

DESIGNING A CONTROL FAILURE SURVIVAL SYSTEM FOR HIGH SPEED TRANSPORT AIRCRAFT USING EIGENVALUE ASSIGNMENT METHOD

ZAIRIL A. ZALUDIN

Abstract. In the event of a control surface failure, the purpose of a reconfigurable flight control system is to redistribute and coordinate the control effort among the aircraft's remaining effective surfaces such that satisfactory flight performance is retained. A major task in control reconfiguration deals with adjusting the controller gains on-line or switching to a different control law to compensate for the failure. In this paper, the former option is considered. The design of a Control Failure Survival System (CFSS) for a hypersonic transport (HST) aircraft is presented. The method is based on eigenvalue assignment which was developed using Linear Quadratic Regulator theory. There are three control inputs available on board the HST; the change in the flaps deflection, the change in the propulsion diffuser area ratio and the change in the total temperature across combustor. Using the method discussed in this paper, the results showed that it was possible to reconfigure the flight control system such that the aircraft stability is regained when either a single or a combination of, control failures occurred simultaneously. In addition, the natural motion characteristics (i.e short period, phugoid and height motion) of the aircraft before the failure occurred are conserved and the transient response of the aircraft state variables after failure was almost the same as before failure occurred. An example is included in this paper using the mathematical model of the longitudinal motion of the HST.

Keywords: Aircraft dynamics; hypersonic flight; optimal control; eigenvalue assignment; LQR Theory

Abstrak. Jika kerosakan berlaku kepada permukaan kawalan penerbangan, tujuan "Sistem Pereka Bentuk Kawalan Penerbangan" ialah untuk membahagi dan menyelaraskan usaha kawalan antara permukaan-permukaan kawalan yang masih aktif untuk tujuan mengekalkan mutu penerbangan yang diinginkan. Tugas utama 'Sistem Pereka Bentuk Kawalan Penerbangan' adalah untuk menyelaraskan unit kawalan semasa kerosakan berlaku ataupun menukar unit kawalan kepada sistem kawalan yang lebih sesuai untuk tujuan memperbaiki kerosakan tersebut. Dalam kertas ini, cara yang kedua dipertimbangkan. Reka bentuk "Sistem Keselamatan Kegagalan Kawalan" untuk pesawat hipersonik dibentangkan. Cara tersebut adalah berdasarkan cara penetapan nilai eigen dan teori pengatur kuadratik linear. Terdapat tiga masukan kawalan ke pesawat tersebut. Jika cara yang dibentangkan di dalam kertas ini digunakan, keputusan analisis yang dibentangkan menunjukkan bahawa sistem kawalan penerbangan untuk pesawat ini boleh direka bentuk sehingga kestabilan pesawat tersebut dicapai semula apabila salah satu ataupun gabungan permukaan kawalan gagal berfungsi pada masa yang sama. Didapati juga gerakan tabii pesawat yang mengalami kerosakan dapat dibaik pulih seperti sebelum kerosakan berlaku. Satu contoh disertakan dalam kertas ini menggunakan model matematik pesawat hipersonik.

Kata kunci: dinamik penerbangan; penerbangan hipersonik; kawalan optimal; penetapan nilai eigen, Teori LQR

* Research Scientist, Department of Aerospace Engineering, Universiti Putra Malaysia, Serdang, Malaysia.

1.0 INTRODUCTION

Considerable attention is being given now in the USA, Europe and Japan to the development of hypersonic aircraft. Though it is some time before hypersonic transport aircraft (HST) will emerge, researchers are currently studying the technology required to tackle some of the biggest challenges in aviation history to produce such aircraft. It will have a capability to fly at a range of hypersonic speeds and at various heights. The big advantage of future HST aircraft, therefore, is their potential for reducing long-range flight times and thereby, increasing aircraft productivity, passenger comfort, and convenience.

Studies have shown that Hypersonic Transport Aircraft (HST), when it exists, will likely suffer some form of control problems [1-3]. Some of these control problems include high dynamic instability, unacceptable handling qualities, and possible structural/rigid body coupled motion. In the event of *any* control surface malfunction, it has been shown that the instability effect on the aircraft dynamics is rapid [4]. To avoid total loss of the aircraft, the onboard flight control system, which will be designed for the aircraft, must be able to at least regain aircraft stability using other control surfaces which are still active, so that the aircraft can be safely landed at the nearest airport.

When using the control reconfiguration method proposed by McLean and Aslam-Mir [5], the optimal feedback gain matrix was reconfigured based on the Minimization Principle [6]. A 'control distribution matrix' was determined which was used to distribute the feedback gain matrix originally obtained when all three control inputs were functioning. As a result, a new reconfigured feedback gain matrix was produced and then used for the aircraft dynamics with a control failure. Using this method the Hyperion dynamic stability could be recovered *only* when the flaps (δ_F) or the ratio of the engine diffuser control (A_D) failed [4]. This is not satisfactory for *Hyperion* because in the event of more than one control fail simultaneously, the loss of the aircraft could not be avoided.

A major task in control reconfiguration deals with adjusting the controller gains online or switching to a different control law to compensate for the failure. This is discussed extensively in [7]. In this paper, a method is proposed which allows those appropriate controller gains to be derived for different combinations of control failures such that the aircraft with the control failure regains dynamic stability and also retains the dynamic characteristics of the aircraft before the failure occurs. The control failure, in this paper, is assumed to be a total loss in control input signal.

2.0 THE AIRCRAFT DYNAMICS

For convenience, the mathematical model defining the dynamics of that aircraft is referred to as *Hyperion* in this paper. The dynamic responses of this aircraft which when subjected to some command input or a disturbance have been digitally simulated and discussed in [4,8,9]. The mathematical model of the aircraft is briefly discussed next.

The equations of longitudinal motion of *Hyperion* were in the form of a single state-space equation,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (1)$$

and an output equation,

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad (2)$$

where \mathbf{x} is the state vector, \mathbf{y} is the output vector, \mathbf{u} is the control vector, \mathbf{A} is the state coefficient matrix, \mathbf{B} is the control driving matrix, and \mathbf{C} is the output matrix. The longitudinal motion of the aircraft involves 7 state variables and 3 control inputs. These variables are shown next.

$$\mathbf{x}' = [\Delta u \quad \Delta \alpha \quad \Delta q \quad \Delta \theta \quad \Delta h \quad \Delta \eta \quad \Delta \dot{\eta}] \quad (3)$$

$$\mathbf{u}' = [\Delta \delta_F \quad \Delta A_D \quad \Delta T_o] \quad (4)$$

u is the forward speed (ft/s), α is the angle of attack (rad), q is the pitch rate (rad/s), θ is the pitch attitude (rad), h is the aircraft altitude (ft), η is the generalised elastic co-ordinate (rad), $\dot{\eta}$ is the rate of change of generalised elastic co-ordinate (rad/s), δ_F is the flap deflection (rad), A_D is the propulsion diffuser area ratio and T_o is the total temperature across combustor ($^{\circ}\text{R}$). Δ denotes a perturbation of a variable from the trim condition.

For the work described in this paper, *Hyperion* was simulated to be flying at Mach 8.0 and at a height of 85 000 ft. The state coefficient matrix, \mathbf{A} , and the control matrix, \mathbf{B} , for the aircraft when it is flying at the stated speed and height are given in Appendix 1. The aircraft was found to be dynamically unstable when no Stability Augmentation System (SAS) was considered [4]. A design based on the Linear Quadratic Regulator (LQR) theory can perform this stabilizing function.

3.0 CHARACTERISTICS OF THE CONTROL FAILURE SURVIVAL SYSTEM FOR HYPERION

The Control Failure Survival System (CFSS) for *Hyperion* should at least regain the aircraft dynamic stability for *any* possible combinations of control failure, with an exception of all controls simultaneously fail. Also, the dynamic characteristics of the aircraft with the control failure but with the CFSS incorporated should retain the dynamic responses which was displayed *prior* to the failure of the controls. The design of the CFSS that delivers these requirements is discussed next. The method is based on the published work by Luo and Lan [10].

3.1 The Method

The usual method of using the optimal LQR theory is to choose the matrices \mathbf{Q} and \mathbf{G} to minimize the performance index, J , defined as [6].

$$J = \frac{1}{2} \int_0^{\infty} (\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{G} \mathbf{u}) dt \quad (5)$$

which results in an optimal feedback gain \mathbf{K} , where:

$$\mathbf{K} = \mathbf{G}^{-1} \mathbf{B}^T \mathbf{P} \quad (6)$$

\mathbf{P} is the solution of the Riccati equation Eq. (7).

$$\mathbf{P} \mathbf{A} + \mathbf{A}^T \mathbf{P} - \mathbf{P} \mathbf{B} \mathbf{G}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = 0 \quad (7)$$

The optimal feedback control is:

$$\mathbf{u}^o = -\mathbf{K} \mathbf{x} \quad (8)$$

It has been published in much of the literature on the subject of LQR theory that the state-weighting matrix \mathbf{Q} must be at least positive semi-definite, the matrix control-weighting matrix \mathbf{G} must be positive definite and the aircraft must be controllable. Both \mathbf{Q} and \mathbf{G} can then be chosen by the designer to obtain the feedback gain matrix, \mathbf{K} . LQR theory guarantees the closed-loop stability of the aircraft, but the process of finding the feedback gain matrix, \mathbf{K} which will cause the aircraft to display some desired flying characteristics is usually an iterative process and requires some experience. The work usually involves changing the matrices \mathbf{Q} and \mathbf{G} until the correct feedback matrix, \mathbf{K} , is obtained that causes the response of the aircraft to satisfy some flying qualities published by the appropriate aviation authority.

However, using the method proposed by Luo and Lan [10], an unique state-weighting matrix, \mathbf{Q} can be calculated to obtain the feedback gain matrix, \mathbf{K} such that the aircraft will display some natural motion as specified by the designer. This is done through specifying the desired closed-loop eigenvalues of the aircraft. The method is shown below for convenience.

The solution of the LQR problem involves an Hamiltonian function

$$\mathbf{H} = \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{G} \mathbf{u} + \boldsymbol{\lambda}_x^T (\mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}) \quad (9)$$

where $\boldsymbol{\lambda}_x$ is the vector of Lagrangian multipliers. The solution to the LQR problem can be obtained by solving the following equations:

$$\dot{\boldsymbol{\lambda}}_x = -\frac{\partial \mathbf{H}}{\partial \mathbf{x}} = -\mathbf{A}^T \boldsymbol{\lambda}_x - \mathbf{Q} \mathbf{x}; \quad \boldsymbol{\lambda}_x(\infty) = 0 \quad (10)$$

$$\dot{\mathbf{x}} = \frac{\partial \mathbf{H}}{\partial \boldsymbol{\lambda}_x} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}; \quad \mathbf{x}(0) = \mathbf{x}_o \quad (11)$$

$$\frac{\partial \mathbf{H}}{\partial \mathbf{u}} = \mathbf{G}\mathbf{u} + \mathbf{B}^T \boldsymbol{\lambda}_x = 0 \quad (12)$$

These equations can be written as

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\boldsymbol{\lambda}}_x \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{G}^{-1}\mathbf{B}^T \\ -\mathbf{Q} & -\mathbf{A}^T \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\lambda}_x \end{bmatrix} = \bar{\mathbf{A}} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\lambda}_x \end{bmatrix} \quad (13)$$

where $\bar{\mathbf{A}}$ is a $(2n \times 2n)$ matrix with n of its $2n$ eigenvalues being the eigenvalues of the closed-loop system that satisfy Eq. (14).

$$\det[\sigma_i \mathbf{I} - \bar{\mathbf{A}}] = 0 \quad \text{where } i = 1, 2, 3 \dots n. \quad (14)$$

σ_i denotes the designer specified closed-loop eigenvalues. Using Bryson's theory [11], the positive definite control-weighting matrix, \mathbf{G} , can be chosen to have a diagonal form having elements given by

$$\mathbf{G}_{kk} = \frac{1}{u_{k,max}^2}, \quad \text{where } \mathbf{G}_{kj} = 0 \text{ and } k \neq j \text{ and } k = 1, 2 \dots m. \quad (15)$$

This choice penalises independently each of the control input $u_1, u_2 \dots u_m$. The values $u_{1,max}, u_{2,max}, \dots u_{m,max}$ represent the maximum limits of each of the control input. The weighting matrix, \mathbf{Q} , is also assumed to have a diagonal form, with its elements given by

$$Q_{ii} = q_i \quad \text{where } Q_{ij} = 0, i \neq j \quad (16)$$

Eq. (17) can be used to determine the n elements, q_i , of the weighting matrix when all the closed-loop eigenvalues are specified. For a specified eigenvalue, $\sigma_1 = \mu_1 + j\omega_1$, Eq. (17) provides one equation for q_i . Hence,

$$f(q_1, q_2 \dots q_n) = \det[(\mu_1 + j\omega_1)\mathbf{I} - \bar{\mathbf{A}}] = 0 \quad (17)$$

As a result, n algebraic equations can be obtained and solved for the unknown elements of the \mathbf{Q} matrix. With the resulting weighting matrices, the Riccati equation can be solved and the optimal feedback control law can be obtained using Eqs. (7) and (6) respectively.

3.2 An Example

In the example shown below, the effectiveness of using the eigenvalue assignment method based on the LQR theory to design a CFSS is demonstrated. The controlled *Hyperion* is required to exhibit a closed-loop response corresponding to those desired closed-loop eigenvalues shown in Table 1.

Table 1 Specified closed-loop eigenvalues for *Hyperion*

Desired Eigenvalues	Natural Frequency (rad/s)	Damping Ratio	Motion Represented
$\sigma_{1,2} = -5 \pm j18$	18.7	0.268	Structural Bending
$\sigma_{3,4} = -40 \pm j12$	41.8	0.958	Short Period
$\sigma_{5,6} = -0.04 \pm j0.012$	0.0418	0.958	Phugoid
$\sigma_7 = -10$	Nil	Nil	Height

When all three controls δ_F , A_D and T_o are functioning properly, the algorithm proposed in [10] is used to generate the unique matrix, \mathbf{Q} , required for the Performance Index, J , to be minimized. The control-weighting matrix chosen was

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

The state-weighting matrix, \mathbf{Q} , obtained from the design, is

$$\mathbf{Q} = \text{diag}[3.5 \times 10^{-2} \quad -2.6 \times 10^4 \quad -1.5 \times 10^2 \\ -6.6 \times 10^3 \quad 8.4 \times 10^{-7} \quad 3.8 \times 10^6 \quad 2.5 \times 10^3] \quad (19)$$

Using this matrix, the optimal feedback gain matrix was obtained as

$$\mathbf{K} = \begin{bmatrix} 1.8 \times 10^{-1} & -1.3 \times 10^2 & -1.6 \times 10^1 & -3.6 \times 10^1 & 7.5 \times 10^{-4} & 4.7 \times 10^2 & -2.0 \times 10^1 \\ -2.0 \times 10^{-2} & 2.0 \times 10^2 & -9.2 \times 10^{-1} & -2.5 \times 10^2 & -6.0 \times 10^{-4} & -1.6 \times 10^3 & -7.6 \times 10^1 \\ 2.5 \times 10^{-5} & 3.4 \times 10^{-2} & -1.7 \times 10^{-3} & 4.7 \times 10^{-7} & 2.5 \times 10^{-7} & 3.6 \times 10^{-2} & 2.3 \times 10^{-4} \end{bmatrix} \quad (20)$$

When the closed-loop eigenvalues of the system are calculated from

$$\det[\zeta \mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{K})] = 0 \quad (21)$$

the eigenvalues were found to be identical to those specified in Table 1. The block diagram of the aircraft dynamics and its Stability Augmentation System (SAS) is shown in Figure 1.

If it is supposed that a sudden lost in the input signal of the flap occurs, then, from an earlier study [4], it was established that aircraft stability was lost immediately. But, if it is supposed that it is possible to recover the aircraft stability by using a new opti-

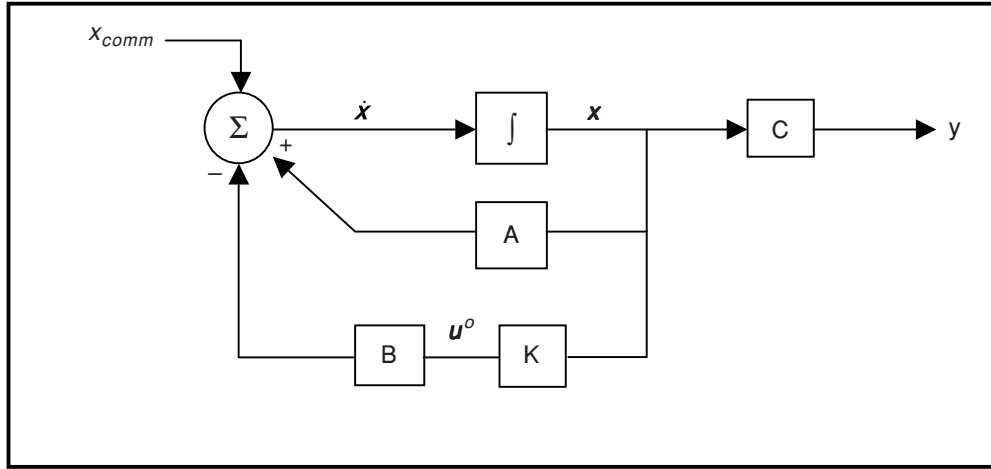


Figure 1 Aircraft Mathematical Model with a SAS

mal feedback gain matrix, which also causes the aircraft with the flaps failure to have the same dynamic response, but involving the use of only A_D and T_o , the question would be how can such a matrix be found? Using the same algorithm in [10], the first column of the matrix, \mathbf{B} , is now set to zero implying that the control, δ_F , has completely failed. This is illustrated as

$$\mathbf{B}_{flap} = \begin{bmatrix} 0 & -1.7159 \times 10^2 & 1.3329 \times 10^{-2} \\ 0 & 4.7726 \times 10^{-3} & -1.6720 \times 10^{-7} \\ 0 & -8.2859 \times 10^{-1} & 6.9090 \times 10^{-5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -9.8249 \times 10^{-1} & 3.4421 \times 10^{-5} \end{bmatrix} \quad (22)$$

A new state-weighting matrix is produced, viz.

$$\mathbf{Q}_{flap} = \text{diag}[7.4 \times 10^2 \quad -1.8 \times 10^{11} \quad 2.3 \times 10^7 \quad 1.8 \times 10^{11} \\ -1.9 \quad 4.9 \times 10^8 \quad -1.8 \times 10^6] \quad (23)$$

As a result of using this matrix, \mathbf{Q}_{flap} , a new optimal feedback gain matrix, \mathbf{K}_{flap} , is found. Note that the 1st row of the matrix, \mathbf{K}_{flap} , is zero confirming that the flaps are not used as one of the active controls, viz.

$$\mathbf{K}_{flap} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 15.0 & 2.5 \times 10^5 & -1.7 \times 10^3 & -2.5 \times 10^5 & -8.0 \times 10^{-1} & 3.1 \times 10^2 & 1.2 \times 10^1 \\ -33.0 & -2.9 \times 10^3 & 6.5 \times 10^3 & 3.3 \times 10^3 & 1.6 & 5.2 \times 10^2 & 1.8 \times 10^2 \end{bmatrix} \quad (24)$$

When the closed-loop eigenvalues of this new system were calculated, they were almost identical to those specified as shown in Table 2.

Table 2 Closed-loop eigenvalues of the aircraft with flaps failure but with CFSS incorporated

Closed-Loop Eigenvalues	Natural Frequency (rad/s)	Damping Ratio	Motion Represented
$\zeta_{1,2} = -4.9996 \pm j17.999$	18.6805	0.2676	Structural Bending
$\zeta_{3,4} = -39.9978 \pm j11.9970$	41.7583	0.9578	Short Period
$\zeta_{5,6} = -0.0399 + j0.0122$	0.04172	0.956	Phugoid
$\zeta_7 = -10.0130$	Nil	Nil	Height

The block diagram in Figure 2 can be used to illustrate the application of this technique. When all controls are functioning properly, switch S connects point m with n. But when the control input for flaps occurs, switch S connects point m with p. For

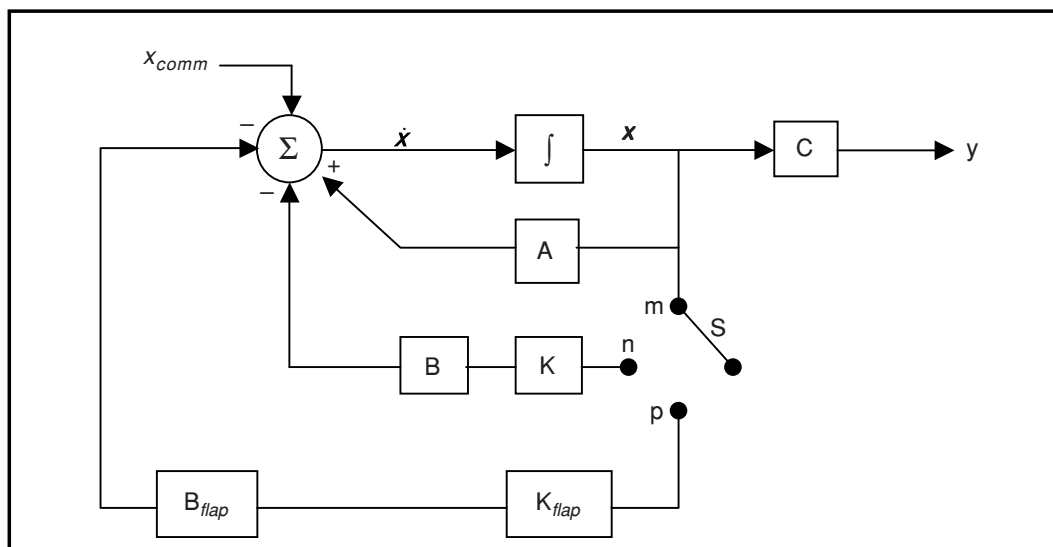


Figure 2 Control Failure Survival System

hypersonic aircraft, it is likely that the control surfaces will use electromechanical actuators [7]. Electromechanical actuators (EMAs) are capable of failure detection and identification within 120 – 160 ms. This capability provides information which control surface has malfunction. When this failure is detected, the CFSS will use the new feedback gain matrix Eq. (24) found from the Luo and Lan algorithm [10].

Using the technique discussed above, it was found that the aircraft dynamic stability could be recovered for all combination of control input failure (except when every control simultaneously failed).

Some responses of *Hyperion* with a flap failure are shown next. The aircraft was subjected to a commanded change in height of 1000 ft. When the aircraft flaps completely failed, the response of the aircraft when using the new feedback gain matrix, \mathbf{K}_{flap} , is determined. Using the same commanded change in height input, the steady-state change in height is only 900 ft (see Figure 3) and a slight increase in steady-state forward speed is observed (see Figure 4).

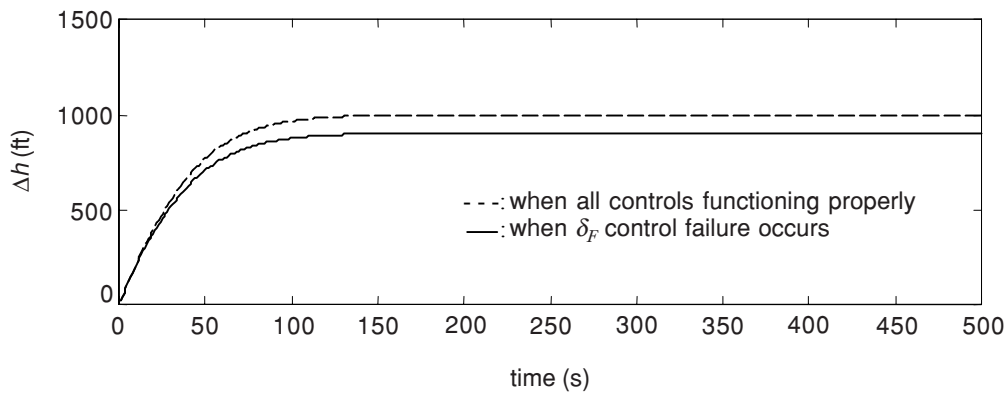


Figure 3 Height Response when δ_F fail

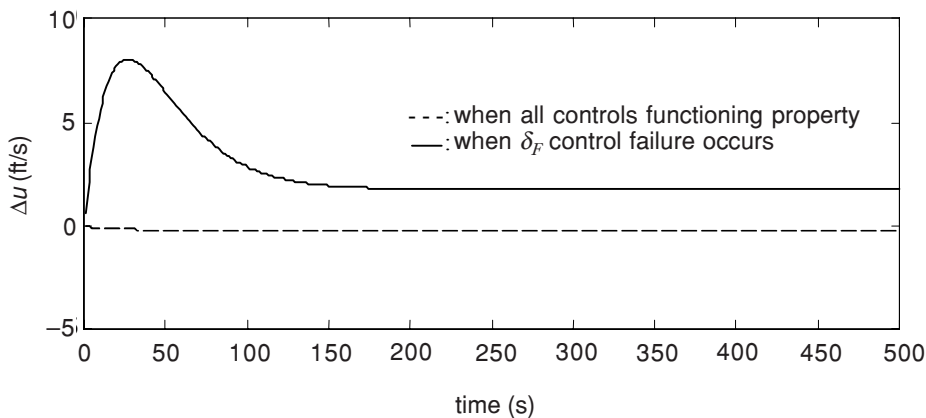


Figure 4 Forward Speed when δ_F fail

There appears to be an increase in the angle of attack, α , as shown in Figure 5. However, when the acceleration of the aircraft at centre of gravity was evaluated, it was found that the increase was small (see Figure 6). These results suggest that the passengers flying in the aircraft may feel very little discomfort when the failure occurs and the CFSS regains aircraft stability.

Figures 7 and 8 show the responses of A_D and T_o respectively. Since the flaps are no longer functioning, the expected increase in activity for A_D and T_o is observed. The root mean square (rms) value for A_D now is 0.1 compared to 0.003 previously (when no flap failure occurred) and for T_o , 1326 compared to 1.85×10^{-4} before failure.

It is interesting to note that the temperature across the engine combustor seem to decrease by 1490°R . It is worth mentioning that the ambient temperature for the engine for *Hyperion* flying at the stated flight condition is 2000°R [4]. Hence, from this simulation, the final temperature across the engine combustor is 510°R .

Suppose that both the δ_F and A_D simultaneously fail. From [4], it was determined that the aircraft will lose its dynamic stability. Using the method described in this paper, it was possible to find the feedback gain matrix that will regain the aircraft stability and still display the closed-loop eigenvalues shown in Table 1. The responses of the aircraft when δ_F and A_D fail are discussed next. When both controls fail, the aircraft lost height by approximately 700 ft (see Figure 9). The response of the aircraft pitch attitude also showed a small decrease of 4.5×10^{-3} rad (see Figure 10). But note that the aircraft still regained stability, consistent with the stated requirement.

And finally, Figure 11 shows a sudden increase in the temperature across the engine combustor which peaked to 240°R at $t = 3$ second, to compensate the loss in δ_F and A_D controls. However, the increase quickly reduced and reached a steady state of -5°R .

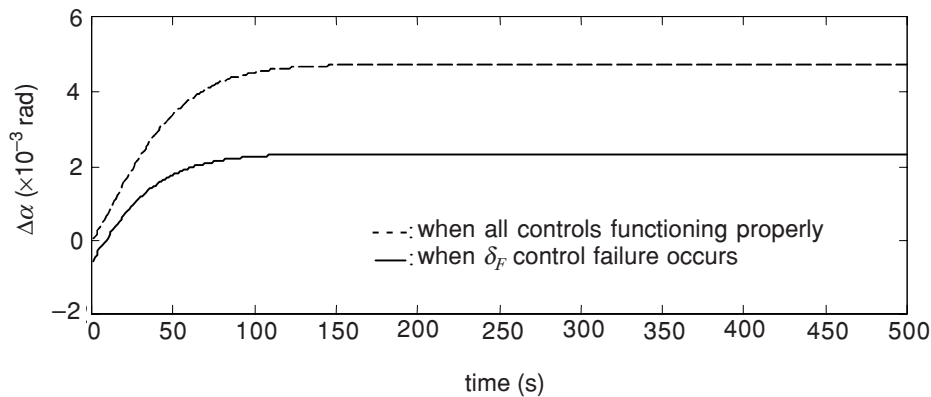


Figure 5 Angle of Attack when δ_F fail

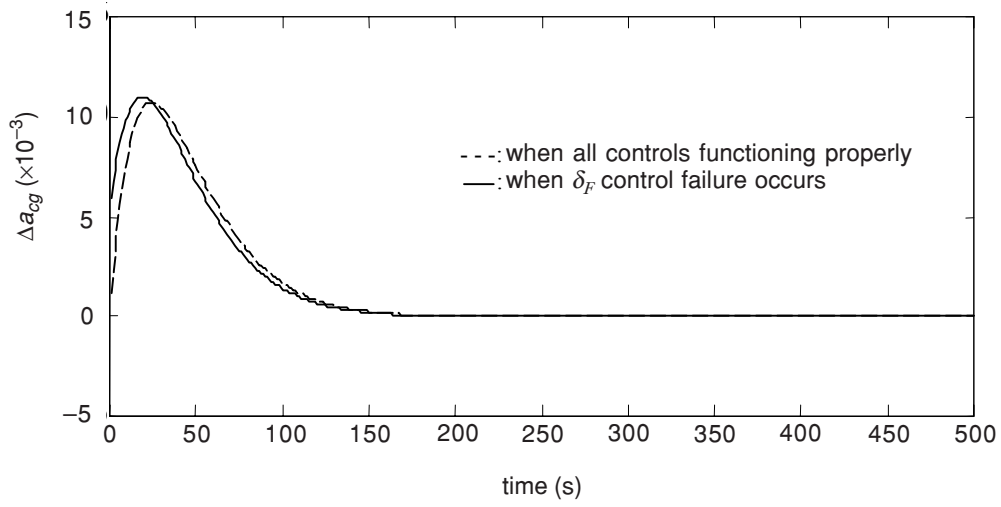


Figure 6 Normal Acceleration Response when δ_F fail

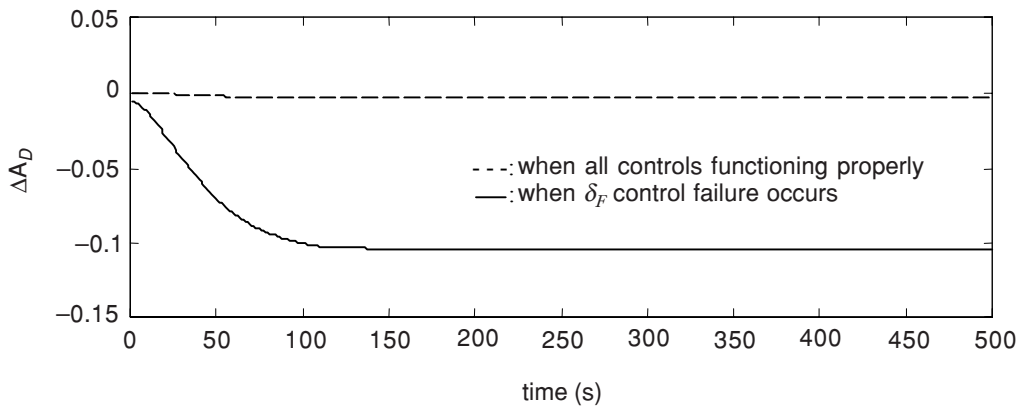


Figure 7 Diffuser Area Ratio Response when δ_F fail

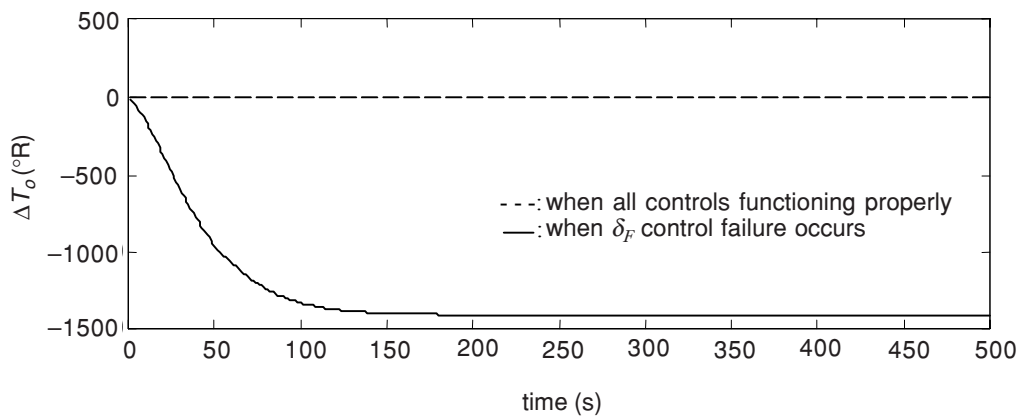


Figure 8 Temperature Across Combustor Control Response when δ_F fail

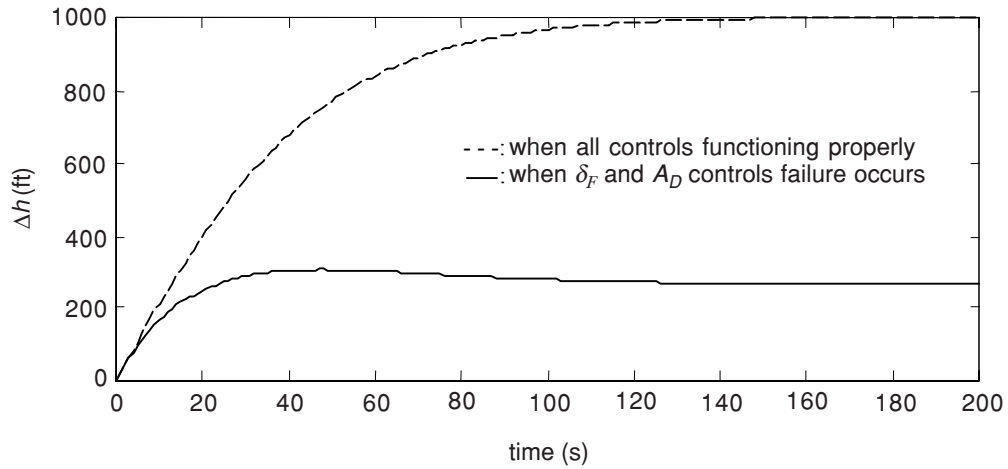


Figure 9 Height Response when both δ_F and A_D fail

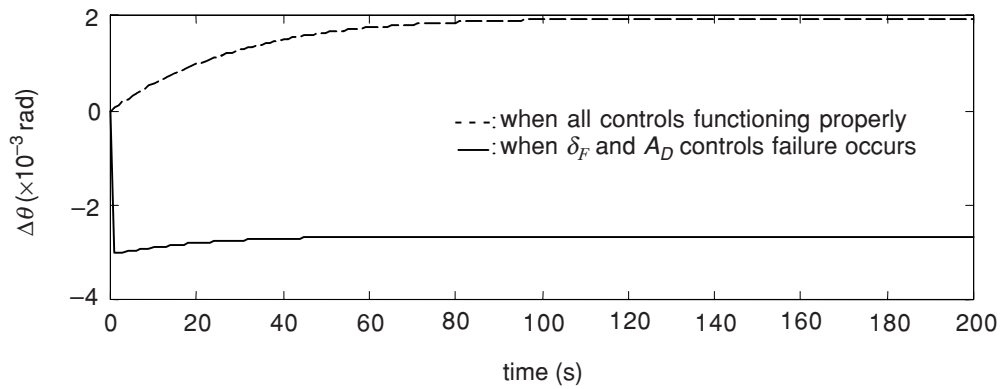


Figure 10 Pitch Attitude Response when both δ_F and A_D fail

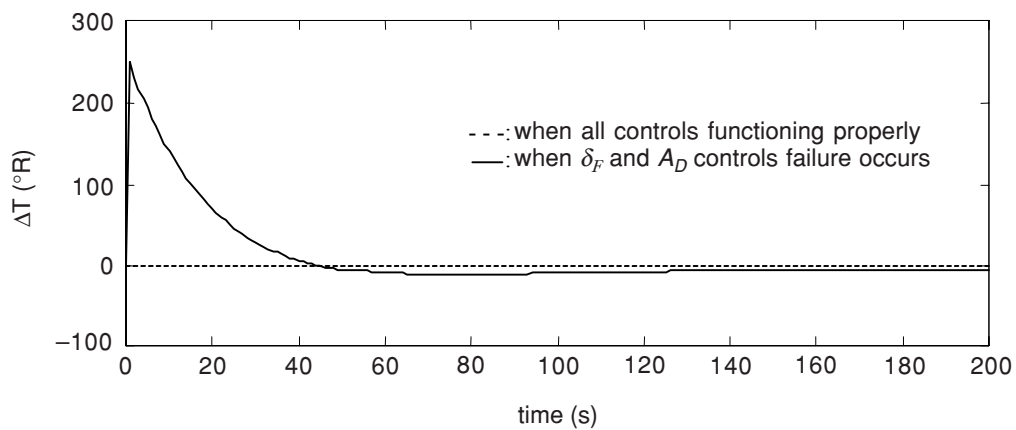


Figure 11 Temperature Across Engine Combustor Response when δ_F and A_D fail

4.0 CONCLUSIONS

The effectiveness of using the eigenvalue assignment method based on the LQR theory for designing CFSS for a Hypersonic Transport Aircraft has been shown here when compared to other methods such as the one proposed in [5]. Using the design, the aircraft dynamic stability and responses can be recovered when either a single or a combination of control failure occurs simultaneously.

5.0 ACKNOWLEDGEMENTS

The author acknowledges the generous financial support of Universiti Putra Malaysia for the researches work in which this paper was based.

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APPENDIX 1

The coefficient matrices A and B for *Hyperion* flying at speed of Mach 8 and at a height of 85000 ft are:

A =

$$\begin{bmatrix} -4.1857 \times 10^{-3} & -3.5030 \times 10^1 & 4.2686 \times 10^{-1} & -3.2200 \times 10^1 & 7.9938 \times 10^{-4} & 1.8614 \times 10^1 & 4.3006 \times 10^{-1} \\ -2.3158 \times 10^{-6} & -5.8716 \times 10^{-2} & 1.0002 & 0 & 4.4227 \times 10^{-7} & -3.9534 \times 10^{-2} & 2.1974 \times 10^{-4} \\ -9.4647 \times 10^{-6} & 4.3430 & -5.7885 \times 10^{-2} & 0 & 1.8076 \times 10^{-6} & 7.2990 & -5.2846 \times 10^{-2} \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & -7.8487 \times 10^3 & 0 & 7.8487 \times 10^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \\ 1.4938 \times 10^{-3} & 5.4953 \times 10^1 & -4.1812 \times 10^{-1} & 0 & -2.8529 \times 10^{-4} & -2.6905 \times 10^2 & -1.1340 \end{bmatrix}$$

B =

$$\begin{bmatrix} -1.1359 \times 10^2 & -1.7159 \times 10^2 & 1.3329 \times 10^{-2} \\ -1.4513 \times 10^{-2} & 4.7726 \times 10^{-3} & -1.6720 \times 10^{-7} \\ -2.3511 & -8.2859 \times 10^{-1} & 6.9090 \times 10^{-5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -9.8249 \times 10^{-1} & 3.4421 \times 10^{-5} \end{bmatrix}$$