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VIBRATION ISOLATION SYSTEM WITH ACTIVE STIFFNESS CONTROL

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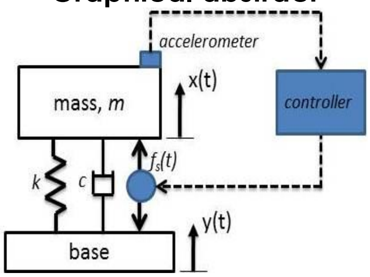
Graphical abstract



Abstract

This paper presents the dynamic analysis of a linear isolator under active stiffness control. Firstly, the literature review on the active stiffness and the concept of skyhook spring are presented. Then, based on the theoretical equation govern the isolation system model, the effect of active stiffness control in the absolute motion transmissibility performance is studied. In addition, the force response of the isolation system under active stiffness control is also examined to highlight its benefits in improving the isolation system performance.

Keywords: Active stiffness, skyhook spring, isolation system, transmissibility, force response

Abstrak

Kertas kerja ini mempersembahkan analisa dinamik bagi sebuah pemencil lelurus dibawah kawalan kekukuhan aktif. Pertamanya, tinjauan literasi bagi kekukuhan aktif dan konsep pegas cangkuk langit dipersembahkan. Kemudian, kesan kawalan kekukuhan aktif bagi kebolehpindahan gerakan mutlak berdasarkan persamaan teori yang menakluk model sistem pemencilan tersebut dikaji. Disamping itu, sambutan daya bagi sistem pemencilan dibawah kawalan kekukuhan aktif juga diperiksa, dan manfaatnya dalam penambahbaikkan prestasi sistem pemencilan diserlahkan.

Kata kunci: Kekukuhan aktif, pegas cangkuk langit, sistem pemencilan, kebolehpindahan, sambutan daya

1.0 INTRODUCTION

The concept of active control for modification of the system response has been reported in [1, 2]. It has been stated that, the mass, damping and stiffness of the system can be independently modified depending on the type of feedback controller. In the case of stiffness modification, displacement feedback should be applied as the control strategy. It is worth to mention that, the robustness of the system to external disturbance will be increased by the stiffening the system. However, as the stiffness of the system increases, the isolation bandwidth will be reduced, due to the increment of the system natural frequency. It is clear that, there is trade-off between robustness to external disturbance and the isolation bandwidth, in the active stiffness application.

Despite the growing interest in active vibration control, the concept of active stiffness or skyhook spring is less highlighted in the literature, in comparison with the concept of active damping or better known as skyhook damping [3]. One of major challenge in the implementation of active stiffness is due to the difficulty in measuring the displacement of the isolated mass. If the accelerometer is used, double integration of the acceleration signal is required to obtain the displacement measurement. In this case, the issue of drift signal and instability which is caused by phase shift and time delay will arise [4].

Collette [5] has reviewed three practical system configurations of skyhook spring which has been developed and patented for commercial products, based on inertial reference. This inertial reference is an oscillator with an extremely low resonance frequency, equipped with sensor for measuring the relative displacement between the inertial mass and the isolated mass. The first configuration is based on the inertial reference mounted on the isolated mass [6, 7]. Meanwhile, in the second configuration, the inertial reference is mounted directly on the ground [8, 9]. The third configuration consists of a small intermediate mass mounted on stiff piezoelectric actuator, and connected in series to the isolated mass by a passive isolator [10, 11]. In contrast, Mizuno [11], mounted the intermediate mass on passive isolator from the base, and connected in series to the isolated mass by a linear actuator. In the study, he introduced displacement cancellation control for achieving system robustness to direct external disturbance.

Mei [12], has proposed concept of skyhook spring as stability control of railway bogies. In the study, the feedback control is based on the absolute yaw movement of the wheel set which can be measured by seismic accelerometer. It has shown that, the proposed control method can provide superior performance in term of stability and robustness in comparison with passive and active damping control. The advantage of displacement feedback control in acquiring system stability is also being reported in [13-17].

Although many studies in the literature has proposed displacement feedback control for achieving a better system stability, little information in term of physical interpretation is provided to justify the obtained result. For this reason, the fundamental concept of active stiffness will be presented in this work. This includes the investigation of the system transmissibility performance and its robustness to external disturbance.

2.0 THEORETICAL MODEL OF ISOLATION SYSTEM WITH SKYHOOK SPRING

A single-degree-of-freedom model of active vibration isolation system subjected to base excitation is shown in Figure 1. It consists of mass *m*, which is mounted on stiffness *k*, and viscous damper *c*, in parallel with secondary force fs for active control purpose. The base and mass displacement are presented by *y(t)* and *x(t)* respectively.

Physically, the secondary force fs is generated by an actuator which is determined by the controller based on the measured signal from the accelerometer. In this study, displacement feedback control is applied to achieve a vibration isolation system with active stiffness. As a result, the stiffness of the system can be modified artificially by the effect of the generated secondary force fs in the isolation system. In particular, the secondary force fs will be proportional to the mass absolute displacement x, which can be expressed as

$$f_s = k_{sky} x \quad (1)$$

where k_{sky} is the control gain from the controller. The configuration of the system can be represented physically as Figure 2. It is obvious that the system can be described as a skyhook spring system, where perfect isolation performance can be achieved if k_{sky} is infinite.

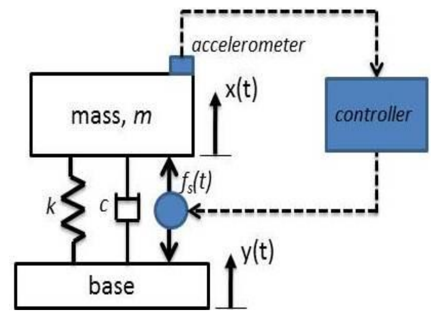


Figure 1 Single-degree-of-freedom model of active vibration isolation system

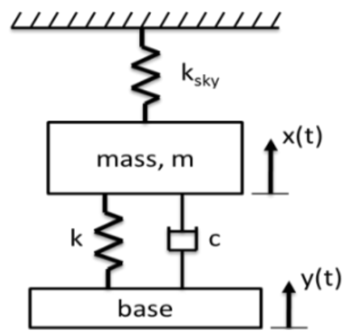


Figure 2 Physical assumption of active stiffness vibration isolation system

3.0 RESULTS AND DISCUSSION

3.1 The Effect of Active Stiffness on the Transmissibility Performance

By referring to Figure 2, the equation of motion of the system can be written as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) + k_{sky}x = 0 \quad (2)$$

Assuming the response of the mass $y=Ye^{j\omega t}$ is harmonic to the base excitation of $x=Xe^{j\omega t}$, the motion transmissibility *T* performance of the system can be obtained as

$$T = \left|\frac{X}{Y}\right| = \left|\frac{k + jc\omega}{(k + k_{sky}) - m\omega^2 + jc\omega}\right| \quad (3)$$

Alternatively, (3) can be written in non-dimensional form as

$$T = \left| \frac{1 + j2\zeta\Omega}{(1+K) - \Omega^2 + j2\zeta\Omega} \right| \quad (4)$$

where

$$\zeta = \frac{c}{2m\omega_n},\ \omega_n = \sqrt{\frac{k}{m}},\ \Omega = \frac{\omega}{\omega_n},\ K = \frac{k_{sky}}{k}$$

In general, there are two significant effects in the absolute motion transmissibility plot of active stiffness in comparison to the passive system as shown in Figure 3. The first effect is in reducing the transmissibility which is below the natural frequency of the system. This effect can be presented mathematically by referring to (4), where the transmissibility expression can be simplified as $T = \left|1/(1+K)\right|$ when $\Omega \to 0$. Therefore, it can be said that as the active stiffness increases, the transmissibility plot at low frequency will be shifted further down. In fact, this is the great benefit in improving the transmissibility performance at the low frequency.

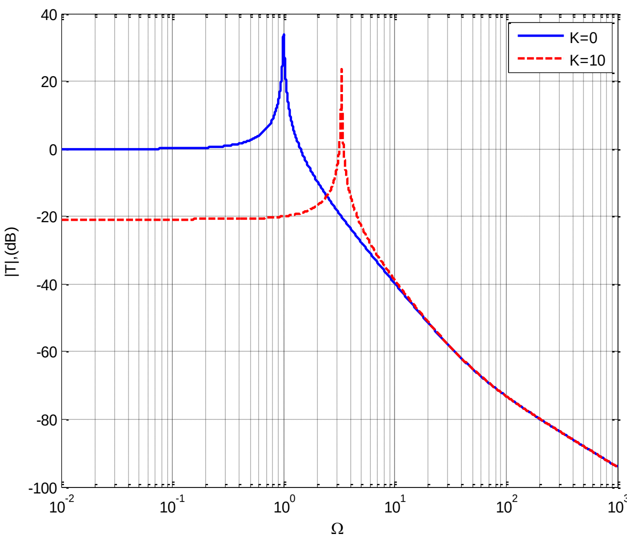


Figure 3 Comparison of absolute motion transmissibility performance between passive and active stiffness vibration isolation. Blue solid line represents passive vibration isolation system, and red dashed line represents active stiffness vibration isolation system

It is also worth to mention that, the active stiffness also has indirectly effect in the changing of the damping ratio of the system as well. As the active stiffness increases, the effective damping ratio of the system decreases. As a result, the resonance peak will become narrower. However, as mentioned earlier, the increment of active stiffness also has the ability to reduce the transmissibility. Note that, the expression of damping ratio is taken from the derivation of the original stiffness $\zeta = c/\left(2\sqrt{km}\right)$, which is the value of damping ratio before applying active stiffness. In fact, the new value of damping ratio as active stiffness being applied is $\zeta^* = \zeta/\sqrt{1+K}$.

The second effect of active stiffness is in changing the natural frequency of the system. This is because the employment of active stiffness will directly modify the stiffness of the system. Therefore, when the active stiffness increases, the natural frequency of the system will be increased. This new natural frequency of the system can be given as $\Omega = \sqrt{1+K}$.

Consequently, the resonance peak of the system with active stiffness can be expressed approximately as

$$T \approx \sqrt{1 + \frac{1}{(2\zeta)^2 (1+K)}} \quad (5)$$

where $\zeta \ll 1$ This expression demonstrates that the damping ratio and active stiffness have major effect in determining the reduction of the resonance peak of the system. As the active stiffness increases, the resonance peak will be decreased. However, it is obvious that the reduction of the resonance peak will not go below 0 dB even the active stiffness is infinite.

It is also noticeable that, at high frequencies both of the transmissibility plots for passive and active stiffness system converge to a same asymptote line of $T = \left|2\zeta/\Omega\right|$ which represents the reduction-20dB/decade. This demonstrates that the active stiffness does not have any effect at high frequencies if the damping coefficient of the system is held constant.

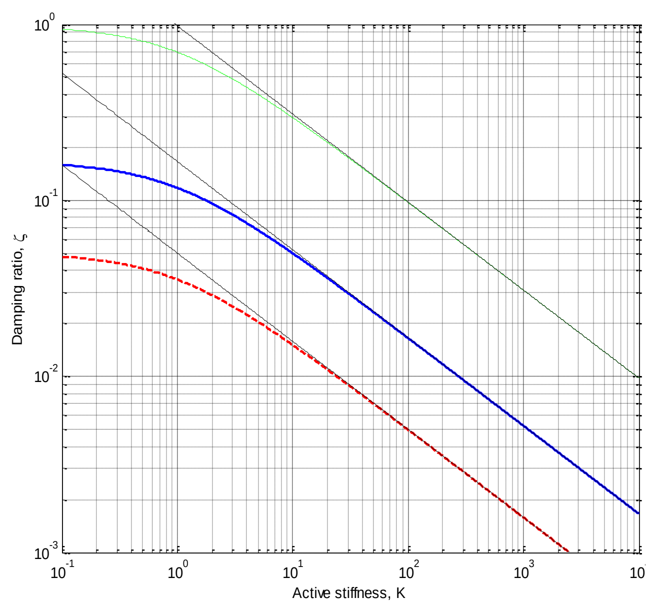


Figure 4 The relationship between damping ratio and active stiffness in achieving for various resonance peak levels. Green line denotes 1dB, bold solid line denotes 10 dB, and red dashed line denotes 20 dB. Black line represents asymptote lines

Based on (5), the relationship between damping ratio and active stiffness in achieving a certain level of resonance peak for $K \gg 1$ can be expressed as

$$\zeta^2 K = \frac{1}{4\left(10^{r_p/10} - 1\right)} \quad (6)$$

where r_p is the level of resonance peak in dB. This inversely proportional relationship between the damping ratio and active stiffness is illustrated in Figure 4, with corresponding asymptote lines.

3.2 The Effect of Active Stiffness on Reducing the Force Response

The force response is very important for measuring the sensitivity or robustness of the system to any direct disturbance. In practice, when the stiffness of the system is low, the force response will be high. This shows that the system is very sensitive to external force. As a result, any external force on the system will make a larger mass displacement. This situation may deteriorate the performance of an isolation system, if it has displacement constraint. Therefore the aim on the application of active stiffness is to overcome this force response problem.

The expression of force response for active stiffness vibration isolation system can be expressed as

$$\left|\frac{X}{F}\right| = \left|\frac{1}{(k+k_{sky}) - m\omega^2 + jc\omega}\right| \quad (7)$$

Equation (7) can be non-dimensionalised as

$$\left|\frac{kX}{F}\right| = \left|\frac{1}{(1+K) - \Omega^2 + j2\zeta\Omega}\right| \quad (8)$$

By having a fully passive system where K=0, the force response in (8) at low frequency $\Omega = 0$ can be determined as $X/F = 1/k$. It is clear that if the stiffness of the system is low, the force response of the system will be high.

For this reason, the function of active stiffness as described in (8) is to increase the stiffness artificially such that the force response will be lowered. The effect of active stiffness in reducing the force response is demonstrated in Figure 5. There are two main purposes in the plot. The first purpose is to show that the force response is high at low frequency when the system has low stiffness. This high force response plot is presented in the blue bold solid, in comparison to the green solid line which denotes the force response plot for system with higher stiffness. Meanwhile, the second purpose in the plot is to demonstrate the function of active stiffness in reducing the force response of the low stiffness system, as illustrated in red dashed line.

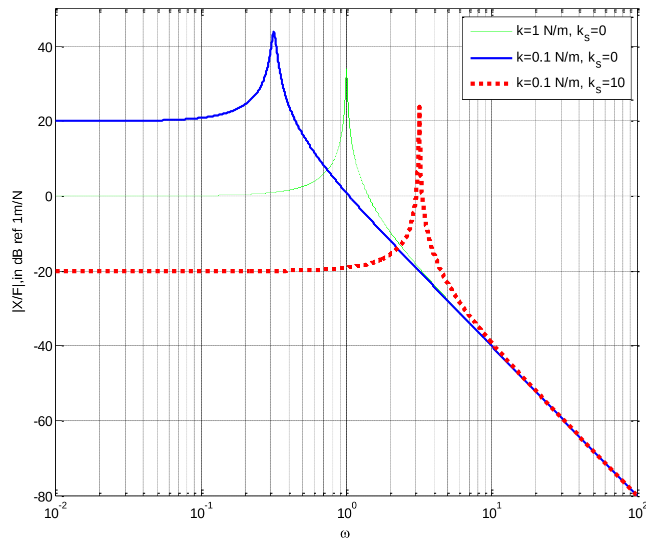


Figure 5 Comparison of receptance plot for three different configurations. Green line represents fully passive system with stiffness (k=1 N/m), blue bold solid line represents fully passive system with low stiffness (k=0.1 N/m), and red dashed line represents hybrid system with the combination of low passive and active stiffness (k=0.1 N/m, ks=10 N/m)

4.0 CONCLUSIONS

This paper has presented a theoretical study on the effect of active stiffness on the performance of vibration isolation system for both of transmissibility and force response. The active stiffness is achieved through the application of actuator into the system using displacement feedback control. The result has shown that active stiffness is beneficial in improving the transmissibility performance at low frequency. Meanwhile, the system becomes more robust to external force disturbance when active stiffness is applied.

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