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VISCOELASTIC ADHESION OF A SPHERICAL TIP INDENTING A FLAT RIGID SURFACE USING LENNARD-JONES INTERACTION

A.K.X. Leong[a], W.W.F. Chong[b*]

[a]Faculty of Engineering and the Environment, University of Southampton Malaysia Campus (USMC), Nusajaya, Johor, Malaysia
[b]National Centre for Advanced Tribology (nCATS), Faculty of Engineering and the Environment, University of Southampton, Southampton, UK

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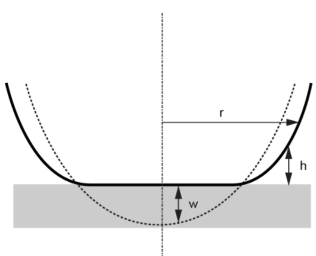
*Corresponding author
w.chong@soton.ac.uk

Graphical abstract



Abstract

Solid and elastic contact problems have been thoroughly investigated before. The most recent efforts incorporate the use of the Lennard-Jones (LJ) potential to describe the inter-surface forces that are present and substantial in micro-sized contact problems. But little work has been done on viscoelastic contact problems. Hence, there is a need to investigate the behaviour of a viscoelastic contact under the LJ interaction. This paper aims to investigate the deformation of an axisymmetric viscoelastic tip that is either pushed onto or pulled from a flat rigid surface. From existing elastic models, a mathematical model was developed to describe the contact problem in a viscoelastic context. This newly developed was solved via numerical means. The result is a model that readily accepts measureable physical properties and gives out the deformation of a viscoelastic tip.

Keywords: Viscoelastic deformation, creep, relaxation, work of adhesion and Lennard-Jones interaction

1.0 INTRODUCTION

The study of contact mechanics started off with the study of the responses of elastic and rigid bodies [1]. However, early models did not account for the attractive and repulsive forces that are always present on surfaces. Hence, different models were proposed to account for the adhesion of these bodies, such as the JKR [2] and the DMT [3] models, to name a few. Later, work was then done to use the Lennard-Jones interaction to model the adhesion of elastic bodies [4, 5, 6, 7]. Since that has already been covered quite extensively, the next logical step would then be to extend the model to viscoelastic materials [8, 9, 10], which is the ultimate aim of this paper.

2.0 THE MATHEMATICAL MODEL

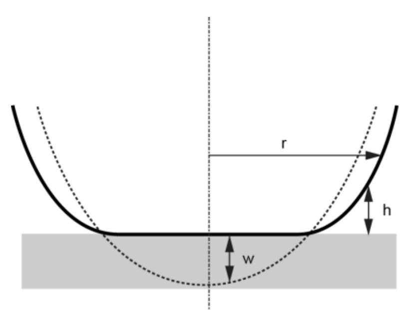


Figure 1 An indenter that is deformed (thick solid line) and its original profile (dotted line) being pressed against a flat rigid surface (grey block). The physical meaning of some variables explained visually

To simplify the problem, some values are non-dimensionalised according to these parameters: $h = H/z_0 - 1$, $r = \rho/\sqrt{Rz_0}$, $\alpha = A/z_0$ [5, 7]. The parameter, H is the distance of the indenter to the surface, ρ is the distance from the centre of the indenter, A is the approach or the relative position of the indenter to the surface, R is the nominal radius of the indenter, and z_0 is the equilibrium spacing between atoms in the lattice. The spherical indenter is modelled to have a profile of a parabola. Adding in the approach of the indenter as well as factoring in the deformation, the deformed indenter, h is

$$h(r,t) = Z - \dot{\alpha}t + \frac{1}{2}r^2 + w(r,t) \qquad \text{(Eq. 1)}$$

where Z is the original position and w is the viscoelastic deformation. The viscoelastic material that is used here is assumed to be linear and has a single relaxation time, τ. The resulting equations [11] govern the material of initial Young's modulus E_0 and long-term modulus E_∞:

$$\frac{1}{E_{crp}(t)} = \frac{1}{E_0} + \frac{E_0 - E_\infty}{E_0 E_\infty}(1 - e^{-\frac{tE_\infty}{\tau E_0}}) \qquad \text{(Eq. 2)}$$

$$E_{rel} = E_\infty + (E_0 - E_\infty)e^{-t/\tau} \qquad \text{(Eq. 3)}$$

The Lennard-Jones force [1, 6] is given to be:

$$p(r) = \frac{8\gamma}{3z_0}\left\{\frac{1}{[h(r)+1]^9} - \frac{1}{[h(r)+1]^3}\right\} \qquad \text{(Eq. 4)}$$

The twist compared to previous methods is that this paper proposes that the surface energy (or work of adhesion), γ, to be modelled using:

$$\gamma(t) = \frac{E_{rel}(t)}{120(1-2\nu)}\sqrt[8]{\frac{M_1}{\rho_B}} \qquad \text{(Eq. 5)}$$

where ν is the Poisson's ratio, M_1 the molecular weight, and ρ_B the bulk density [12].

Combining the ideas presented above, using [13] we have:

$$w(r,t) = \int_{t_0}^{t} \frac{4(1-\nu^2)}{45\pi E_{crp}(t-\xi)}(E_0 E_\infty)e^{-(t-\xi)/\tau}\sqrt[8]{\frac{M_1}{\rho_B}}\sqrt{\frac{R}{{z_0}^3}}\int_0^{r_\infty}\left\{\frac{1}{[h(r,\xi)+1]^9} - \frac{1}{[h(r,\xi)+1]^3}\right\}\frac{r}{r+\bar{r}}\mathrm{K}\left(\frac{2\sqrt{r\bar{r}}}{r+\bar{r}}\right)\mathrm{d}\bar{r}\,\mathrm{d}\xi + \frac{4(1-\nu^2)}{45\pi(1-2\nu)}\sqrt[8]{\frac{M_1}{\rho_B}}\sqrt{\frac{R}{{z_0}^3}}\int_0^{r_\infty}\left\{\frac{1}{[h(r,t_0)+1]^9} - \frac{1}{[h(r,t_0)+1]^3}\right\}\frac{r}{r+\bar{r}}\mathrm{K}\left(\frac{2\sqrt{r\bar{r}}}{r+\bar{r}}\right)\mathrm{d}\bar{r} \qquad \text{(Eq. 6)}$$

where K is the complete elliptic integral of the first kind.

2.1 The Algoritm

For every position r, the solution is found numerically using the following steps:

1. Assume initial deformation to be 0.
2. Use resulting value of h to compute the next estimation of the deformation via Eq. 6.
3. Compare the previous estimation of the deformation to the next estimation. If the difference (see Eq. 7) is not small enough, let the next estimate of h be $h_{i+1} = h_i - (\Omega \times \mathrm{Difference})$, and then repeat step 2 until the difference is negligible.

$$\mathrm{Difference} = h(r,t) - Z + \dot{\alpha}t - \frac{1}{2}r^2 - w(r,t) \qquad \text{(Eq. 7)}$$

3.0 RESULTS AND DISCUSSION

The simulation was performed with the following typical elastomer properties:

Table 1 Physical properties of a typical elastomer [14, 15]

Property	Value
Bulk density, ρ_B	920 kg/m³
Molecular weight, M_1	1.661×10^{-25} kg
Poisson's ratio, ν	0.05
Starting Young's modulus, E_0	3 MPa
Final Young's modulus, E_∞	1 MPa
Relaxation time, τ	1 s
Equilibrium distance, z_0	1.65 Å

3.1 Case 1

Pushing an indenter against the flat surface. The response of the indenter is defined by the creep and relaxation functions of the material (see Fig. 2). It can be observed that the adhesion or the 'stickiness' of the indenter grows with time, even though it is initially not that 'sticky'. The simulation in Fig. 2 is an indenter that starts off from rest at an approach of 3.0 and moves downwards at a speed of 0.7units/s. Each subsequent solid line represents 0.20s on the clock. The dashed line (t=0.35s) represents the position before the jump-on at t=0.40s. Note the decreasing gap sizes between the lines after the jump-on, representing the saturation of the creep and relaxation moduli as defined in Eq. 2 and Eq. 3.

3.2 Case 2

Pulling off the indenter from the surface. The response of the viscoelastic indenter for this case is not as visually simple as Case 1. In the beginning, the adhesive force grows; hence, the contact area grows in the initial stages. But as the indenter continues to be pulled away from the surface, the contact area naturally shrinks. But it can be seen that, even though the speed of the upward pull in Fig. 3 is twice the speed of the downward push of Fig. 2, the final pull-off approach and time is greater than that of the pull-on. This could probably be the reason why hysteresis (assuming only elastic deformation) occurs between pushing and pulling of an indenter to/from a surface. As a visualization of this simulation, the indenter in Fig. 3 is initially at rest at approach 1.95 (the thickest line) and moves up at 1.4units/s. The solid lines represent the initial growth of the contact area as the indenter is pulled off, whilst the dashed lines represent the subsequent shrinking of the contact area and the general upward motion. Both these lines are spaced 0.50s apart. The points represent the times just before and just after the jump-off (t=2.60s and t=2.65s respectively).The whole system is setup by connecting the PI camera module to the CSI port on the Raspberry PI board via ribbon cable while the LCD screen is connected to the board via HDMI cable. The wireless keyboard and mouse is connected to the board using wireless USB adapter. This is only needed when manipulation of code is required. The power is supplied to the board by connecting a micro USB to USB cable to a wall socket USB adapter or power bank.

3.3 Case 3

Testing the time dependence on the viscoelastic response. A viscoelastic indenter was pushed against the flat surface with speeds of 0.7units/s and 0.35units/s. The result was then compared to the rigid and the elastic indenter cases. The elastic indenter uses the same method as [5, 7] to get the result. The result of this comparison is that the slower the indenter, the more it behaves as though the deformation is purely elastic. This is because the slower indenter has more time to be attracted by the adhesive force between the surfaces. A snapshot of this phenomenon can be seen in Fig. 4. The contact radius of the slower indenter is closer to the contact radius of the elastic indenter compared to the faster indenter. The same trend can be seen throughout all approach values. Even the jump-on positions are different for different speeds. Hence, the model is shown to be able to capture the time-dependence of viscoelastic materials.

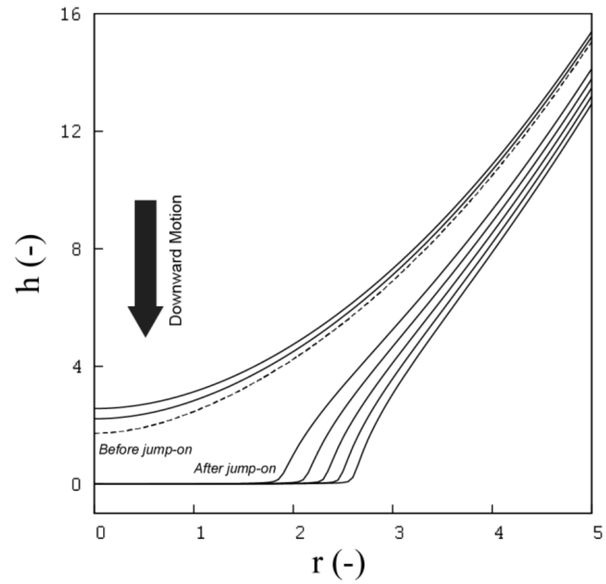


Figure 2 The response of a viscoelastic indenter that is pushed on a flat rigid surface

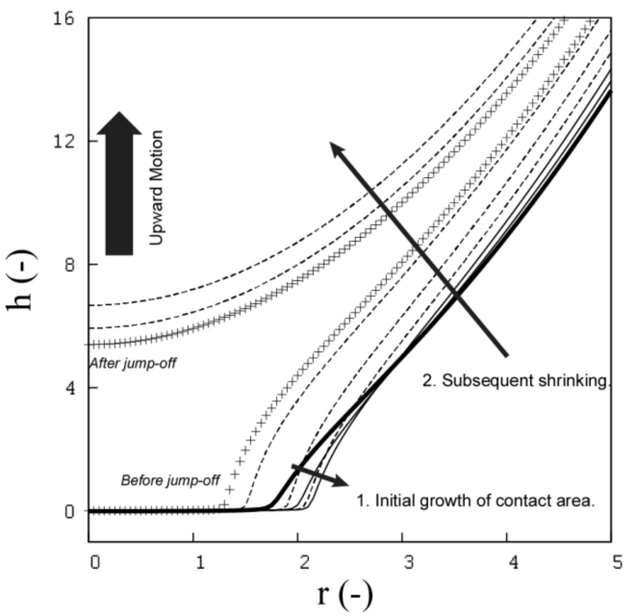


Figure 3 The response of a viscoelastic indenter being pulled off a flat rigid surface

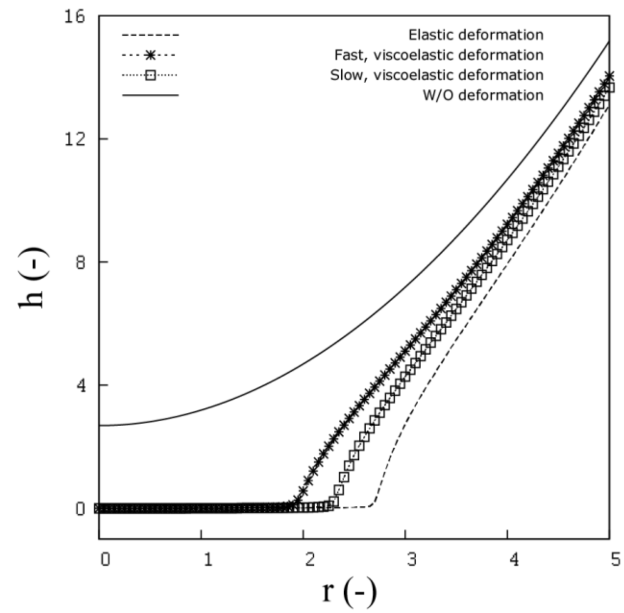


Figure 4 The difference between a slow moving and a fast moving viscoelastic indenter with respect to a rigid and an elastic indenter at the same approach

4.0 CONCLUSION

This paper describes a work where the basic mathematics behind viscoelastic responses was combined with inter-surface forces modelled under the Lennard-Jones interaction with a twist of using Weir's method of approximating the work of adhesion. The result of this amalgamation is a mathematical model capable of predicting the behaviour of a viscoelastic indenter that is pressed against and pulled off from a flat rigid surface. The model is also able to account for the time dependency of the responses of viscoelastic materials. In future work, the robustness of this model will be tested and compared to real life materials.

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