Using Extreme Value Theory to Evaluate Conditional VaR for Risk Management in Electricity Markets

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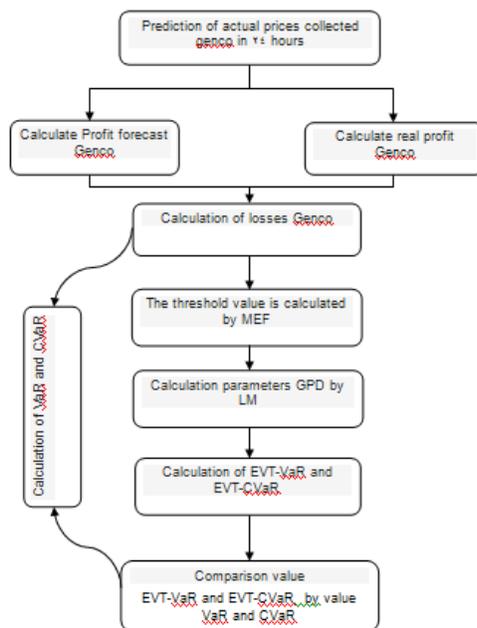
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Graphical abstract

Abstract

In a deregulated power market, generation companies attempt to maximize their profits and minimize their risks. This paper proposes a risk model for bidding strategy of generation companies based on EVT-CVaR method. Extreme Value Theory can overcome shortcomings of traditional methods in computing financial risk based on value-at-risk and conditional value-at-risk method. Also, generalized Pareto distribution is suggested to model tail of an unknown distribution and parameters of the GPD are estimated by likelihood moment method. Numerical results for risk assessment using the proposed approach are presented for IEEE 30-bus test system. According to the findings, this method can be used as a robust technique to calculate the risk for bidding strategy of generation companies.

Keywords: Extreme value theory (EVT); generalized Pareto distribution (GPD); Conditional Value-at-Risk (CVaR); Likelihood Moment Estimation

1.0 INTRODUCTION

In an electricity market, price of electricity is the most important signal for all market participants. All activities of generation companies (GENCOs) are affected by electricity price. An important characteristic of electricity price is its volatility. A discussion about the reasons of electricity price volatility can be found in [1, 2]. The objective of a GENCO in electricity markets is to maximize its own profit from the sale of power and ancillary services. However, electricity price volatility and fluctuations may lead to financial loss of GENCOs. Thus, financial risks always threat GENCOs in electricity markets. Consequently, GENCOs need effective bidding strategy for contributing in electricity markets. This strategy should maximize the profit of the GENCO and at the same time consider forecast price uncertainty and risk management [3]. Therefore, appropriate analytical tools are required for modeling financial risk of GENCOs in electricity markets.

In [4], objective is to maximize utility function, which is obtained from combining the risk and profit. Risk penalty factor represents the relation between the risk and profit, which is depend on the GENCO's risk aversion. In [4], risk is modeled through mean variance method, which is a model in portfolio selection. In [5],

an extended version of mean variance method, named mean variance skewness model is proposed. This method maximizes the expected profit and skewness, while minimizing the variance of return as the risk. Expected downside risk is used as risk measure in [1]. Another tool for risk assessment is value at risk (VaR). It determines Financial risk associated with a production schedule, one can estimates that at a specified confidence level, GENCO due to fluctuations LMPs how sees loss [6]. Confidence level depends on the amount of risk aversion GENCO. Normally, confidence level for a medium risk aversion is 95%, the highest confidence level is 99% (high risk aversion), the lowest confidence level is 92.5% (low risk aversion) [6]. When VaR is accepted that base on the normal distribution. This assumption for the market price may be not acceptable.

CVaR is the average loss that has exceeded the VaR value. Since CVaR is larger than the VaR, scheduling with less CVaR necessarily, will lead to less VaR [6]. CVaR can be models for discrete random variables. It is an advantage of CVaR in portfolio optimization [7].

In [6] VaR is calculated using the safety parameter and security constrained self-scheduling method is proposed that financial risk is measured using the VaR and CVaR. The other security constrained self-scheduling method in the day ahead market is proposed in [8] that unit commitment of generators is considered in a mixed integer programming (MIP) model. In [9] using expected downside risk (EDR) as constraint to modeling of financial risk. The formulated problem is mixed integer linear program.

In [7] a review of risk management methods in the power market is performed. Hydropower scheduling model is considered in power market and stochastic programming method is used in Hydropower scheduling.

Recently the new method has been proposed for risk management, uses fuzzy numbers for modeling the price uncertainty. In [10] a stochastic profit maximization problem was presented in which price uncertainty is modeled using fuzzy number.

It is clear that previous methods of calculating VaR and CVaR that was used for market risk analysis; the key point was to describe the loss distribution function F, which is usually used from the normal distribution. As mentioned, the normal distribution for the power market price may be not acceptable.

Extreme value theory (EVT) is a branch of statistics theory [11]. For data analysis are used that have abnormal behavior and it considers an unknown loss distribution function [12].

McNeil studied estimation of the tail of loss distribution and estimated the measure of risk for financial time series using EVT [13].

In this paper, to model the tail of unknown distribution is used GPD distribution. Most often Maximum Likelihood (ML) method is used to calculate the parameters of GPD but this method has problems. This method has not answer for some samples. Calculation is complex and has a convergence problem. This method for $\xi \geq 1$ has not answer because

the likelihood function has not local maximum. A new method for calculating the parameters is likelihood moment method. This method has simple calculation, always exists and asymptotic effects are high. It has a little sensitivity to the chosen threshold [8, 14].

This paper is organized as follows: section II describes EVT model and compute VaR and CVaR based on EVT; section III is threshold selection and describes two methods of parameters estimation: Maximum likelihood and likelihood moment estimators; section IV and V are numerical results and conclusions.

2.0 METHODOLOGY

2.1 Extreme Value Theory and Risk Management

There are two ways to model the distribution tail. The older method is Block maxima (BM) method. In this method, time is divided into blocks or periods and the maximum sample in each period will be selected and considered. This method is useful when data include of a maximum set. The second method is peak over threshold (POT). This method considers higher values of the threshold, more effective and more used in financial applications [12]. So our choice of method in this paper is POT.

We consider an unknown loss distribution function $F(x)$ of the variables x. we focus on the distribution function F_u. Distribution function F_u is called conditional excess distribution function (cedf) as follows:

$$F_u(y) = \Pr\{x - u \leq y | x > u\} \tag{1}$$

Where $y = x - u$ is called excess number.
This probability can be expressed as follows:

$$F_u(y) = \frac{\Pr\{x - u \leq y | x > u\}}{\Pr(x > u)} = \frac{\Pr\{u < x \leq y + u\}}{\Pr(x > u)}$$

$$= \frac{F(y + u) - F(u)}{1 - F(u)} \tag{2}$$

Since $x = y + u$. So we can write the following formula for $x > u$ [16].

$$F(x) = [1 - F(u)]F_u(y) + F(u) \tag{3}$$

Blakema (1974) and De Haan, Pickand (1975) provided a result about conditional excess distribution that can be expressed as the following theorem:
Theorem1. For a large group of underlying distribution functions, conditional excess distribution function $F_u(y)$ for a large value of u, is estimated as follows:

$$F_u(y) \cong G_{\xi,\sigma}(y); \qquad u \to \infty \tag{4}$$

Where

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}} & \xi \neq 0 \\ 1 - e^{-y/\sigma} & \xi = 0 \end{cases} \tag{5}$$

For $y \in [0, x_F - u]$ if $\xi \geq 0$ and $y \in [0, -\sigma/\xi]$ if $\xi < 0$. $x_F \leq \infty$ is the right endpoint of F. $G_{\xi,\sigma}$ is called generalized Pareto distribution (GPD). ξ is called tail index, σ is called scale parameter [12].

According to the above theorem, we can use the GPD instead of $F_u(y)$ in (3).

$$F(x) = [1 - F(u)]F_u(y) + F(u) \quad (6)$$

We require an estimate of $F(u)$. An obvious candidate is empirical estimate $(n - N_u)/n$, where n is the total number of observations and N_u is the number of observations above the threshold. The result is as follows:

$$F(x) = 1 - \frac{N_u}{n}(1 + \xi\frac{x-u}{\sigma})^{-1/\xi} \quad (7)$$

Equation (7) is tail of estimator and it is valid only for $x > u$ [15].

If F is the loss distribution function and β is the confidence level, VaR is calculated from the following equation:

$$VaR_\beta = F^{-1}(1 - \beta) \quad (8)$$

$$F^{-1}(\alpha) = u + \frac{\sigma}{\xi}[\left(\frac{n}{N_u}\alpha\right)^{-\xi} - 1] \quad (9)$$

If $\alpha = 1 - \beta$, according to equation (8), equation (9) defines VaR for us[17].

$$VaR_\beta = u + \frac{\sigma}{\xi}\left[\left(\frac{n}{N_u}(1 - \beta)\right)^{-\xi} - 1\right] \quad (10)$$

$$CVaR_\beta = VaR + E(x - VaR_\beta|x > VaR_\beta) \quad (11)$$

Where $E(x - VaR_\beta|x > VaR_\beta)$ is the mean of excess distribution $F_{VaR_\beta}(y)$ over the threshold VaR_β. The EVT model for excess distribution above a given threshold is stable. If a higher threshold is taken, the excess distribution above the higher threshold is also a GPD with the same shape parameter but a different scale parameter. Consequence is as follow:

$$F_{VaR_\beta}(x - VaR_\beta) = G_{\xi,\acute{\sigma}}(x - VaR_\beta) \quad (12)$$

The mean of the above distribution is given by $(\beta + \xi(VaR_\beta - u))/(1 - \xi)$. The CVaR is estimated as [12]

$$CVaR_\beta = \frac{VaR_\beta}{1 - \xi} + \frac{\beta - \xi u}{1 - \xi} \quad (13)$$

Now, should ξ and σ parameters in GPD distribution and the threshold u are calculated. Then market risk can be calculated.

2.2 Parameters Estimation

Threshold selection is important. If we choose too low a threshold, there might be biased estimation. Otherwise, choose high threshold will cause estimation with high standard errors because Number of observations decreases (variance of parameter estimation will be too high) [17].

We usually use Mean Excess Function (MEF) to estimate threshold.

$$e(u) = E(X - u|X > u) \quad (14)$$

Where u is the threshold and $e(u)$ is the mean excess function.

For GPD, the mean excess function is a linear function given by de Rozario.

$$e(u) = \frac{\sigma + \xi u}{1 - \xi} \quad (15)$$

This is increasing if ξ is positive [12].

The mean excess plot, introduced by Davidson and Smith (1990), which graphs the conditional mean of the data above different thresholds, the sample mean excess function which is defined by

$$e_{n_u}(u) = \frac{\sum_{i=1}^{n_u}(x_i - u)}{\sum_{i=1}^{n_u} I_{(x_i > u)}} \quad (16)$$

Where I=1 if $(x_i > u)$ and 0, otherwise. n_u is the number of data points which exceed the threshold u. If the empirical MEF has a positive gradient above a certain threshold u, it is an indication that the data follows the GPD with a positive shape parameter ξ [15].

3.2 Estimation of Parameters ξ, σ

After threshold selection, next step is the estimation of parameters ξ, σ of the GPD. The estimation can be obtained using the method of maximum likelihood.

3.3 Maximum Likelihood Estimation (MLEs)

To estimation value ξ and σ, process is following:
Get derivative through $G_{\xi,\sigma}$, and then we have density function:

$$g_{\xi,\sigma}(x) = \frac{1}{\sigma}(1 + \xi\frac{x}{\sigma})^{-(\frac{1}{\xi}+1)} \quad (17)$$

For $x_1, x_2, \ldots, x_n$, density function is as follow:

$$\frac{1}{\sigma^n}\prod_{i=1}^{n}(1 + \xi\frac{x_i}{\sigma})^{-(\frac{1}{\xi}+1)} \quad (18)$$

Likelihood function is as follow:

$$L(x, \xi, \sigma) = -n(\ln\sigma) - (\frac{1}{\xi} + 1)\sum_{i=1}^{n}\ln(1 + \frac{\xi x_i}{\sigma}) \quad (19)$$

To calculate ξ, σ, get derivative through $L(x, \xi, \sigma)$:

$$\frac{\partial L(x,\xi,\sigma)}{\xi} = \frac{1}{\xi^2}\sum_{i=1}^{n}\ln\left(1 + \frac{\xi x_i}{\sigma}\right) - \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{n}\frac{x_i}{\sigma + \xi x_i} = 0$$

$$\frac{\partial L(x,\xi,\sigma)}{\sigma} = \frac{-n}{\sigma} + \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{n}\frac{\xi x_i}{\sigma^2 + \sigma\xi x_i} = 0 \quad (20)$$

With solving above equations, we can get ξ , [11].

2.3 Likelihood Moment Estimation (LMEs)

We can use (b, ξ) instead of (σ, ξ) where b=ξ/σ.
Let X= $(x_1, x_2, \ldots, x_n)$. The likelihood function is
$$l(x, b, \xi) = n\ln\left(\frac{b}{\xi}\right) - (\frac{1}{\xi} + 1)\sum_{i=1}^{n}\ln(1 + bx_i) \quad (21)$$
The equation satisfied by the MLEs for (b, ξ) is equivalent to

$$n^{-1}\sum_{i=1}^{n}(1 - bx_i)^{-1} - (1 - \xi)^{-1} = 0, \quad \xi = -\frac{1}{n}\sum_{i=1}^{n}\ln(1 - b x_i) \quad (22)$$

Eliminating ξ, the equation is as follow:

$$n^{-1}\sum_{i=1}^{n}(1 - bx_i)^{-1} - (1 + \frac{1}{n}\sum_{i=1}^{n}\ln(1 - b x_i))^{-1} = 0 \quad (23)$$
The numerical solution (23) is complex and may have convergence problems.

Note that $E((1 - bX)^r) = (1 + r\xi)^{-1}$ for any constant r satisfying $1 + r\xi > 0$, the sample version which is equivalent to

$$n^{-1} \sum_{i=1}^{n} (1 - bx_i)^r - (1 + r\xi)^{-1} = 0 \qquad (24)$$

The equations satisfied by the MLEs in (22) correspond to $r = -1$.

Now it can be see that the condition ξ<1 exists for ML, the condition $1 + r\xi > 0$ for (24) to hold when $r = -1$. Replacing the r in (24) by $-\frac{r}{\xi}$ together with the second equation in (22) gives the estimating equation for b:

$$\frac{1}{n} \sum_{i=1}^{n} (1 - bx_i)^p - (1 - r)^{-1} = 0 \qquad (25)$$

Where $p = rn / \sum_{i=1}^{n} \ln(1 - bx_i)$ and r < 1.

The LMEs equal the MLEs approximately if the r chosen to the true parameter ξ. The LMEs with $r = -\frac{1}{2}$ has high asymptotic efficiencies, and should be recommended if there is no information about ξ. Unlike (23), the numerical solution of (25) becomes easy due to the following theorem.

Theorem2. Let g (b) is the left side of (25). Then,

  (a) g (b) is a smooth monotone function of b unless $r = 0$ or $x_1, x_2, ..., x_n$.

  (b) $lim_{b \to -\infty} g(b) < 0$ And $lim_{b \to x_{(n)}^{-1}} g(b) > 0$ if $r < \frac{1}{2}$, $r \neq 0$ and $n > 2$.

Using theorem 2, (25) has a unique solution $\hat{b} \in (-\infty, x_{(n)}^{-1})$ which can be easily obtained by numerical iterative approaches. Moreover, $\hat{b}_0 = -1$ can simply is used as an initial value of the procedure, but any value belonging to $(-\infty, x_{(n)}^{-1})$ is qualified.

When $\hat{b}$ is obtained, ξ is estimated by

$$\hat{\xi} = -\frac{1}{n} \sum_{i=1}^{n} \ln(1 - \hat{b}x_i)$$

Then σ can be estimated by $\hat{\sigma} = \hat{\xi}/\hat{b}$ [8, 14].

3.0 RESULTS AND DISCUSSION

This model is tested on the IEEE 30-bus system [18]. The data used in the numerical simulation of this paper have been obtained from PJM market. Prices are for the year 2009. Prices for a week, equivalent to 168 hour are predicted. Using this information the optimization program is written in Gams software and loss is calculated. Then using Matlab software risk is calculated using EVT-CVaR. According to what was stated in the past, we need to estimate parameters and threshold selection for the calculation of EVT-CVaR. Figure 1 shows the diagram that is obtained from the MEF. As can be seen υ = 147 is chosen, because after it, plot is linear and has a positive slope.
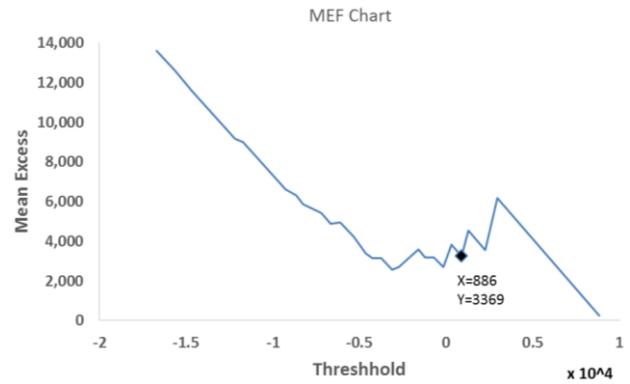

Figure 1 MEF chart to select the threshold

After estimating the parameters using likelihood moment method in Matlab software following results were obtained:

ξ = 0.3486

σ = 2261.4

Now we can calculate the CVaR and VaR using EVT. Results are given in the Table 1 for various confidence levels.

Table 1 CVaR and VaR calculation results using EVT

β	VaR	CVaR
0.90	1410.8	5163.2
0.92	1978	6033.9
0.95	3327.3	8105.3

Results are shown on the chart in Figure 2.

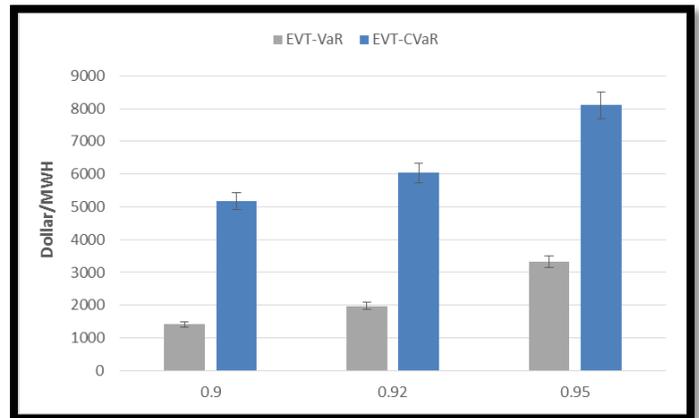**Figure 2** CVaR and VaR calculation results using the EVT

For all β values should CVaR value is larger than the VaR. results obtained from calculation the VaR and CVaR using EVT in Table 1, it shows.

According to the Table 1, will be seen that increasing β increases VaR and CVaR. Since β shows the level of GENCO risk, increasing β is increasing VaR and CVaR and is leading to lower profits.

4.0 CONCLUSION

This paper proposes a risk model for bidding strategy of generation companies based on EVT-CVaR method. Extreme Value Theory (EVT) can overcome shortcomings of traditional methods in computing financial risk measures of value-at-risk (VaR) and conditional VaR (CVaR). Also, generalized Pareto distribution (GPD) is suggested to model tail of an unknown distribution and parameters of the GPD are estimated by likelihood moment (LM) method. Numerical results for risk assessment are presented for IEEE 30-bus test system and The results of the proposed approach compared with traditional methods(CvaR,VaR).

Based on these results the calculated risk of the proposed method is equal to the amount of risk calculated by the CvaR. These results prove the accuracy of this method.

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