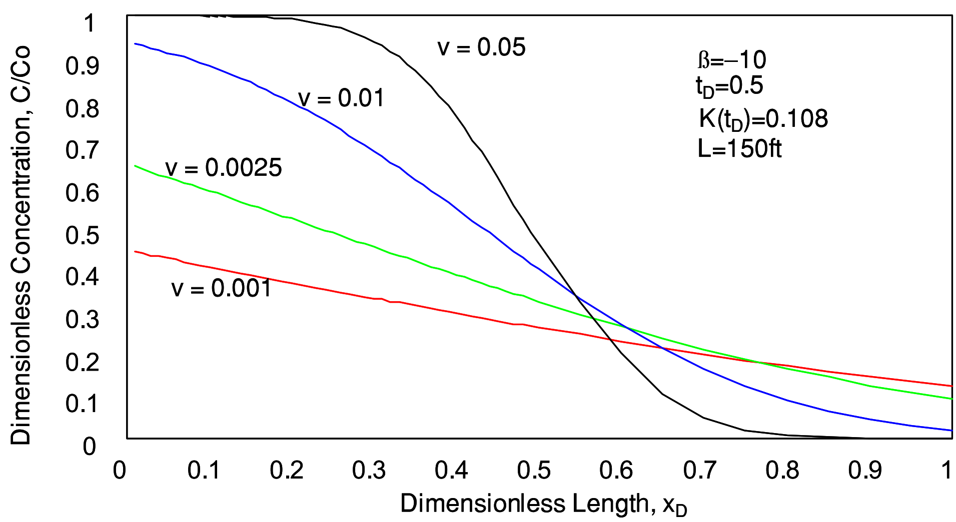


Figure 9 Dimensionless concentration as a function of dimensionless length at different values of velocity - Second Type Boundary

It clearly indicates that the size of mixing zone increases when v decreases. Lower fluid velocity causes more time for mixing to occur, hence, longer mixing zone.

3.0 CONCLUSION

Present knowledge of dispersion coefficient, K, is based on the assumption of reservoir homogeneity, which leads to a constant value of dispersion coefficient. This is due to the difficulty of characterizing complex heterogeneities with mathematical equations. It has been found that K is a function of time, and the heterogeneity of porous media can be characterized by a fractal exponent, β using a tracer test. It was discovered that as β becomes smaller (porous medium becomes increasingly heterogeneous), the size of the mixing zone increases. Another factor affecting Δx_D is time dependent dispersion coefficient, $K(t_D)$. In a heterogeneous reservoir, mixing zone increases with increasing value of dispersion coefficient at relatively short duration of flow. For relatively long period of flow, however, Δx_D continues to increase even though $K(t_D)$ remains constant. The third factor is average fluid velocity, v. Mixing zones have inverse relationship with fluid velocity in that Δx_D increases as v decreases.

Continuous injection of solvent at extremely long test time in order to obtain series of concentration ratio generally is not economical due to the amount of solvent required, operating hours involved and overhead costs that is incurred. Characterizing the reservoir and identifying the size of mixing zone by determining β at any flow time now become more practical. The use of the time dependent dispersion coefficient is practical since the fractal exponent obtained from the test can be used to predict the concentration profile and size of mixing zone at a longer time frame.

Efficient recovery of oil requires that reservoir factors such as β, L, and v be increased or decreased accordingly in order to achieve a desired mixing zone. Optimum value of v that justifies the economics and at the same time recovers maximum oil must be obtained. This can only be done if the heterogeneity of the reservoir or porous medium is known. Knowing the proper value of β enables the desired value of v be estimated.

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NOMENCLATURE

C = concentration
Co = initial concentration
K = dispersion coefficient
$K(t)$ = time dependent dispersion coefficient
L = characteristic length
ν_x = average linear flow velocity in a longitudinal direction
t = time
t_D = dimensionless flow time based on distance
β = fractal exponent
ϕ = porosity
ϕ_e = effective porosity
C_D = dimensionless concentration, C/Co
x_D = dimensionless distance, x/L
t_D = dimensionless time, $\nu.t/L$