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THE INTEGRATED DETERMINISTIC MODEL FOR A VENDOR MANAGED INVENTORY IN A TWO-STAGE SUPPLY CHAIN

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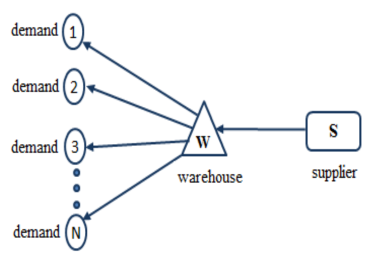
Graphical abstract



Abstract

In a two-stage supply chain system, vendor managed inventory (VMI) policies is an integrating decisions between a supplier and his retailers. The supplier assumes the responsibility of maintaining inventory at its retailers and ensuring that they will not run out of stock at any moment. This paper discusses an optimization approach, considering the model of static demand on the inbound as well as the outbound inventory for a two-stage supply chain implementing VMI. In the proposed solutions for coordinating the single-warehouse multiple-retailers (SWMR) system, retailers are first clustered to minimize the within-cluster travel costs and distances and are then replenished using an optimal direct shipping strategy satisfying some additional restrictions.

Keywords: Inventory control, distribution, supply chain optimization, deterministic demand

1.0 INTRODUCTION

Vendor Managed Inventory (VMI) is an inventory management policy, in which the supplier assumes, in addition to its inbound inventory, the responsibility of maintaining inventory at the retailers and ensures that they will not run out of stock at any moment. The delivery times and quantities to deliver to a retailer is no longer done after the retailers' orders, the supplier determines the quantity and when the delivery takes place. The replenishment is thus proactive as it is based on the available inventory information instead of being reactive in response to retailers' orders. This policy has many advantages for both the supplier and the retailers. The supplier has the possibility of combining multiple deliveries to optimize the truck loading and the routing cost. Moreover, as the deliveries become more uniform, the amount of inventory that must be held at the supplier can be drastically reduced. On the other hand, the retailers need no longer to dedicate resources to the management of their inventories. Also, the service level (i.e. product availability) increases, as the supplier can track inventory levels at the retailers to determine the precise replenishment urgency.

The reason VMI gains more popularity is the current enabling technologies to monitor retailer inventories through an online system and cost effective manner. Inventory data can be made available much easier. However, implementing VMI does not in all cases lead to improved results. Failure can happen due, for example, to the unavailability of the necessary information or the inability of the supplier to make the right decisions. The large amount of data makes it extremely hard to optimize this problem. It involves managing inventory in supply chains and optimizing distribution, which are two particularly challenging problems.

In this paper, we analyze the model of deterministic demand on a two-stage supply system implementing VMI. We focus the problem of coordinating the single-warehouse multiple-retailers (SWMR) system. We also consider the inventory routing problem (IRP) where a single warehouse serves n retailers. Each retailer j faces deterministic demand at a constant rate d_j. Inventories are kept at the warehouse as well as at each retailer. Whenever the orders are placed by the retailers, the deliveries are made from the warehouse with a homogenous fleet of vehicles of limited capacity. Then, the warehouse in turn places orders to an outside supplier to fill the orders of the retailers (see Figure 1). For instance, we try to consider the simplest case of SWMR as follows: a fixed charge is incurred whenever the warehouse places an order. Similarly, for each order placed by each retailer, a facility-dependent setup cost is charged. Also, there is a facility-dependent holding cost of inventory at each facility in the system. In this simplest case, there is no known polynomial method for solving SWMR problem under a given information environment. So, the power-of-two approximation of Roundy [1] is currently the best heuristic available.

An approach is proposed to minimize the overall inventory and distribution costs of the SWMR system while taking into account retailers' demands at the supplier. The problem is attempted by repeating the steps described below. In the first step, retailers are clustered to minimize the traveled distance or equivalently distribution costs. Then, a direct shipping procedure is used to determine the optimal replenishment schedule for the fixed retailer groups. In a third step, retailers can be switched from group to group to again optimize the total costs by local search combined with a simulation.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the integrated deterministic model for SWMR system and section 4 describes the proposed solution approach. Finally, section 5 provides conclusions and some direction for future research approach.

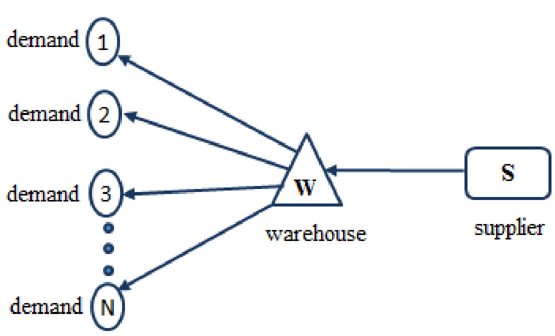


Figure1 A two-echelon inventory system

2.0 LITERATURE REVIEW

One stream of research related to this problem is the single warehouse and multiple-retailers inventory models taking transportation cost into account. Examples of such studies were carried out by [2, 3, 4, 5, 6, 7, 21]. Gallego and Simchi-Levi [8] showed that direct shipping policies which each vehicle visits a single retailer, are within 6% of optimality under certain restricted parameter settings. Furthermore, good empirical performance for the so-called power-of-two strategies under which each retailer is replenished at constant intervals, which are power-of-two multiples of a common base planning period were described in [9, 10]. The effectiveness of a large class of policies, called zero inventory ordering (ZIO) policies, for the single warehouse multiple-retailers system was analyzed in [11]. In this class, a retailer receives an order when its inventory level is down to zero. This analysis is motivated by the observations that direct shipping, power-of-two policies.

An extension of this research line is concerned with the integrated models, involving location-inventory network design that integrates the location and inventory decisions. Some studies on a practical distribution network design problem for computer spare parts have been reported in [12]. Their model takes into account the inventory cost at the various warehouses. Also, an analytical model was developed to minimize the total fixed operating costs and inventory holding costs incurred by warehouses, together with the transportation costs [13]. The model is solved heuristically. Shen et al. [14] and Daskin et al. [15] considered the case where retailers are facing uncertain demands following a Poisson distribution, and Shu et al. [16] solved the problem for general demand distribution.

In all models, the inventory holding costs at the retailers are ignored. The model considered here does not consider the design issue. However, it takes all inventories at the warehouse as well as at the retailers into account. In this paper, we also take into account the loading and unloading costs of the homogenous vehicle. The VMI policy addresses the issue of coordinating the warehouse and retailers inventory replenishment activities to minimize the system-wide multi-echelon ordering and holding costs.

3.0 DETERMINISTIC MODEL

For the model development, let R be the set of retailers, indexed by i. Let $R^+=R\cup\{0\}$, where 0 indicates the warehouse and V the set of available vehicle. We also define the following notations:

- ψ^v: the fixed operating and maintenance costs of vehicle $v\in V$;
- t_{ij}: the duration of a trip from retailer $i\in R^+=R\cup\{0\}$ to retailer $j\in R^+$;
- τ_{ij}: a per unit transportation cost from the warehouse or retailer i to retailer j;
- φ_0: a fixed ordering cost incurred by the warehouse each time it places an order; the ordering cost is independent of the order quantity;
- φ_j: a fixed ordering cost incurred by each retailer $j\in R$ each time it places an order from the

warehouse; the fixed ordering cost is independent of the order quantity;

- h_0: the per unit per year inventory holding cost rate in warehouse 0;
- h_j: the per unit per year inventory holding cost rate in retailer j;
- d_j: a constant demand rate per year faced by retailer j;
- T_0: the replenishment interval at warehouse 0;
- T_j: the replenishment interval at retailer j;
- θ_Δ: the fixed vehicle loading and dispatching cost;
- θ_j: the fixed unloading cost at the retailers.

Assume that retailers are clustered and served by the set of vehicles v in V^*, and let R^v be the set retailers served by vehicle v. If customer j is served by vehicle v, then $T^v = T_j$. The objective function to be optimized is:

SWMR:

$$Z_D = \sum_{v\in V^*}\psi^v + \frac{\varphi_0}{T_0} + \sum_{v\in V^*}\left(\frac{1}{2}h_0\left(\sum_{j\in R^v} d_j\right)\left[\max\left(T_0, T^v\right) - T^v\right]\right) + \sum_{v\in V^*}\left(\frac{1}{T^v}\left(\tau^v + \sum_{j\in R^v}\varphi_j\right) + \frac{1}{2}\left(\sum_{j\in R^v} h_j d_j\right)T^v\right) \quad (1)$$

where $\tau^v = \Sigma_{(i,j)\in \mathrm{Trip}(v)}\tau_{ij}$ is the total travel cost of the complete trip made by vehicle v, satisfying the restrictions that $\Sigma_{(i,j)\in \mathrm{Trip}(v)}t_{ij} \le T^v$ and that the total amount delivered to the retailers in each tour made by the vehicle during its trip, Trip(v), should not exceed the vehicle's capacity.

The most effective way in terms of travel distance to supply these retailers is to travel along the shortest tour that visits the warehouse and all of the retailers in R, or the TSP-tour through R plus the warehouse. This tour gives a solution for the infinite time horizon. The cycle time T is the time between two consecutive iterations of the tour. The tour cannot be restarted before it is finished, so the total time needed to complete a tour gives a lower bound on the cycle time. Therefore, the minimal cycle time for replenishing the set of retailers R in a single tour served by the vehicle v, denoted by $T^v{}_{\min}$, is given by the following formula:

$$T^v{}_{\min} = \sum_{j\in R^v} T_{TSP}(R^v \cup \{0\}) \quad (2)$$

On the other hand, the capacity of vehicle v induces an upper bound on the replenishment cycle of the retailers it serves. The upper bound on this cycle time is called maximal cycle time, which is denoted by $T^v{}_{\max}$. The following formula gives the maximal cycle time for replenishing the set of retailers R in a single tour by the vehicle v.

$$T^v{}_{\max} = \min_{j\in R^v}\left\{\frac{k}{\sum_{j\in R^v} d_j}\right\} \quad (3)$$

Where k is the capacity of the vehicle and d_j is the demand rate of retailer $j\in R$. Obviously, a tour is only feasible if its minimal cycle time is not greater than its maximal cycle time: $T^v{}_{\min} \le T^v{}_{\max}$.

Assuming the power-of-two inventory stationary policy, in which each retailer is replenished at equally distant time intervals which are power-of-two multiples of a common base planning period, T_B. In the absence of the travel and vehicle capacity restrictions on T^v, Roundy [1] showed that the convex programming relaxation of (1) approximates the optimal solution value to 98% accuracy.

If we also assume that (h_j–h_0) > 0 for every retailer j, we can summarize the main results of Roundy [1] as is done in [17] for the basic model single-warehouse multiple-retailers (SWMR): The solution of (1) is a lower bound on the average cost of any feasible inventory control policy, and the solution can be rounded off to obtain a feasible integer-ratio policy with a cost within 98% of the minimum of (1). Such a policy can be computed in $O(n\log(n))$ time (see the algorithm below). Furthermore, in the solution to (1), the retailers can be divided into three groups: G, L, and E.

For retailers in G, the replenishment interval is given by:

$$\hat{T}^v = \sqrt{\frac{2\left[\theta_\Delta + \tau^v + \sum_{j\in R^v}\varphi_j + \theta_j\right]}{\sum_{j\in R^v} h_j d_j}} > \hat{T}_0 \quad (4)$$

For retailers in L, the replenishment interval is given by:

$$\hat{T}^v = \sqrt{\frac{2\left[\theta_\Delta + \tau^v + \sum_{j\in R^v}\varphi_j + \theta_j\right]}{\sum_{j\in R^v}\left(h_j - h_0\right)d_j}} < \hat{T}_0 \quad (5)$$

Finally, for retailers in E, the replenishment interval is the same as that at the warehouse and given by:

$$\hat{T}^v = \hat{T}_0 = \sqrt{\frac{2\left(\varphi_0 + \sum_{v\in E}\left[\theta_\Delta + \tau^v + \sum_{j\in R^v}\varphi_j + \theta_j\right]\right)}{\sum_{v\in E}\sum_{j\in R^v} h_j d_j + \sum_{v\in L}\sum_{j\in R^v} h_0 d_j}} \quad (6)$$

If we start from a feasible partition $(R^v)_{v\in V^*}$ of retailers, that satisfies $T^v{}_{\min} = \Sigma_{(i,j)\in \mathrm{Trip}(v)}t_{ij} \le T^v{}_{\max}$ defining the smallest cycle obtained from the total amount delivered to the retailers served during each sub-tour made by the vehicle. We can determine the optimal values for each vehicle v, as follows:

$$T_{opt}^{v} = \begin{cases} \hat{T}^{v}, & \text{if } T_{\min}^{v} \le \hat{T}^{v} \le T_{\max}^{v} \\ T_{\min}^{v}, & \text{if } T_{\min}^{v} > \hat{T}^{v} \\ T_{\max}^{v}, & \text{if } \hat{T}^{v} > T_{\max}^{v} \end{cases} \quad (7)$$

To complete the procedure we need to develop an algorithm that determines the optimal feasible partition of retailers. This can be achieved by means of a combined solution method for SWMR problem combined with any effective heuristic for the vehicle routing problem. In this paper, the constrained vehicle routing problem (VRP) is solved using a constructive local search procedure that maintains for each vehicle the condition $T^{v}_{\min} = \Sigma_{(i,j)\in \mathrm{Trip}(v)} t_{ij} \le T^{v}_{\max}$ on its cycle time.

4.0 SOLUTION APPROACH

For the solution of the complete SWMR problem, firstly, we adapted the algorithm proposed by [1] to minimize inventory cost and determine the possible retailers set partitions *G, E, L*. Then, we use a saving heuristic algorithm to solve the constrained VRP problem for elements in *E* to cluster retailers as much as possible with the objective of minimizing transportation cost.

4.1 Roundy's Algorithm

In the first step, we adapted the algorithm proposed by [1] to solve the SWMR problem. The objectives are to minimize inventory cost and determine the possible retailers set partitions *G, E* and *L*. Before the paper by [1], most researchers attacked this problem by restricting themselves to special policies such as nested policy and stationary policy. A policy is stationary if the order intervals are constant for each facility. A policy is nested if a facility orders every time any of its immediate suppliers does. Unfortunately, he showed that the effectiveness of an optimal nested policy can be arbitrarily close to zero and a nested policy may have a rather poor cost performance. The effectiveness is defined as the ratio of the optimal value and the heuristic value of the objective function.

Therefore, Roundy [1] introduced two types of policies, namely, the integer-ratio policy and the power-of-two policy. An integer-ratio policy presented how to compute the average costs. This policy is a stationary policy in which the order interval of each facility in the system is an integer multiple of a base planning period, T_B. The power-of-two policy is a subset of integer-ratio policy that each facility orders at a power-of-two multiple of a base planning period, T_B. He showed that for multi-retailer inventory model, the average cost of the optimal power-of-two policy is within 6% of the average cost of any feasible policy. This result has made power-of-two policies very attractive. The complexity of both the policies developed by [1] is O(*n* log *n*), where *n* is the total number of retailers.

Now, we discover in more detail the solution procedure introduced by [1]. Firstly, we assume that no shortage or backlogging is permitted. Without loss of generality, replenishment is assumed to be instantaneous. Moreover, let us assume that the base planning period, T_B, is fixed which is one hour and that only power-of-two policies are employed. The order interval of each facility in the system is a power-of-two multiple of T_B.

Case 1: Retailer Order Interval Greater than Warehouse Order Interval.

When determining the total average costs of the system, if $T_0 \le T_j$, the warehouse places an order for retailers *j* at the same time when retailer *j* make an order. Therefore, no inventory of product *j* is held at the warehouse, and the only costs to consider are those incurred at the retailer *j*. So, the average total costs to retailer *j* is:

$$c_j(T_j) = \frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j \quad (8)$$

Case 2: Retailer Order Interval Less than Warehouse Order Interval.

If $T_0 \le T_j$, the warehouse orders $T_0 d_j$ for retailer *j* every T_0. Both the warehouse and retailer *j* carry inventories in this case. The system inventory of product *j* is the sum of the inventory of product *j* at retailer *j* and the inventory at the warehouse. Therefore, the average inventory total costs to retailer *j* is:

$$g_j(T_j) = \frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j + \frac{1}{2} h_o d_j (T_o - T_j) \quad (9)$$

So, given T_0 and T_j , we can calculate the total average costs to retailer *j*, c(T_0; T_j), is:

$$C(T_o, T_j) = \frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j + \frac{1}{2} h_o d_j [\max(T_o, T_j) - T_j] \quad (10)$$

As a result, the formula for solving the problem of the SWMR system with the objective function of minimizing total average cost at both the warehouse and retailers is given below:

$$\min c(T) = \frac{\varphi_o}{T_o} + \sum_{i=1}^{n} \frac{1}{2} h_o d_j [\max(T_o, T_j) - T_j] + \sum_{i=1}^{n} \frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j \quad (11)$$

Subject to

$$T_0 = 2^{k0}\, T_B \quad (12)$$
$$T_j = 2^{kj}\, T_B \quad (13)$$

Constraints (12) and (13) restrict the retailer and warehouse reorder intervals to satisfy the power-of-two policy. So, we solve the relaxed problem where the power-of-two constraints (12) and (13) are ignored. By definition, τ'_j is the optimal solution to $c_j(T_j)$ and τ_j is the optimal solution to $g_j(T_j)$, it is easy to verify that $\tau'_j \le \tau_j$. Both c(*T*) and c(T_0; T_j) are convex in

T_0, and the optimal solution to the relaxed problem, given T_0, as follows:

$$T_j = \begin{cases} \tau'_j & \text{if } T_0 < T_j \\ T_0 & \text{if } \tau'_j \leq T_0 \leq \tau_j \\ \tau_j & \text{if } \tau_j < T_0 \end{cases} \tag{14}$$

4.2 A Close Optimal Power-of-Two Policy

In the second step, we adapted the algorithm developed by [18]. They considered only a feasible power-of-two policies multiples of the base order interval (T_B). From this algorithm we can determine the possible set partitions of retailers in G, E and L. Firstly, they started by letting T_0 be a power-of-two policy, the proposed method finds the corresponding optimal power-of-two policy, T_j , for each retailer j, and calculates the corresponding total average costs of the system. Then, they increased T_0 to the next power-of-two period until the total average costs of the system increases at which point the optimal power-of-two policy is found (see Figure 2).

The optimal power-of-two solution, t'_j and t_j , are used instead of τ'_j and τ_j because the proposed method only considers the power of-two policies. By definition, $T_j = 2^{kj}T_B$ and $T_j = 2^{mj}T_B$ are the optimal power-of-two solutions to $c_j(T_j)$ and $g_j(T_j)$, respectively. The optimal power-of-two solutions t'_j and t_j, is obtained by rounding the solution to the power-of-two multiple of T_B.

This algorithm has proven that for a given T_0, the optimal power-of-two policy is given by: (See more detailed algorithm in [18].

$$T_j = \begin{cases} t'_j & \text{if } T_0 < T_j \\ T_0 & \text{if } t'_j \leq T_0 \leq t_j \\ t_j & \text{if } t_j < T_0 \end{cases} \tag{15}$$

Based on (15), and the fact that $c(T)$ is convex T_0, they proposed an iterative heuristic that monitors the changes in total average costs if policy T_i is used instead of policy T_j. Define $t_{min} = \min\{ t'_j : j = 1, ,n\}$ and $t_{max} = \min\{ t_j : j = 1, , n\}$, the heuristic for SWMR system that exploits (15) is summarized as follows:

SWMR Heuristic proposed by [18]:
Step 0: Calculate t'_j and t_j for $j=1,\ldots,n$. Find $t_{min} = \min\{ t'_j: j=1,\ldots,n\}$ and $t_{max} = \max\{ t_j: j=1,\ldots,n\}$. Let $j = 0$, $T_0 = t_{min}$, $T^0=\{\emptyset\}$, and $c(T^0) = \infty$.
Step 1: Choose T_j according to condition (16). Let $T^j=\{ T_0, T_1,\ldots, T_n\}$ and calculate $c(T^j)$ using (11). If $\Delta(T^j, T^{j-1}) < 0$, go to step 2. Otherwise, stop, and the best power-of-two policy is $T^*= T^{j-1}$.
Step 2: If $T_0 < t_{max}$, set $j=j+1$, $T_0 = 2\,T_0$ and go to step 1. If $T_0 = t_{max}$, it means that the optimal T_o falls in the region $[t_{max}, \infty]$. Since for any $T_0 > t_{max}$, the optimal T_j, for all j, remains the same. Therefore, given the optimal T_j, for all j, T_0 can be found by first minimizing (11) with respect to T_0 and then rounding the solution such that $2^{k-1}\sqrt{2T_B} \leq T_o = kT_B \leq 2^k\sqrt{2T_B}$. Stop.

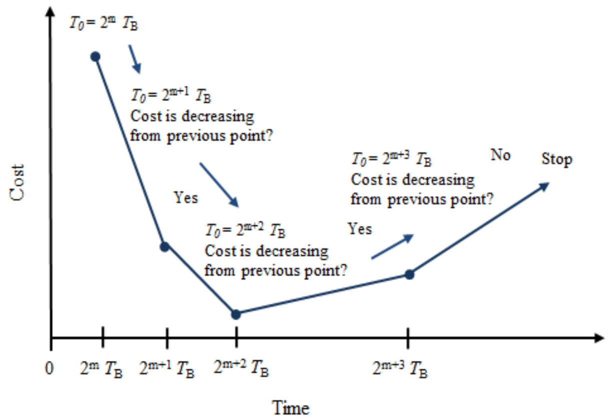


Figure 2 Steps of the algorithm, Chu and Leon (2008)

4.3 A Saving Heuristic Algorithm

In the next step, we use a saving heuristic algorithm developed by [19] to solve the constrained VRP problem. This algorithm is based on a saving concept. The purpose of this algorithm is to select the retailers that will be included in a route by grouping them into a cluster. The saving algorithm finds pairs of retailers that are beneficial in a route and links as many of the pairs as possible. The objective is to find a solution which minimizes the total transportation costs. Moreover, the solution must satisfy the restrictions that every retailer is visited exactly once, where the demanded quantities are delivered, and the total demand on every route must be within the vehicle capacity restriction. The transportation costs are specified as the cost of driving from the warehouse to any other point of the retailers. The costs are not necessarily identical in the two directions between two given points.

Clarke and Wright [19] published an algorithm for the solution of that kind of vehicle routing problem. This saving algorithm is an exchange procedure that was originally developed for the VRP. The algorithm has been designed for VRP is characterized as follows. From the warehouse product must be delivered in given quantities to given retailers. For the transportation of the product a number of vehicles are available, each with a certain capacity with regard to the quantities. Every vehicle that is applied in the solution must cover a route, starting and ending at the warehouse.

Clarke and Wright [19] method is a heuristic algorithm, and therefore it does not provide an optimal solution to the problem with certainty. The method does, however, often yield a relatively good solution. The basic savings concept expresses the cost savings obtained by joining two routes into one route as illustrated in Figure 3, where point 0 represents the warehouse and point i; j represent retailers.

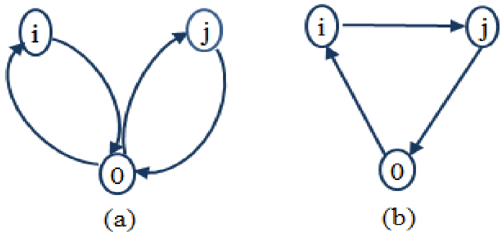


Figure 3 Illustration of the savings concept

Figure 3(a) shows the initial solution consists of a separate route to each of the retailers *i* and *j*. Then, routes in the solution are combined pairwise in order to obtain a better solution as illustrated in Figure 3(b). Because the transportation costs are given, the savings that result from driving the route in Figure 3(b) instead of the two routes in Figure 3(a) can be calculated. Therefore, the total transportation cost D_a in Fig. 3(a) is:

$$D_a = C_{0i} + C_{i0} + C_{0j} + C_{j0} \quad (16)$$

Equivalently, the transportation cost D_b in figure 3(b) is:

$$D_b = C_{0i} + C_{ij} + C_{j0} \quad (17)$$

By combining the two routes, one obtains the savings S_{ij}:

$$S_{ij} = C_{i0} + C_{0j} - C_{ij} \quad (18)$$

The savings S_{ij} indicate that it is attractive with regard to costs, to visit retailers *i* and *j* on the same route such that retailer *j* is visited immediately after retailer *i*. There are two versions of the savings algorithm, a parallel and a sequential version. In the parallel version of the algorithm, the merge yielding the largest saving is always implemented, whereas the sequential version keeps expanding the same route until this is no longer feasible.

In the first step of this saving heuristic algorithm, the savings for all pairs of retailers are calculated and all possible pairs of retailers are sorted in descending order of the savings. Next, from the top of the sorted list of retailer pairs with the largest saving, one pair of retailers is considered at a time. Then, when a pair of retailers *i* - *j* is considered, the two routes that visit *i* and *j* are combined. If this can be done without deleting a previously established direct connection between two retailer pairs, and if the total demand on the resulting route does not exceed the vehicle capacity. So, in the sequential version one must start a new from the top of the list every time a connection is established between a pair of retailers, while the parallel version only requires one pass through the list.

The detailed steps of the Clarke and Wright [19] algorithm for the solution of VRP are as follows:
Step 1: Compute the savings , $S_{ij} = C_{i0} + C_{0j} - C_{ij}$, of combining every possible pair of retailers I and j.
Step 2: Order the savings S_{ij} in a decreasing order.
Step 3: Starting at the top of the list does the following.

Parallel version

Step 4: If a given link result in a feasible route according to the constraints of the VRP. Then append this link to the solution. If not, reject the link.
Step 5: Try the next link in the list and step 4 is repeated until no more links can be chosen.

Sequential version

Step 4: Find the first feasible link in the list, which can be used to extend one of the two ends of the currently constructed route.
Step 5: If the route cannot be expanded further, terminate the route. Choose the first feasible link in the list to start a new route.
Step 6: Step 4 and 5 are repeated until no more links can be chosen.

4.4 An Improvement Heuristic Algorithm

An improvement algorithm begins with an arbitrary solution and ends up in a local minimum where no further improvement is possible. An improvement heuristic is proposed that can be applied to an existing solution at any time to improve the solution quality. The improvement heuristic consists of removing and re-inserting a route. The route is inserted into the existing distribution patterns of the solution, both in a separate tour and in the existing tours, and the cheapest alternative is kept. If a route ends up in the same position as before, the solution is restored and no improvement is found. If the route ends up in a different position, it means that the solution has improved. So, routing plans with lower costs can then be obtained using improvement heuristics that try to apply elementary modifications to the current solution.

The best known improving heuristics for VRP are the edge exchange heuristics. The 2-opt exchange is a very simple, yet very useful, improvement heuristic. It involves exhaustively considering exchanges of two retailers in different routes. The 2-opt exchange method is a local search heuristic was introduced by [20] consists of eliminating two edges and reconnecting the two resulting paths in order to obtain a new tour (see Figure 4). The 2-opt procedure for the VRP is given:
Step 1: Let *T* be the current tour.
Step 2: For every node *i* = 1; 2 , , *n*: Examine all 2-opt moves involving the edge between *i* and its successor in the tour. If the move reduces total cost of the route, implement it and then choose the best such 2-opt move and update *T*.
Step 3: If no improving move could be found, then stop.

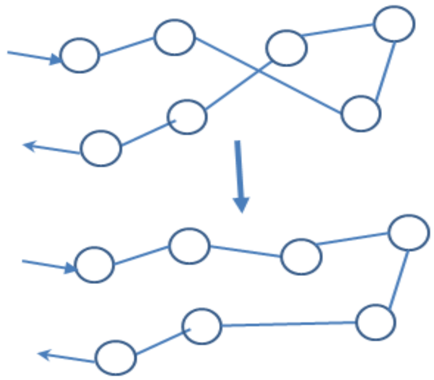


Figure 4 A basic arc interchange in the 2-opt procedure

5.0 CONCLUSION

Managing inventory and routing in a supply network is a very challenging optimization problem. In this paper, we propose a global solution approach for a two-stage supply chain implementing vendor managed inventory (VMI). We focus the problem of coordinating the single-warehouse multiple-retailers (SWMR) system. The SWMR is a simple form of supply chain where retailers draw required material from a single warehouse to satisfy their given individual demands. The warehouse in turn places orders to an outside supplier to fill the orders of the retailers.

An approach is proposed to minimize the overall inventory and distribution costs of the SWMR system while taking into account retailers' demands at the supplier. The approach is based on some effective algorithms for inventory and routing sub-problems. In particular, the algorithm to solve the SWMR problem, proposed by [1], and adapted the method by [18] and [19] for the solution of that kind of vehicle routing problem (VRP), are taken advantage of in our approach. The complex component in the proposed approach is still the VRP sub-problem which is heuristically solved in this paper.

Further research approach will consist of adapting the existing method and solution to some numerical experiments and to the real life application problems, including a large set of retailers, driving-time restrictions for the vehicles and their drivers, delivery time windows at the retailers, heterogeneous vehicle fleets, multiple warehouses, and multiple products. Finally, the basic assumption of constant demand rates is not always valid. Therefore, it is worthwhile to investigate how the approach can be extended to explicitly take some demand variability into account. We will be extending the approach developed by [17] and [18] for the stochastic case in the future research.

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