

TEMPERATURE PROFILE COMPARISON THROUGH
CONCRETE STRUCTURES USING ANALYTICAL TECHNIQUE

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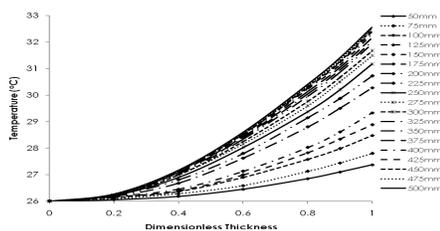
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Graphical abstract



Abstract

Concrete structures are always exposed to the influence of environmental conditions. Mechanical properties of concrete structures undergo changes due to temperature variation. Temperature reading varies with time, season, location and point. This manuscript compared temperature profile of various sizes of rectangular concrete structure. The thickness of the concrete structures ranges from 50 mm to 500 mm at regular intervals of 25 mm. One-dimensional transient heat conduction analytical method was used to generate data for the analysis. Findings showed that time taken for unit change of temperature at centre of rectangular concrete structure varies in a polynomial order with the thickness of the structure. Similarly, the gradient of the dimensionless surface temperature and thickness of structure is steeper as the thickness approaches zero. Furthermore, temperature values at specific points vary as thickness changes. Therefore, mechanical properties of concrete structures also vary from point to point since the properties are temperature dependent. More studies to accommodate temperature distribution through various concrete shapes should be done since temperature gradient through concrete structures depends on geometry.

Keywords: Temperature profile, concrete structure

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1.0 INTRODUCTION

The importance of concrete structures in the construction industry and environmental development is not questionable. Their applications are seen everywhere at different seasons amid varying environmental conditions. It is obvious that concrete structures respond to the changing environmental conditions, mostly temperature and moisture. Several researchers reported variation of mechanical properties of concrete structures due to temperature changes [1-15]. However, the ways temperature changes affect mechanical properties of concrete

structures from point to point with respect to time and location are yet to be fully understood.

Temperature and moisture content level in concrete structures using embedded nanotechnology/micro-electromechanical systems sensors was monitored [16]. They reported increase in concrete temperature as the hydration temperature increases to its maximum of 26°C and then decreased.

The development of mechanical properties of concrete when cured under different environmental conditions has been investigated [17]. They constructed environmental chamber in the laboratory that works in conjunction with a freezer to provide

chilled air and a heat gun to provide hot air. They found out that concrete strength and modulus of elasticity vary inversely to temperature.

A simply supported slab in the laboratory was studied to extract the vibration properties with modal testing under varying temperature [18]. They also considered the response of the slab to varying degrees of damage at changing temperature. They discovered that there was decrease in temperature which can result in the increase of the second frequency, as compared with the undamaged slab when the slab was slightly damaged. Changes of mechanical properties of concrete structures at slight temperature variation were eminent.

The effect of the non-uniform temperature field on the structural vibration characteristics of reinforced concrete structures has been investigated [19]. They conducted an experiment with a simply supported reinforced concrete slab to determine the effect of temperature on vibration properties of the structure across the slab thickness within a day. It was found that the vibration properties of the structure changes from one point to the other across the slab thickness. The changes of the properties were attributed to differences of temperature values across the thickness of the slab.

Hence, there is need to understand temperature distribution through cross-sections of concrete structures properly. This would give room to create some relationships that could promote comprehension of interactions between temperature and concrete structures to promote sustainable development. In this paper, a transient heat conduction analysis was carried out by analytical method to determine the temperature profile through concrete structures of various thickness sizes. Information provided could be relevant in damage assessment for enhancement of structural rehabilitation.

2.0 GOVERNING EQUATION

The heat transfer equation was used to formulate the problem. One-dimensional transient heat conduction differential equation is expressed as in [20].

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (1)$$

Where T , x , t and α are temperature, position, time and thermal diffusivity of the material respectively.

A typical slab can be modeled as a plane wall of thickness $2L$, with the length and breadth being too large compared to the thickness. The centre of the wall is at the point L and the wall is symmetric about its centre point. Assume that the slab has an initial temperature T_i , at $t = 0$ and is placed in an environment of constant temperature T_∞ for $t > 0$. Again, there is heat transfer between the slab and its environment by convection with constant and uniform heat transfer coefficient h . Furthermore, let the

radiation heat transfer effect be incorporated into h . Considering $0 \leq x \leq L$ part of the slab, where points 0 and L correspond to the slab centre and slab surface respectively, boundary conditions and initial condition can be expressed as in equations 2 and 3 respectively.

$$\frac{\partial T(0,t)}{\partial x} = 0 \quad \text{and} \quad -k \frac{\partial T(L,t)}{\partial x} = h[T(L,t) - T_\infty] \quad (2)$$

$$T(x,0) = T_i \quad (3)$$

$$\text{Let } x_d = \frac{x}{L} \quad (4)$$

$$T_d(x,t) = \frac{T(x,t) - T_\infty}{T_i - T_\infty} \quad (5)$$

$$t_d = \frac{\alpha t}{L^2} = Fo \quad (6)$$

$$Bi = \frac{hL}{k} \quad (7)$$

$$\text{Note that } \alpha = \frac{k}{\rho c_p},$$

where k , ρ , c_p , Bi , x_d , Fo , T_d and t_d are thermal conductivity, density, specific heat capacity, dimensionless heat transfer coefficient, dimensionless position, Fourier number, dimensionless temperature and dimensionless time respectively. The dimensionless position (thickness), temperature, time and heat transfer coefficient are the normalized forms of thickness, temperature, time and heat transfer coefficient respectively. The essence was to assess the relative weight of the variables. Hence, appropriate substitutions into equations 1 and 2 give equations 8 and 9.

$$\frac{\partial^2 T_d}{\partial x_d^2} = \frac{L^2}{\alpha} \frac{\partial T_d}{\partial t} \quad (8)$$

$$\frac{\partial T_d(1,t)}{\partial x_d} = \frac{hL}{k} T_d(1,t) \quad (9)$$

Equations 1, 2 and 3 can be written in their dimensionless forms as equations 10, 11 and 12 respectively.

$$\frac{\partial^2 T_d}{\partial x_d^2} = \frac{\partial T_d}{\partial t_d} \quad (10)$$

$$\frac{\partial T_d(0, t_d)}{\partial x_d} = 0 \quad \text{and}$$

$$\frac{\partial T_d(1, t_d)}{\partial x_d} = -BiT_d(1, t_d) \quad (11)$$

$$T_d(x_d, 0) = 1 \quad (12)$$

Where $T_d(x_d, t_d) = \frac{T(x, t) - T_i}{T_\infty - T_i}$
 = dimensionless temperature. (13)

And x_d , Bi and t_d are as shown in equations 4, 6 and 7 respectively.

Thus $T_d = f(x_d, Bi, Fo)$ (14)

3.0 ANALYTICAL SOLUTION OF THE PROBLEM

The dimensionless temperature function in equation 13 can be expressed using the method of separation of variable as

$$T_d(x_d, t_d) = M(x_d)N(t_d) \quad (15)$$

Substituting equation 15 into equation 10 gives

$$\frac{\partial^2 MN}{\partial x_d^2} = \frac{\partial MN}{\partial t_d} \quad (16)$$

Dividing by the product MN gives

$$\frac{1}{M} \frac{\partial^2 M}{\partial x_d^2} = \frac{1}{N} \frac{\partial N}{\partial t_d} \quad (17)$$

Bearing in mind that both x_d and t_d can be varied separately, the equality in equation 17 can be applicable for any value of x_d and t_d only if equation 17 is equal to a constant.

Let equation 17 be equated to $-\lambda^2$ so that $N(t_d)$ will be physical. Hence,

$$\frac{\partial^2 M}{\partial x_d^2} + \lambda^2 M = 0 \quad \text{and}$$

$$\frac{\partial N}{\partial t_d} + \lambda^2 N = 0 \quad (18)$$

Then, $M = C_1 \cos(\lambda x_d) + C_2 \sin(\sin \lambda x_d)$ and

$$N = C_3 e^{-\lambda^2 t_d} \quad (19)$$

So that $T_d = MN = C_3 e^{-\lambda^2 t_d} [C_1 \cos(\lambda x_d) + C_2 \sin(\sin \lambda x_d)]$

Let $A = C_1 C_3$ and $B = C_2 C_3$, (where A and B are arbitrary constants) so that

$$T_d = e^{-\lambda^2 t_d} [A \cos(\lambda x_d) + B \sin(\sin \lambda x_d)] \quad (20)$$

Applying the boundary conditions in equation 11 gives

$$\frac{\partial T_d(0, t_d)}{\partial x_d} = 0$$

$$\rightarrow -e^{-\lambda^2 t_d} (A \lambda \sin 0 + B \lambda \cos 0) = 0 \rightarrow B = 0 \rightarrow$$

$$T_d = A e^{-\lambda^2 t_d} \cos(\lambda x_d)$$

$$\frac{\partial T_d(1, t_d)}{\partial x_d} = -BiT_d(1, t_d)$$

$$\rightarrow -A e^{-\lambda^2 t_d} = -BiA e^{-\lambda^2 t_d} \cos \lambda \rightarrow \lambda \tan \lambda = Bi$$

Since the transcendental equation $\lambda \tan \lambda = Bi$ has an infinite number of roots, then

$$\lambda_n \tan \lambda_n = Bi \quad (21)$$

Hence, $T_d = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 t_d} \cos(\lambda_n x_d)$ (22)

A_n can then be deduced from the initial condition in equation 12

$$T_d(x_d, 0) = 1 \rightarrow 1 = \sum_{n=1}^{\infty} A_n \cos(\lambda_n x_d) \quad (23)$$

Multiplying both side of equation 23 by $\cos(\lambda_n x_d)$ and integrating from $x_d = 0$ to $x_d = 1$ gives

$$\int_0^1 \cos(\lambda_n x_d) \partial x_d = A_n \int_0^1 \cos^2(\lambda_n x_d) \partial x \rightarrow A_n$$

$$= \frac{4 \sin \lambda_n}{2 \lambda_n + \sin(2 \lambda_n)} \quad (24)$$

Provided that $t_d > 0.2$ and at $n = 1$, equation 22 can be rearranged to form

$$\frac{T(x, t) - T_i}{T_\infty - T_i} = A_1 e^{-\lambda_1^2 t_d} \cos(\lambda_1 x_d) \quad (25)$$

4.0 TEMPERATURE GREDIENT THROUGH STRUCTURES

Temperature gradients of different sizes of concrete structures were obtained analytically. Rectangular concrete structures of thickness range of 50-500 mm at regular intervals of 25 mm each were used. The structures were assumed to maintain uniform initial temperature of 25 °C and kept in an environment of uniform medium temperature of 35 °C. Temperature distributions through the concrete thickness at the time the temperature at the centre changes by one degree were determined using equation 25 in conjunction with equations 4, 6, 7, 21 and 24. Values of temperature at dimensionless thicknesses of 0, 0.2, 0.4, 0.6, 0.8 and 1 for the various nineteen slab thicknesses were deduced step by step using equation 25 and plotted in Figure 3. Figure 1 shows the time taken for the temperature at the centre of the structures to change by one degree. The graph is a polynomial

function of the second order. It indicates that ratio of change in time taken for unit degree change at centre of structures to change in thickness of the structures is not constant. The thicker the concrete structures, the more the time lag. The relationship between the surface temperatures of the concrete structures is not proportional but has good correlation as shown in Figure 2. The gradient increases in steepness as the thickness tends to zero. Again, temperature gradients through concrete thicknesses of 50 mm to 500 mm at constant intervals of 25 mm and at time when a unit degree change occurred at the slab centre were shown in Figure 3. It indicates that temperature values at different points of a structure are never the same in practice. Temperature values at points also vary as thickness changes. However, the temperature gradients lie closer to each other as thickness increases. This would result to variation of mechanical properties of concrete structures since the properties are temperature dependent. Therefore, it is noticeable that mechanical properties of concrete structures would differ from point to point with respect to time as temperature changes [19]. However, Figure 3 shows no significant difference between temperature profiles across various concrete thickness sizes.

5.0 CONCLUSION AND RECOMMENDATION

Temperature is one of the major environmental conditions that affect mechanical properties of concrete structures. The compressive strength, tensile strength, elastic modulus and natural frequencies of concrete structures reduce as temperature increases. One-dimensional transient heat conduction through different sizes of rectangular concrete structure showed variation in temperature values across the thickness of the structure. Time taken for unit change of temperature at centre of rectangular concrete structure varies in a polynomial order with the thickness of the structure. Again, the gradient of the dimensionless surface temperature and thickness of structure is steeper as the thickness approaches zero. Temperature values at specific points vary as thickness changes. Hence, mechanical properties of concrete structures vary from point to point since the properties are temperature dependent. The paper showed that geometry of concrete structure affects temperature profile through the structure. Therefore, more studies should be carried out to cover various geometries.

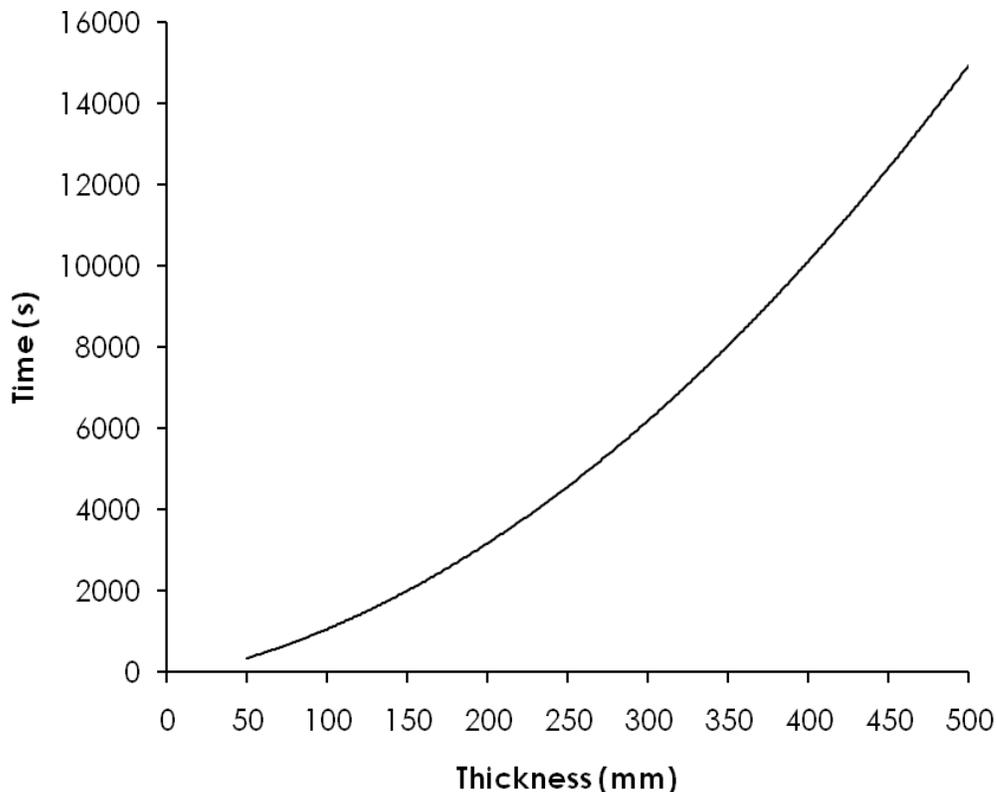


Figure 1 Time-thickness curve per degree change at centre

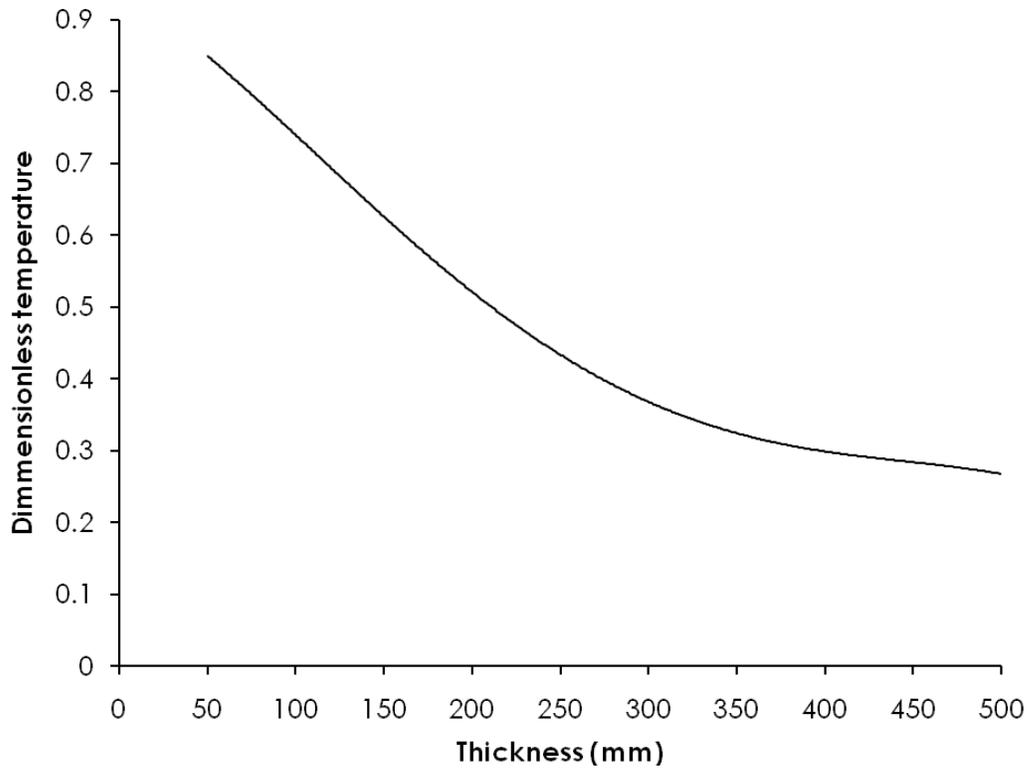


Figure 2 Relationship between dimensionless surface temperatures

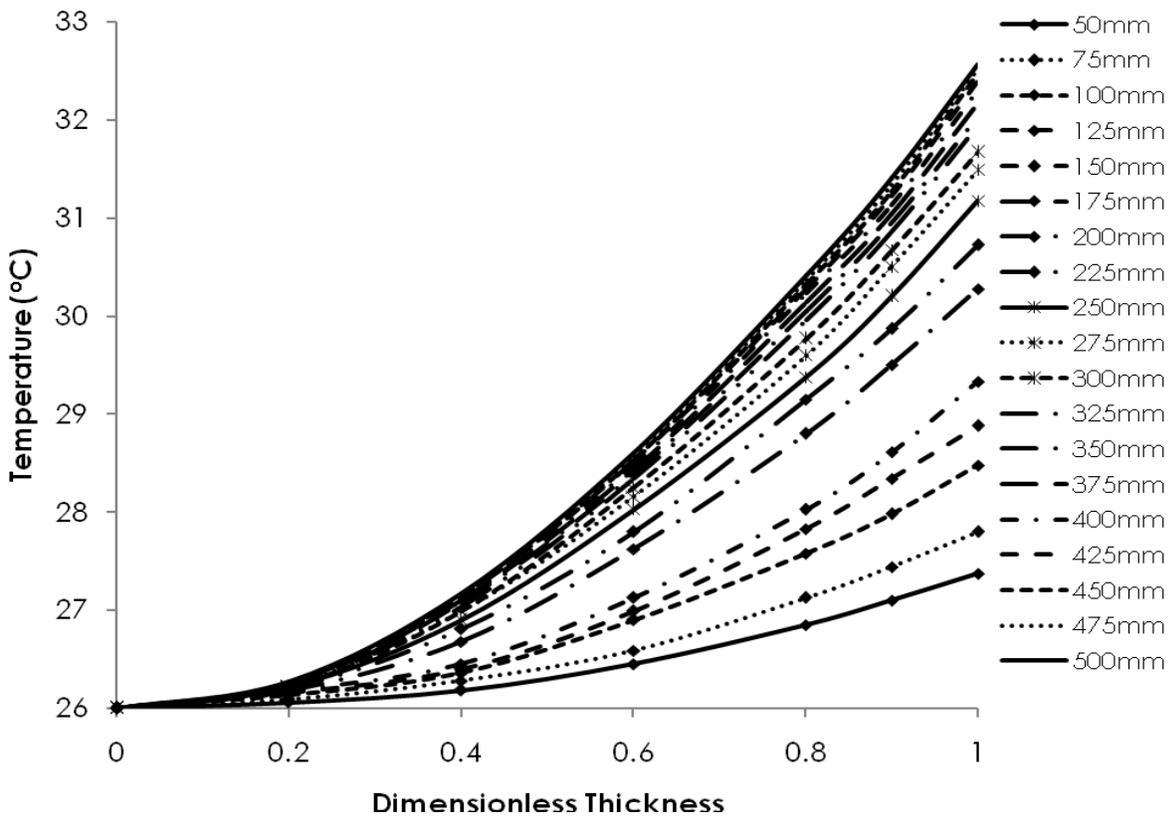


Figure 3 Gradient of temperature through concrete thickness of various sizes

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