FORCE APPROPRIATION METHOD FOR TWO DEGREES OF FREEDOM NONLINEAR SYSTEM

Mohd Shahrir Mohd Sani[a]*, Muhamad Norhisham Abdul Rani[b], Mohd Azmi Yunus[b], Ahmad Azlan Mat Isa[b], Liyana Roslan[b]

[a]Advanced Structural Integrity of Vibration Research (ASIVR), Faculty of Mechanical Engineering, University Malaysia Pahang, 26600 Pekan, Pahang, Malaysia
[b]Dynamics and Control Laboratory, Faculty of Mechanical Engineering, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia

Graphical abstract
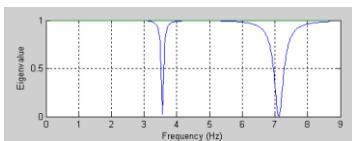
Abstract

In this paper, the vibrations of structures are investigated via force appropriation method in which modes are excited individually by forces. Multivariate mode indicator function (MMIF) is used in the modal testing for investigating the prospective nonlinearities in the structures. The method is applied to simulate two degree of freedom with simple nonlinearities. The results are presented and evaluated to indicate that the method has advantages when it comes to involving in mode by mode identification. Results calculated from modal assurance criteria (MAC) and modal purity indicator (MPI) show that the qualities of evaluating a pure normal mode are in satisfactory. This suggests that force appropriation method for nonlinear structures is reliable and efficient, particularly in terms of the isolation of individual modes and determination of high quality modal parameters.

Keywords: Force appropriation, nonlinear, vibration, multivariate mode indicator function

1.0 INTRODUCTION

The identification process for linear structural system has been widely explored and has become theoretically mature. In real practice, most structures behave some nonlinear characteristic [1-3]. Therefore, identification of non-linear structure through detection, localisation and estimation is a necessity for engineering structures [4-5]. However, the process of model identification for non-linear systems is very challenging and not as well developed as linear systems.

The force appropriation method or the phase resonance approach is used to determine the multi-point force vector that will induce single-mode behaviour, thus allowing each normal mode to be identified in isolation. Based on a nonlinear extension of phase resonance testing, the proposed methodology excites the structure to isolate a single nonlinear normal modes during the experiments [6]. The force appropriation method or the phase resonance approach is used to determine the multi-point force vectors that will induce single-mode behaviour, thus allowing each normal mode to be identified in isolation. The method can be classified into two categories, direct or iterative approaches. Iterative methods are complicated to apply because of the long computing time and convergence problems [7]. Therefore, most researchers have only considered the direct force appropriation method to extract eigenvalues and eigenvectors of structures.

This paper presents a method for the identification of two degrees of freedom non-linear system using force appropriation method to allow mode by mode identification.

2.0 THEORY OF FORCE APPROPRIATION METHOD

Force appropriation method is a technique of exciting a single mode of structure by applying multiple forces. This method involves the analysis of the Frequency Response Function (FRF) matrix in order to obtain a set of multiple exciter force patterns and un-damped natural frequencies. Normal modes of the system can be excited in isolation. In the steady state, the complex displacement or amplitude, **x** and response at the r transducer measurement positions on the structure is given by

$$x = [A(\omega) + iB(\omega)]f \qquad \text{(Eq. 1)}$$

where **f** is the mono-phase force vector applied at e excitation points, while **A** and **B** are the real and imaginary parts of FRF matrix at frequency ω. In order to excite a normal or pure mode, the response in mono-phase and excitation must be in quadrature (90° phase). At this condition, the real part of the response will be zero while the imaginary part corresponds to the undamped normal-mode shape, $\emptyset_j$. Therefore, for undamped normal mode;

$$\mathrm{Re}(x) = Af_j = 0 \qquad \text{(Eq 2)}$$

$$\mathrm{Im}(x) = Bf_j = \phi_j \qquad \text{(Eq 3)}$$

This occurs at each natural frequency of the structure. The force vector derived for a corresponding normal mode will excite only that mode. The force vector of proportionality damped systems is derived from a modal force input to mode of interest and no contribution to the other modes. However, for non-proportionally damped system, modal force contributions are included for any coupled mode in order to cancel the unnecessary modal responses because of the modal cross-damping terms. For rectangular FRF matrices, where number of responses more than number of excitation points ($m > e$), an exact solution is not obtainable. In practice the real part of the response can be minimised across all of the response measurements, with different specific cost functions being used in each of the methods. Juang and Wright [8] stated that this method used eigen-properties but different expressions. Ibanez [9] used the extended Asher method to minimise the sum of the squares of the real part of the response with respect to the force vector, leading to the eigenvalue problem. In this formulation, author stated that the eigenvalues λ dropped to zero at undamped natural frequencies. In addition, zero eigenvalues will only be produced if a quadrature response is recognizing on all r responses simultaneously. Instead, minima in the Extended Asher eigenvalue trace should be sought. Williams *et al*. [10] employed the MMIF to minimise the ratio of the sum of the squares of the real part of the response to the sum of the squares of the moduli, as defined as;

$$A^T A f = \lambda (A^T A + B^T B) f \qquad \text{(Eq. 4)}$$

An estimated mass matrix or called as weighting matrix may also be included in the MMIF formulation. Minima of eigenvalues λ can be identified through undamped natural frequencies. Modal purity indicator (MPI) is used for evaluating numerically the degree to which of normal mode has been appropriated [11]. MPI is defined as:

$$MPI = 1 - \frac{\sum_1^{no} |Y_R(x)||Y(x)|}{\sum_1^{no} |Y(x)|^2} \qquad \text{(Eq. 5)}$$

where $Y(x)$ is the response of a system at the xth measurement position, $Y_R(x)$ is the real part of response $Y(x)$ and the excitation of the system is purely real. If the MPI equal to 1, that is indicates a perfectly appropriated normal mode such that all points on the mode shape will be in quadrature. Naylor [12] stated that if a value of MPI ≥ 0.9 is considered good, MPI ≥ 0.95 is very good and MPI ≥ 0.99 to be an excellent result.
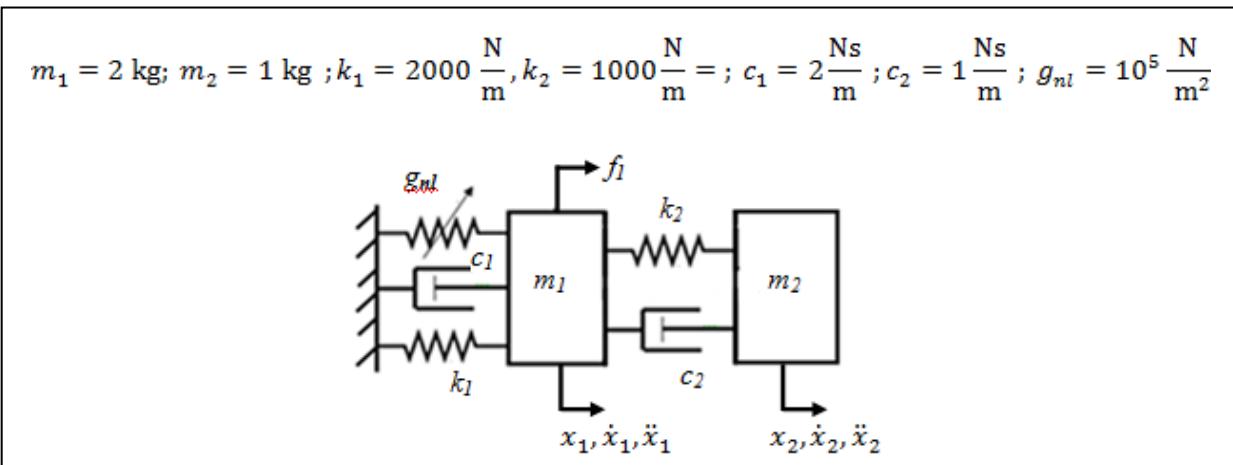
3.0 RESULTS AND DISCUSSION

The force appropriation and MMIF method were applied on a simulated two degree of freedom system as shown in Figure 1.

Matlab coding for force appropriation was developed to run eigenvalues problem of two degrees of freedom with different frequency steps. Table 1 shows the force appropriation and MMIF result for two degrees of freedom. A forward analysis for the normal mode test of this system gave the natural frequency for mode 1 as 22.3607 rad/s (3.5588 Hz) and mode 2 as 44.7214 rad/s (7.117 Hz). Higher frequency step (0.1 rad/s) provides satisfactory results for natural frequencies and mode shapes, but not for force vector and minimum eigenvalue. Smaller frequency step (0.001 rad/s) gives more accurate results for natural frequencies, mode shapes and force vector, but has longer computational time.

The target of this method was to determine the mono-phase force vector, f_1 and f_2 when they were applied at undamped natural frequencies and excited the corresponding undamped pure mode. Modal assurance criteria (MAC) and modal purity indicator (MPI) shows good correlation between each other. The function of the modal assurance criterion (MAC) was to provide a measure of consistency (degree of linearity) between estimates of a modal vector, which provides an additional confidence factor in the evaluation of a modal vector from different excitation (reference) locations or different modal parameter estimation algorithms. Perfect purity (MPI=1) of the mode shape was

obtained when two degree of freedoms were applied at effective natural frequencies.

Furthermore, Figures 2 to 4 show the MMIF graphs of eigenvalues, λ versus frequency, ω for different frequency steps.

$$m_1 = 2 \text{ kg}; m_2 = 1 \text{ kg}; k_1 = 2000 \frac{N}{m}, k_2 = 1000 \frac{N}{m} = ; c_1 = 2\frac{Ns}{m}; c_2 = 1\frac{Ns}{m}; g_{nl} = 10^5 \frac{N}{m^2}$$

Figure 1 A nonlinear two degrees of freedom system

Table1 Force appropriation and MMIF result for a system of two degrees of freedom

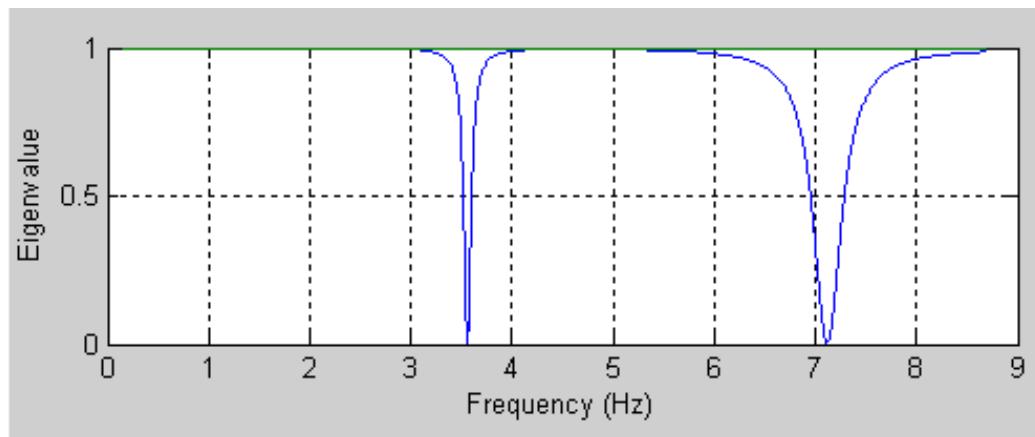|	Frequency Step (rad/s)					
	0.1		0.01		0.001	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
Frequencies, ω (Hz)	3.5651	7.1142	3.5587	7.1174	3.5589	7.1176
Minimum Eigenvalue	0.0217	0.0004	6.65×10^{-6}	1.66×10^{-6}	1.48×10^{-6}	1.16×10^{-7}
f_1(N)	94.95	-117.33	-11.90	-125.89	-8.53	-126.42
f_2(N)	-62.70	72.35	-9.05	63.84	-10.74	63.31
Computational Time (s)	0.355	0.348	0.710	0.700	36.18	36.41
MAC	1		1		1	
MPI	1		1		1	

4.0 SUMMARY

The study of using force appropriation method on a simple two degrees of freedom nonlinear system has been presented and discussed. The proposed method revealed that the desired mode that possesses the frequency and forcing vector of the structure was efficiently and accurately determined.

Therefore, the proposed method is perceived to be an effective method for the identification of nonlinear large complex structures that have more degrees of freedom with large number of modes. It is proposed that the force appropriation approach can be extended in combination with nonlinear identification technique for multi degree nonlinear systems.
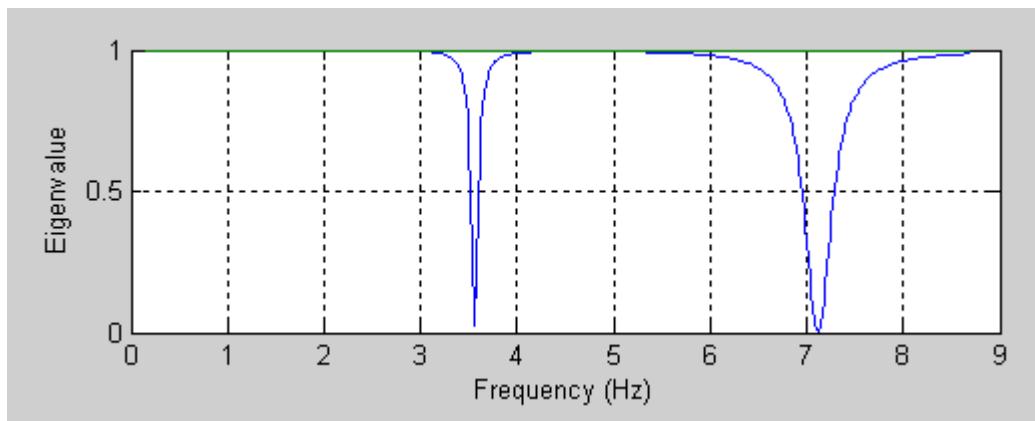
Figure 2 MMIF: Eigenvalue, λ versus Frequency, ω (0.1 rad/s frequency step)

Figure 3 MMIF: Eigenvalue, λ versus Frequency, ω (0.01 rad/s frequency step)
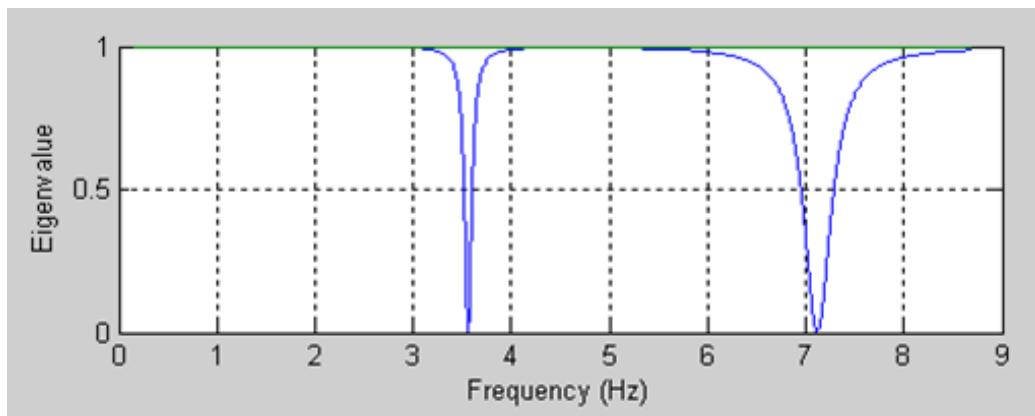
Figure 4 MMIF: Eigenvalue, λ versus Frequency, ω (0.001 rad/s frequency step)

Acknowledgement

References

[1]   Dearson, R. K. 1994. *Discrete Time Dynamic Model*. Oxford University Press.
[2]   Worden, K. & Tomlinson, G. R. 2001. *Nonlinearity in Structural Dynamics: Detection, Identification And Modelling*. University of Sheffield: Institute of Physics Publishing.

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[3]   Ewins, D. J. 1999. *Modal Testing: Theory, Practice and Application.* Hertfordshire :Research Studies Press Ltd.

[4]   Hot, A., Kerschen, G., Foltête E. & Cogan, S. 2012. Detection and Quantification of Non-linear Structural Behavior Using Principal Component Analysis. *Mechanical System and Signal Processing*. 26: 104-116.

[5]   Kerschen, G., Worden, K., Vakakis, A. F. & Golinval, J. C. 2006. Past, Present and Future of Nonlinear System Identification in Structural Dynamics. *Mechanical System and Signal Processing*. 20: 505-592.

[6]   Peeters, M., Kerschen, G. & Golinval, J. C. 2011. Modal Testing of Nonlinear Vibrating Structures Based on Nonlinear Normal Modes: Experimental Demonstration, *Mechanical System and Signal Processing*. 25: 1227-1247.

[7]   Wright, J. R., Cooper, J. E. & Desforges. M. J. 1991. Normal-mode Force Appropriation–theory and Application. *Mechanical Systems and Signal Processing*. 13: 217-240.

[8]   Juang, J. N. & Wright, J. R. 1991. A Multi-point Force Appropriation Method Based Upon a Singular Value Decomposition Approach. *Journal of Vibration and Acoustics*. 113: 176-181.

[9]   Ibanez, P. 1967. Force Appropriation by Extended Asher's Method. SAE Aerospace Engineering and Manufacturing Meeting. SAE Paper No 760873.

[10]   Williams, R., Crowley, J. & Vold, H. 1986. The Multivariate Mode Indicator Function in Modal Analysis. *Proceeding of the 4th International Modal Analysis Conference*, Los Angeles. 66-70.

[11]   Desforges, M. J., Cooper, J. E. & Wright, J. R. 2004. Force Appropriation for flight flutter Testing. *Proceeding Institute Mechanical Engineers*. 218: 189-198.

[12]   Naylor, S., Platten, M. F., Wright, J. R. & Cooper, J. E. 2004. Identification of Multi-degree of Freedom Systems with Nonproportional Damping Using the Resonant Decay Method. *Journal Vibration Acoustics*. 126: 298-306.