

PI SLIDING MODE CONTROL FOR MISMATCHED UNCERTAIN SYSTEMS

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Abstract. This paper focused on the proportional-integral sliding mode control for uncertain dynamic systems with mismatch uncertainties. First, the switching surface condition for the sliding mode control is synthesized. Then the control law is designed to drive the state trajectories of the system onto the sliding surface and the system remains in it thereafter. The proposed control law is applied to a numerical example and its performance is compared to the state variable feedback control system design methodology.

Keywords: sliding mode control, state variable feedback regulator, mismatched uncertainties

1.0 INTRODUCTION

The control of dynamical systems, whose mathematical models contain uncertainties, has occupied the attention of researchers in recent times and has been extensively studied. These uncertainties could be due to parameters, constant or varying, which are unknown or perfectly known, or due to unknown or imperfectly known inputs into the system [1]. Sliding mode control (SMC) has been widely applied to system with uncertainties since it was introduced about three decades ago. A salient feature of this control is that it is completely robust to systems with matched uncertainties [2]. It is certainly true that many systems can be classified under this category. However, there are many systems which unfortunately are affected by uncertainties which do not satisfy the matching condition. A sliding mode control scheme for mismatched uncertain systems has been recently developed [3,4,5,6]. All these researches used the traditional method to design the sliding surface. However, [7] has developed a sliding mode control scheme using an integral-type sliding surface.

In this paper we considered a class of uncertain-dynamical system in mismatched condition. The sliding surface is designed based on proportional-integral sliding mode control (PISMC). We also proposed a new control scheme to control such a system with mismatched uncertainties. The proposed control scheme performance is compared with the state variable feedback regulator (SVFR) system design methodology to show the effectiveness of the control design.

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2.0 SYSTEM DEFINITION

Consider the uncertain dynamic system which can be modeled by the equation:

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x, t) \quad (1)$$

where $x(t) \in \mathfrak{R}^n$ is the n-dimensional state vector, $u(t) \in \mathfrak{R}^m$ is the controlled input signal, and the constant matrices A and B are of appropriate dimensions. The vector $f(x, t)$ represents the uncertainties with the matched and mismatched parts in the plant. Note that $f(x, t)$ is uniformly bounded with respect to time t , and locally uniformly bounded with respect to state x . The following assumptions are taken as standard:

Assumption i: There exists a known non-negative scalar function such that $\|f(x, t)\| \leq \beta(x, t)$, where $\|\bullet\|$ denotes the standard Euclidean norm.

Assumption ii: The pair (A, B) is controllable and the input matrix B has full rank.

3.0 SWITCHING SURFACE DESIGN

In this study, we utilized the PI sliding surface defined as follows:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau) d\tau \quad (2)$$

where $C \in \mathfrak{R}^{m \times n}$ and $K \in \mathfrak{R}^{m \times n}$ are constant matrices. The matrix K satisfies $\lambda_{\max}(A + BK) < 0$ and C is chosen so that CB is nonsingular. It is well known that if the system is able to enter the sliding mode, $\alpha(t) = 0$. Therefore the equivalent control, $u_{eq}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ [8] i.e,

$$\dot{\sigma}(t) = C \dot{x}(t) - \{CA + CBK\}x(t) = 0 \quad (3)$$

If the matrix C is chosen such that CB is nonsingular, this yields

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(x, t) \quad (4)$$

Substituting equation (4) into system (1) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C\}f(x, t) \quad (5)$$

Theorem 1: If $\|\tilde{F}(x, t)\| \leq \beta_1(x, t) = \|I_n - B(CB)^{-1}C\| \beta(x, t)$, the uncertain system in equation (5) is boundedly stable on the sliding surface $\sigma(t) = 0$.

Proof:

For simplicity, we let

$$\tilde{A} = (A + BK) \quad (5a)$$

$$\tilde{F}(x, t) = \{I_n - B(CB)^{-1}C\} f(x, t) \quad (5b)$$

and rewrite (5) as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(x, t) \quad (6)$$

Let the Lyapunov function candidate for the system is chosen as

$$V(t) = x^T(t)Px(t) \quad (7)$$

Taking the derivative of $V(t)$ and substituting equation (5) into it, gives

$$\begin{aligned} \dot{V}(t) &= x^T(t)[\tilde{A}^T P + P \tilde{A}]x(t) + \tilde{F}^T(x, t)Px(t) + x^T(t)P \tilde{F}(x, t) \\ &= -x^T(t)Qx(t) + \tilde{F}^T(x, t)Px(t) + x^T(t)P \tilde{F}(x, t) \end{aligned} \quad (8)$$

where P is the solution of $\tilde{A}^T P + P \tilde{A} = -Q$ for a given positive definite symmetric matrix Q . It can be shown that equation (8) can be reduced to:

$$\dot{V}(t) \leq -\lambda_{\min}(Q)\|x(t)\|^2 + 2\|P\|\|x(t)\|\beta_1(x, t) \quad (9)$$

Since $\lambda_{\min}(Q) > 0$, consequently, $\dot{V}(t) < 0$ for all t and $x \in B^c(\eta)$, where $B^c(\eta)$ is the complement of the closed ball $B(\eta)$, centered at $x = 0$ with radius $\eta = \frac{2\beta_1(x, t)\|P\|}{\lambda_{\min}(Q)}$.

Hence, the system is boundedly stable.

Remark: For the system with uncertainties satisfy the matching condition, i.e., $\text{rank}[B \mid f(x, t)] = \text{rank}[B]$, then equation (5) can be reduced to

$\dot{x}(t) = (A + BK)x(t)$ [9]. Thus, asymptotic stability of the system during sliding mode is assured.

We now design the control scheme that drives the state trajectories of the system in equation (1) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter.

4.0 VARIABLE STRUCTURE CONTROLLER DESIGN

For the uncertain system in equation (1) satisfying assumptions (i) and (ii), the following control law is proposed:

$$u(t) = -(CB)^{-1}[CAx(t) + \phi\sigma(t)] - k(CB)^{-1} \frac{\sigma(t)}{\|\sigma(t)\| + \delta} \quad (10)$$

where $\phi \in \mathfrak{R}^{m \times m}$ is a positive symmetric design matrix, k and δ are positive constants.

Theorem 2: The hitting condition of the sliding surface (2) is satisfied if

$$\|A + BK\| \|x(t)\| \geq \|f(x, t)\| \quad (11)$$

Proof:

In the hitting phase $\sigma^T(t) \dot{\sigma}(t) > 0$; using the Lyapunov function candidate

$$V(t) = \frac{1}{2} \sigma^T(t) \sigma(t), \text{ we obtain}$$

$$\begin{aligned} \dot{V}(t) &= \sigma^T(t) \dot{\sigma}(t) \\ &= \sigma^T(t) \left[-(CA + CBK)x(t) - \phi\sigma(t) - \frac{k\sigma(t)}{\|\sigma(t)\| + \delta} + Cf(x, t) \right] \\ &\leq -\left[\|\phi\| + \left\| \frac{k}{\|\sigma(t)\| + \delta} \right\| \right] \|\sigma(t)\|^2 + \left\{ \|C\| \|A + BK\| \|x(t)\| - \|C\| \|f(x, t)\| \right\} \|\sigma(t)\| \end{aligned} \quad (12)$$

It follows that $\dot{V}(t) < 0$ if condition (11) is satisfied. Thus, the hitting condition is satisfied.

5.0 EXAMPLE

To illustrate the performance of the proposed controller, consider the third order single input system as given in references [2,3,4], i.e.,

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x, t), \text{ where}$$

$$A = \begin{bmatrix} -0.03 & 0.01 & 0.01 \\ -0.05 & -0.15 & 0.05 \\ -0.09 & 0.03 & -0.17 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and $f(x,t) = \Delta Ax(t) + f_1(t)$ where

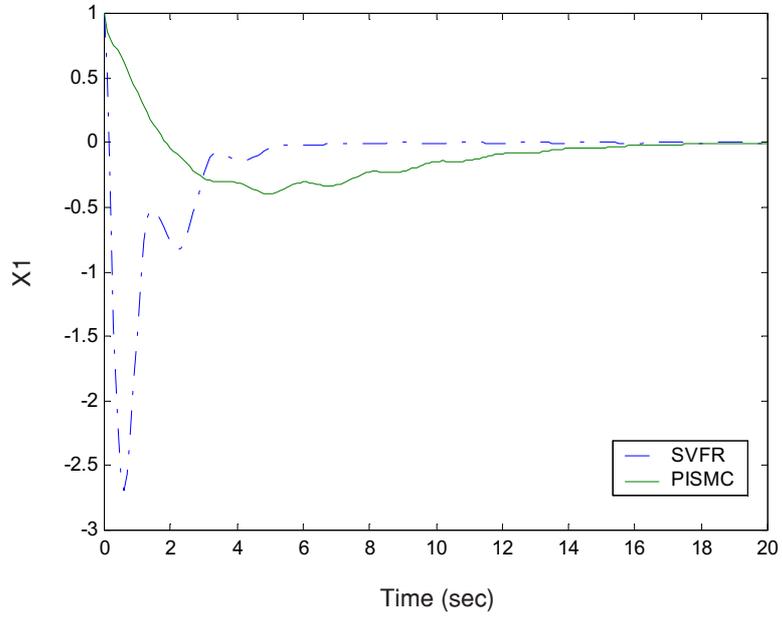
$$\Delta A = \begin{bmatrix} 0.01 + 0.44 \sin(3.14t) & 0.01 + 0.004 \cos(3.14t) & 0.008 + 0.002 \sin(6.28t) \\ 0.55 + 0.220 \sin(3.14t) & 0.05 + 0.020 \cos(3.14t) & 0.04 + 0.010 \sin(6.28t) \\ 0.50 \sin(3.14t) & 0 & 0 \end{bmatrix}$$

$$f_1(t) = \begin{bmatrix} 0 \\ 0 \\ 5 \sin(3.14t) \end{bmatrix}$$

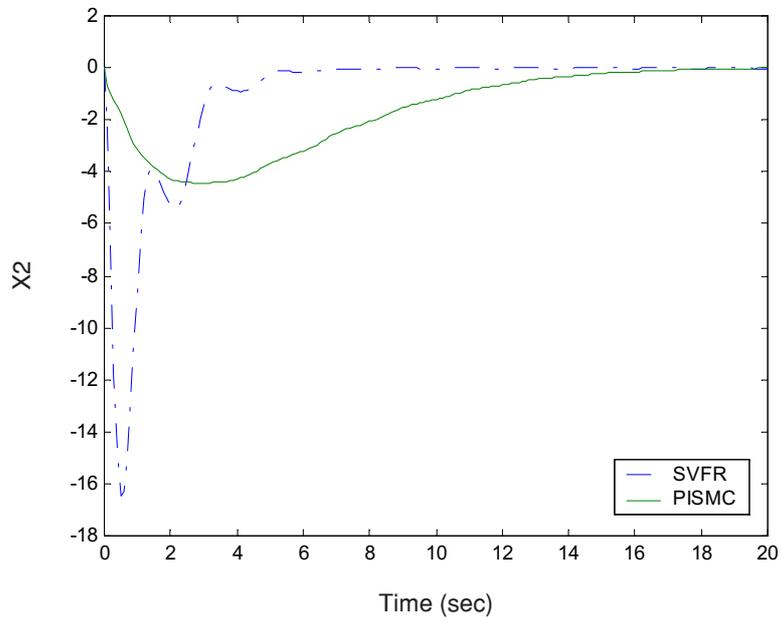
Note that the matrix $f_1(t)$ satisfies the matching condition while the matrix $\Delta Ax(t)$ does not. For the proportional-integral sliding mode control (PISMC), we utilize the pole placement method to determine the value of λ which yields $K = [-6449.4 \ 938.13 \ -7.65]$ such that $\lambda(A + BK) = \{-1, -3, -4\}$. In this simulation, the following values are selected: $C = [10 \ 50 \ 0.01]$, $\phi = 500$, $k = 1$ and $\delta = 0.01$. For comparison purposes, the performance of the PISMC is compared to the SVFR of the form $u = -Kx$ where K is the state feedback gain. The values of the state feedback gain for the SVFR chosen is similar to the values of K in PISMV, i.e., $k_1 = -6449.4$, $k_2 = 938.13$ and $k_3 = -7.65$. Figures 1(a), 1(b) and 1(c) show the state response subjected to the initial condition $x(0) = [1 \ 0 \ 0]^T$ for both controllers. Figure 2 displays the variation in the sliding surface with respect to time. The corresponding control input for both controllers is shown in Figure 3. It can be seen from the simulation results that the system with mismatch uncertainties utilizing both controllers are practically stable but the PISMC performed better as compared to the SVFR control technique. The result also shows that the state trajectory hits and slides on the sliding surface as intended.

6.0 CONCLUSIONS

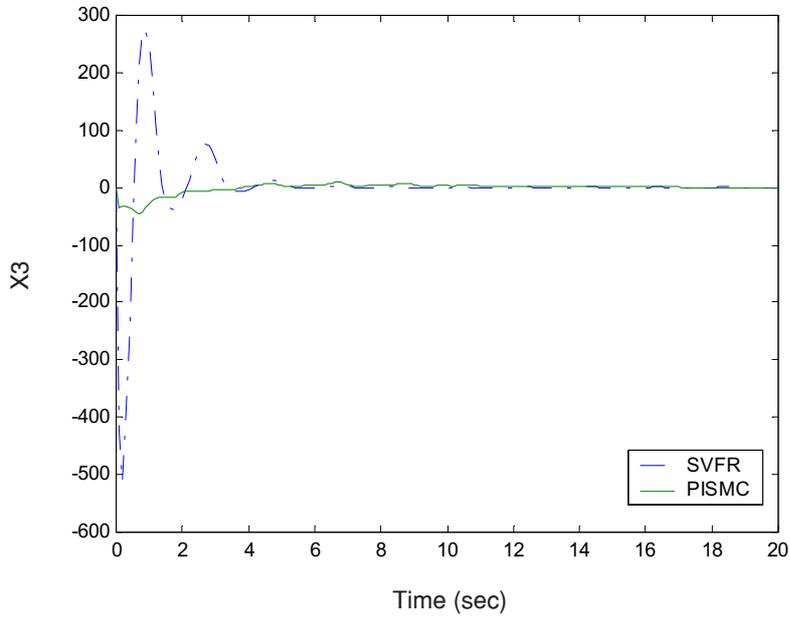
In this paper, the PI sliding mode control technique is proposed for controlling uncertain system where the uncertainties do not satisfy the matching condition. The proposed control law is compared with the state variable feedback regulator. It has been shown mathematically and through computer simulations that the proposed control scheme is capable of controlling the uncertain system and is practically stable with respect to the mismatched uncertainty condition and performed better as compared to the state variable feedback control technique.



(a)



(b)



(c)

Figure 1(a),(b),(c) States responses for both controllers

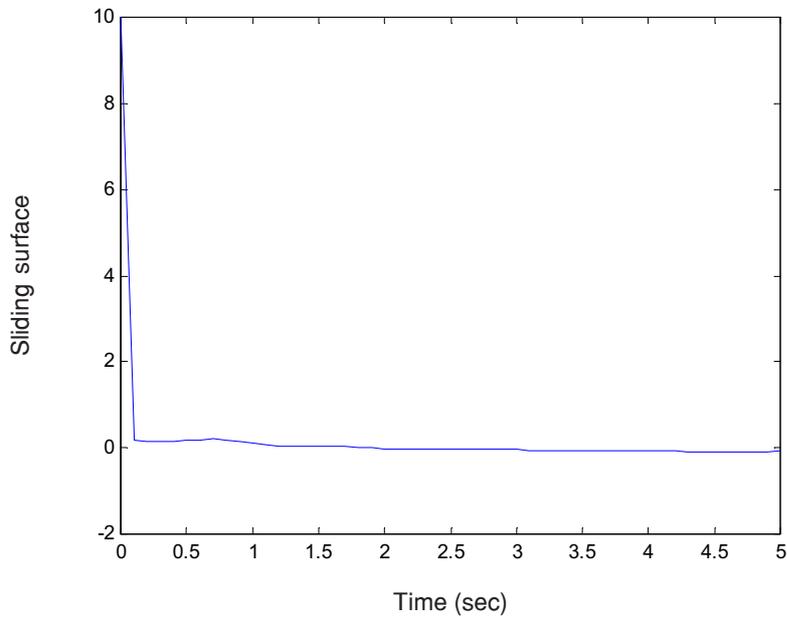


Figure 2 Sliding surface for PISM

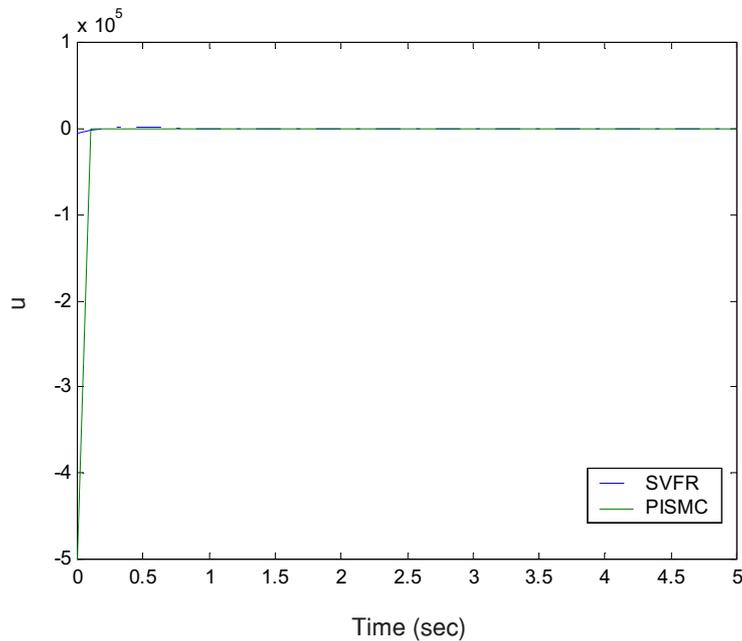


Figure 3 Controlled input of the system

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