

## STATE-DEPENDENT BOUNDARY LAYER METHOD FOR ATTITUDE CONTROL OF SATELLITE

Nurul Syazwani Hussain<sup>a\*</sup>, Hassrizal Hasan Basri<sup>a</sup>, Sazali Yaacob<sup>b</sup>

<sup>a</sup>School of Mechatronic Engineering, Universiti Malaysia Perlis, Perlis, Malaysia

<sup>b</sup>Universiti Kuala Lumpur Malaysian Spanish Institute, Kulim Hi-TechPark, 09000 Kulim, Kedah, Malaysia

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\*Corresponding author  
ha\_niey88@yahoo.com

### Graphical abstract



### Abstract

Sliding mode control is known to be robust against parameter uncertainties and external disturbances. Based on the dynamic equation of motion, a sliding mode controller is designed to solve this problem. However, for the sliding surface to be attractive, a switching function is used in the control law, which caused chattering of the control signal. In order to avoid this, a boundary layer method is considered in the modified controller. So this paper proposes new boundary layer designs that resolve the problem in control accuracy and control signal smoothness in sliding mode control. The propose design improve the system state to almost zero with no chattering in the control signals.

*Keywords:* Sliding mode control, mathematical model, attitude control, state dependant boundary layer

### Abstrak

Gelongsor kawalan mod dikenali sebagai teguh terhadap ketidaktentuan parameter dan gangguan luaran. Berdasarkan persamaan dinamik gerakan, mod kawalan gelongsor direka untuk menyelesaikan masalah ini. Walau bagaimanapun, bagi permukaan gelongsor untuk menarik, fungsi pensuisan digunakan dalam undang-undang kawalan, yang menyebabkan gelugutan isyarat kawalan. Untuk mengelakkan ini, satu kaedah lapisan sempadan dianggap di dalam alat kawalan diubah suai. Jadi kertas ini mencadangkan reka bentuk lapisan sempadan baru yang menyelesaikan masalah dalam ketepatan kawalan dan kawalan isyarat kelancaran dalam gelongsor kawalan mod. Yang mencadangkan reka bentuk memperbaiki keadaan sistem kepada hampir sifar tanpa gelugutan dalam isyarat kawalan.

*Kata kunci:* Gelongsor kawalan mod, model matematik, kawalan sikap, keadaan lapisan sempadan bergantung

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## 1.0 INTRODUCTION

The attitude of a satellite can be as important to control as its position. The motion is given by the satellite body rotation with respect to different frames of motion. This research starts by choosing the suitable methods that can be apply and to get the suitable nonlinear mathematical model of satellite attitude is

described by the dynamic equations of motion and kinematic equations of motion. There are a several methods that can be used to determine the controller. For spacecraft, sliding mode control is widely used in design control system. It has features like disturbance rejection, uncertainty insensitivity, strong robustness and fast response.

Sliding mode control is a nonlinear technique which possesses two important features. First, it can reject the external disturbances and second, it is robust to the system parameter variations. The sliding-mode control is developed for the spacecraft attitude reorientation and detumbling. For sliding surface to be more attractive, a switching function must be used in the control law, which cause chattering of the control signal. To eliminates chattering, one can introduce is boundary layer method, that is apply around the sliding surface. The boundary layer can cause the control accuracy not precise. So this paper proposes new boundary layer designs that resolve the problem in control accuracy and control signal smoothness in sliding mode control.

### 2.0 MATHEMATICAL MODEL OF SATELLITE ATTITUDE DYNAMICS

In a two-component gel, it is easy to modify the molecular structure of either of the two components. The modeling of changing angular position and orientation rate of satellite, due to external forces acting on the body is known as attitude dynamics as shown in Figure 1. The satellite dynamics model describes the dependences between external torques and the satellite's angular velocity.

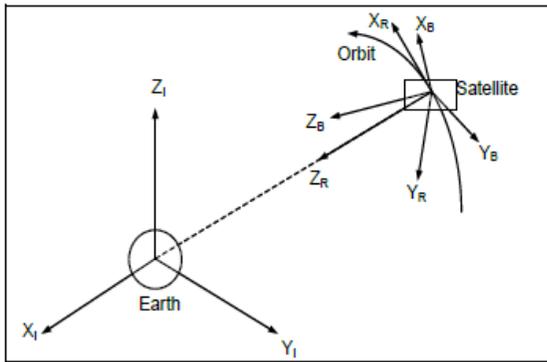


Figure 1 Definition of the orbit reference frame [1]

The attitude motion of the satellite is modeled by the Euler equation for the motion of a rigid body under the influence of external torques. The total external torques acting on the body is equal to the inertial momentum change of the system. In this section the dynamic equation for general satellite with momentum exchange devices like reaction wheels for example will be developed. Thus, the attitude dynamics equation of satellite in the satellite body frame is derived from Euler's moment equation (1):

$$T = \dot{h} = \dot{h} + \omega * h \tag{1}$$

is denotes the external torque, is breakdown into two principal parts;  $T_c$ , control torques can be used for controlling the attitude motion of the satellite; and  $T_d$ , those torques due to the different disturbing

environmental phenomena. The momentum of the entire system is divided into the angular momentums of the rigid body,  $h = [h_x \ h_y \ h_z]^T$  and the moment exchanges devices  $h_w = [h_{wx} \ h_{wy} \ h_{wz}]^T$ . Finally,  $\omega$  is the angular velocity vector of the body frame with respect to inertial frame. The general equation of motion becomes:

$$\begin{bmatrix} T_{cx} + T_{dx} \\ T_{cy} + T_{dy} \\ T_{cz} + T_{dz} \end{bmatrix} = \begin{bmatrix} \dot{h}_x + \dot{h}_{wx} + (\omega_y h_z - \omega_z h_y) + (\omega_y h_{wz} - \omega_z h_{wy}) \\ \dot{h}_y + \dot{h}_{wy} + (\omega_z h_x - \omega_x h_z) + (\omega_z h_{wx} - \omega_x h_{wz}) \\ \dot{h}_z + \dot{h}_{wz} + (\omega_x h_y - \omega_y h_x) + (\omega_x h_{wy} - \omega_y h_{wx}) \end{bmatrix} \tag{2}$$

The angular velocity of the satellite, expressed in the body axes is given by:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \ddot{\phi} - \phi \omega_o \\ \dot{\theta} - \omega_o \\ \dot{\phi} + \dot{\theta} \omega_o \end{bmatrix} \tag{3}$$

And its first derivative:

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} \ddot{\phi} - \dot{\phi} \omega_o \\ \ddot{\theta} \\ \ddot{\phi} + \dot{\theta} \omega_o \end{bmatrix} \tag{4}$$

Where the Euler angles  $\phi$ ,  $\theta$ , and  $\phi$  are the rotational angles about the  $X_B$ ,  $Y_B$  and  $Z_B$  axis, respectively. While  $\omega_o$ , is the orbital angular velocity of the satellite.

From the definition of angular momentum i.e.  $h = I\omega$ , then the component wise of the angular momentum is given by

$$\begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} I_x \omega_x \\ I_y \omega_y \\ I_z \omega_z \end{bmatrix} \tag{5}$$

Hence, by using definition in (5) together with equation (3) and (4), equation (2) becomes

$$\begin{aligned} T_{dx} + T_{cx} &= I_x \ddot{\phi} + \omega_o (I_y - I_z - I_x) \dot{\phi} + \omega_o^2 (I_y - I_z) \phi \\ &+ \dot{h}_{wx} - \omega_o h_{wz} - \dot{\phi} h_{wy} - \phi \omega_o h_{wy} + (I_z - I_y) \dot{\phi} \dot{\theta} + \omega_o (I_z - I_y) \dot{\theta} \dot{\phi} + \dot{\theta} h_{wz} \end{aligned} \tag{6a}$$

$$\begin{aligned} T_{dy} + T_{cy} &= I_y \ddot{\theta} + h_{wy} + (I_z - I_y) \dot{\phi} \dot{\theta} + \omega_o (I_x - I_z) \dot{\theta} \dot{\phi} \\ &+ \omega_o (I_z - I_x) \phi \dot{\phi} + \omega_o^2 (I_z - I_x) \phi \dot{\phi} + \dot{\phi} h_{wx} + \omega_o \dot{\theta} h_{wx} - \dot{\theta} h_{wz} + \omega_o \phi h_{wz} \end{aligned} \tag{6b}$$

$$\begin{aligned} T_{dz} + T_{cz} &= I_z \ddot{\phi} + \omega_o (I_z + I_x - I_y) \dot{\phi} + \omega_o^2 (I_y - I_x) \phi + \dot{h}_{wz} \\ &+ \omega_o h_{wx} + \dot{\theta} h_{wy} - \phi \omega_o h_{wy} + (I_y - I_x) \dot{\theta} \dot{\phi} - \omega_o (I_y - I_x) \phi \dot{\theta} - \phi h_{wx} \end{aligned} \tag{6c}$$

These equations are nonlinear.

### 3.0 SLIDING MODE CONTROLLER DESIGN

$$\dot{x} = Ax + B(u + \Delta E x + d), \quad x(0) = x_0 \tag{7}$$

Where  $x = [x_1, x_2, \dots, x^{n-1}, x^n]^T$  the state variable vector, uncertainty is  $\Delta E$  is possibly time-varying, and  $d$  an unknown disturbance. Note that one can always perform a state transformation such that the

controllable pair (A,B) is in the controller canonical form

The objective of sliding mode control is to regulate the state  $x$  in (7) to zero, and this is achieved by a two-stage control design.

### 3.1 Design of the Sliding Variable

Augmented state are define as

$$\dot{v} = x_1 \quad \text{or} \quad v = \int_0^t x_1 d\tau \quad (8)$$

$$\begin{aligned} s &= Cx + c_0 v \quad \text{where} \quad C = [c_0, c_1, \dots, 1] \\ &= x_n + c_{n-1}x_{n-1} + \dots + c_1x_1 + c_0 \int_0^t x_1 d\tau \\ &= x_1^{(n-1)} + c_{n-1}x_1^{(n-2)} + \dots + c_1x_1 + c_0 \int_0^t x_1 d\tau \end{aligned} \quad (9)$$

Next, the differential equation (8) and (10) can be cast into a state space

$$\dot{z} = Tz + Gs \quad (10)$$

Where,

$$z = \left[ \int_0^t x_1 d\tau \quad x_1 \quad \dots \quad x_{n-1} \right]^T \quad (11)$$

Where matrices T and G are in the controller canonical form

$$T = \begin{bmatrix} 0 & 1 & \cdot & \cdot \\ \cdot & 0 & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -m_0 & \cdot & \cdot & -m_{n-1} \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix} \quad (12)$$

Then the Lyapunov inequality equation is used to determine the positive definite matrix (P) matrix by using the Lyapunov inequality shown in (13) where  $\sigma$  positive constant is.

$$(-T - \sigma I)^T P + P(-T - \sigma I) \leq 0 \quad (13)$$

### 3.2 Design of the Control Input

The stable state space equation (5) suggest that if the sliding variable  $s$  can driven to zero by some control design, the state  $z$  will decay to zero. Therefore, the switching sliding mode control to drive  $s$  to zero shown

$$u = -\sigma s - c_0 x_1 - CAx - \rho(x)f_2(s) + \eta_1^2 G^T Pz + \eta_0 \eta_1 G^T P e_z \quad (14)$$

Where  $\sigma > 0$ ,  $s$  is the sliding variable,  $\rho(x) = \rho(u)$  and  $\rho > 1$ ,  $f_2(s)$  is the switching function, and finally,  $\eta_0$  and  $\eta_1$  represent the boundary layer function

$$f_2(s) = \frac{s}{|s| + \epsilon_1} \frac{1}{|z| + \epsilon_0} \quad , \quad \|z\|_p \triangleq \sqrt{z^T P z} \quad (15)$$

In which  $\epsilon_1 > 0$  and  $1 \gg \epsilon_0 > 0$ .

## 4.0 RESULT

This section, a simulation using MATLAB software is carried out to demonstrate the

- State dependant boundary layer control
- Constant/Decaying-width boundary layer control

The satellite considered in the simulation is a mini-satellite of Malaysia, RazakSAT. The satellite characteristics of RazakSAT are given in the Table 1, which is was provided by Astronautic Technology Sdn Bhd (ATSB), the Malaysian company who's incharged for RazakSAT's mission.

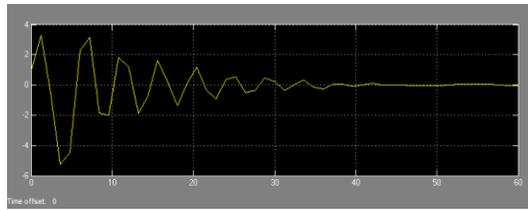
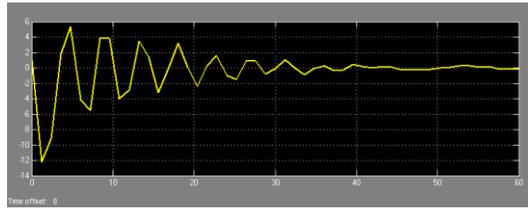
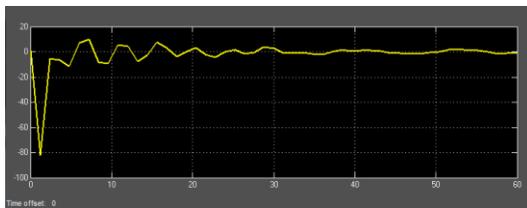
Table 1 RazakSAT's characteristics

Moment of Inertia, $I_x$	$2.54 \times 10^7 \text{ kgmm}^2$
Moment of Inertia, $I_y$	$2.62 \times 10^7 \text{ kgmm}^2$
Moment of Inertia, $I_z$	$2.10 \times 10^7 \text{ kgmm}^2$
Orbital rate, $\omega_0$	0.0609 deg/s

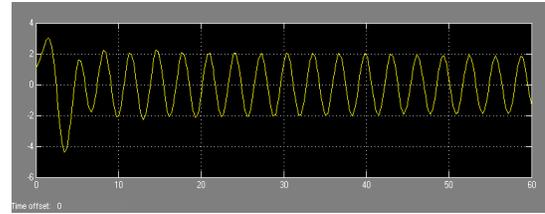
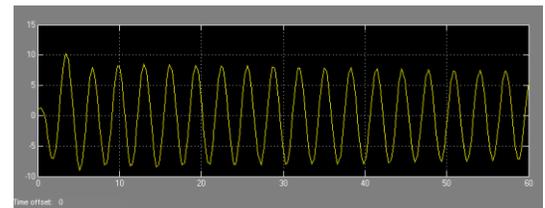
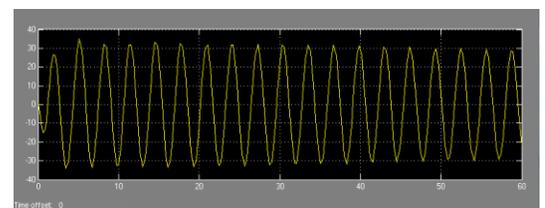
The comparison is made between the state-dependant boundary layer and constant/decaying-width boundary layer control. The results show in Figure 2 and 3.

## 5.0 CONCLUSION

This paper is proposes a new boundary layer design that resolves the long existent design dilemma in sliding mode control between the requirement of control accuracy and control signal smoothness. The elimination of chattering effect in Sliding Mode Control by using state-dependent boundary layer method is chosen. This technique will manipulated the state variables as the component to determine the boundary layer width depends on its current state value as the main concept. Besides that, this improved existing state-dependent boundary layer method is proven capable in handling multiples types of disturbances and most importantly this method also able to reduce the chattering effect inside boundary layer while maintaining the robustness of the attitude control of Mini-Satellite. As for chattering problem, state dependent boundary layer method is use to eliminates of chattering effect.

a) Time response of roll angle,  $\phi$ b) Time response of pitch angle,  $\theta$ c) Time response of yaw angle,  $\phi$ 

**Figure 2** Chattering elimination inside the boundary layer with state-dependent boundary layer

a) Time response of roll angle,  $\phi$ b) Time response of pitch angle,  $\theta$ c) Time response of yaw angle,  $\phi$ 

**Figure 3** Chattering elimination inside the constant/decaying-width boundary layer control

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