The effect of GPS Satellite Geometry on the Precision of DGPS Positioning In Minna, Nigeria

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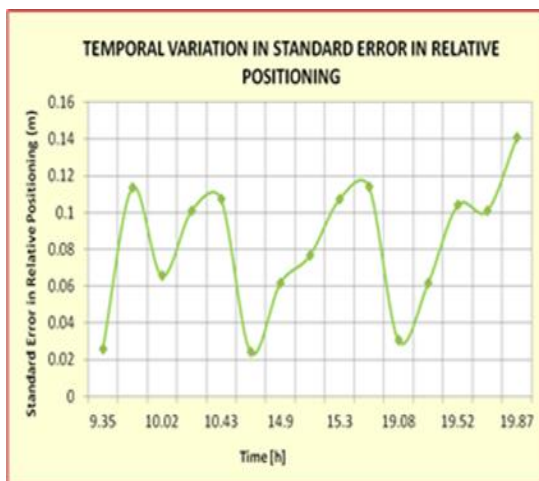Department of Surveying & Geoinformatics, Federal University of Technology Minna, Nigeria

Graphical abstract

Abstract

This paper investigates the effect of Satellite Geometry in the Precision of DGPS positioning, since it has an amplifying effect on other sources of errors associated with GPS positioning. DGPS positional data were acquired at three epochs of a day using Promak3 DGPS and post-processed using GNSS solution software. The temporal variations in PDOP, number of visible satellites and the standard errors in relative positioning, (which are all functions of the satellite geometry) were analysed both graphically and statistically (using statext v1.0 software) to ascertain the impact of satellite geometry on the derived positions. The graphical results indicated various temporal variations in the parameters defining a GPS-based position on the earth surface; but the statistical tests conducted show no significant differences in the means of the PDOP and standard error in relative positioning obtained in three epochs at 0.05 significant level. Also, for the short ranges, an average standard error of 0.046m, 0.043m and 0.092m were obtained at the three epochs respectively; while for the medium ranges an average standard error of 0.112m, 0.096m, and 0.123m were obtained at the three epochs respectively.it was concluded that the longer the occupation time, the better the satellite geometry and thus the higher the precision in DGPS positioning

Keywords: GPS Satellite Geometry, DGPS Positioning, PDOP, Precision

1.0 INTRODUCTION

The development of GPS (Global Positioning System) has made position determination easy and very fast both in absolute and relative mode. Satellite based positioning is improving continuously and various efforts are being made towards making the accuracy and precision of the system better. In GPS application, the accuracy and the precision of the determined position cannot be overemphasized, since inaccuracy could result in wasted efforts and resources. This underscores the need for great accuracy and precision in position determination.

It is important to note that the precision of GPS positioning depends on the errors in the range measurement and the Dilution of Precision (DOP); these are functions of the satellite geometry [1]. The satellite geometry has an amplifying effect on the impact of GPS error sources from observations to adjusted parameter and this effect varies with the time of observation. Thus, the satellite geometry determines the level of propagation of GPS error sources [2]. The science of position determination

using the GPS has gained applications in various disciplines and fields of human endeavor such as roads and highway, railway, telecommunication, aviation, marine, agriculture, mapping, recreation, public safety and relief etc. We live in an environment surrounded by both man-made and natural features. It is therefore impossible to overlook accuracy in determining the location of these features as it affects proper planning and development of the environment. In view of these, this paper seeks to investigate the effect of satellite geometry on the precision of DGPS (Differential Global Positioning System) positioning in the study area. This is achieved based on the following objectives:

i.   To investigate how the number of visible satellites results in the temporal variation of PDOP within the study area.

ii.   To examine the effects of satellite-range geometry on the resultant positions (xyz) in the observers domain within the study area.

iii.   To ascertain, statistically, if there are significant differences in the means of PDOP and the computed standard error of the resultant positions obtained at various epochs.

2.0 THE CONCEPT OF GPS POSITIONING

The GPS system currently consists of 31 satellites; each continually transmits a signal that comprised of the satellite position and the signal transmission time. Based on this information, pseudorange equations are formed for a user whose position is unknown [3]. The principle behind GPS is the measurement of distance (or "range") between the satellites and the receiver on the earth surface. The satellites tell us exactly where they are in their orbits by broadcasting data the receiver uses to compute their positions. The GPS receiver processes the satellite range measurements and produces its position. GPS uses a system of coordinates called WGS 84 (World Geodetic System 1984).

Positioning with GPS can be performed in two ways: point positioning and differential (relative) positioning. GPS point positioning employs one GPS receiver (Figure 1), while differential positioning employs two (or more) GPS receivers.
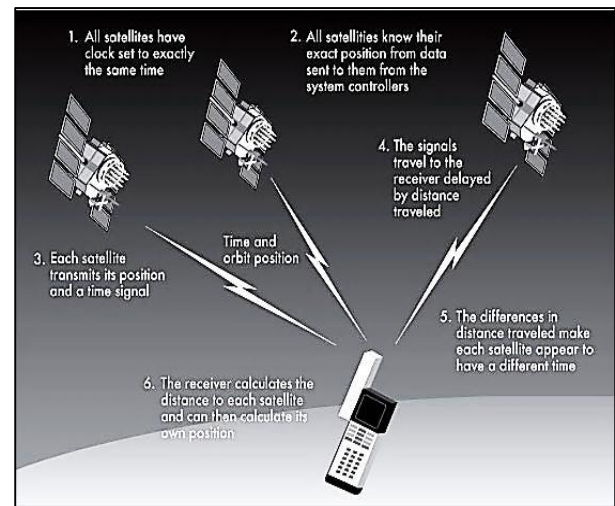

Figure 1 The Concept of GPS point positioning [4]

The underlying premise of differential GPS (DGPS) requires that a GPS receiver, known as the base station, be set up on a precisely known location (Figure 2). The base station receiver calculates its position based on satellite signals and compares this location to the known location; the difference is applied to the GPS data recorded by the roving GPS receiver [5].
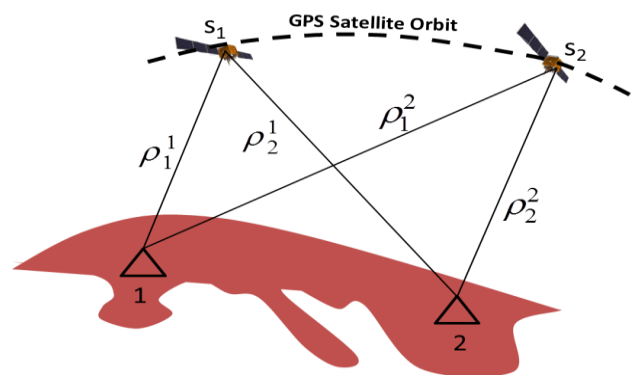

Figure 2 GPS differential positioning

Surveying works with GPS have conventionally been carried out in the differential positioning mode. This is mainly due to the higher positioning accuracy obtained with the differential positioning mode compared to that of the GPS point positioning. A major disadvantage of GPS differential positioning, however, is its dependency on the measurements or corrections from a reference receiver; i.e. two or more GPS receivers are required to be available [5]. New developments in GPS positioning show that a user with a single GPS receiver can obtain positioning accuracy comparable to that of differential positioning (i.e., centimetre to decimetre accuracy) [6].

2.1 GPS Satellite Geometry

One factor affecting GPS accuracy is satellite geometry. In simple terms, satellite geometry refers to where the satellites are located relative to each other (from the perspective of the GPS receiver). If a GPS receiver is locked onto four satellites and all four of these satellites are in the sky to the north and west of the receiver, satellite geometry is rather poor. If those same four satellites are spread out in all directions and separated equally at approximately 90-degree intervals (north, east, south, and west), positional accuracy improves dramatically. Therefore, the geometry of satellites, or lack of it, has obvious implications with regard to positioning [7].

2.2 Dilution of Precision (DOP)

DOP is an indicator of the quality of the geometry of the satellite constellation. Computed position can vary depending on the satellites used for the measurement. Different satellite geometries can magnify or lessen the errors in the error budget of GPS positioning. A greater angle between the satellites lowers the DOP, and provides a better measurement. A higher DOP indicates poor satellite geometry, and an inferior measurement configuration [8].

Some GPS receivers can analyze the positions of the satellites available, based upon the almanac, and choose those satellites with the best geometry in order to make the DOP as low as possible. Another important GPS receiver feature is to be able to ignore or eliminate GPS readings with DOP values that exceed user-defined limits. Other GPS receivers may have the ability to use all of the satellites in view, thus minimizing the DOP as much as possible.

3.0 DERIVATION OF THE EFFECT OF SATELLITE GEOMETRY ON GPS-DERIVED POSITION

Generally speaking, in GNSS relative positioning, the observation equation for carrier phase *L* and pseudorange *P* measurements is respectively given as [7, 9]:

$$L = \rho + c\left(dt^s - dt_R\right) - d^{ion} + d^{trop} +$$
$$\left(dH^s + dH_R\right) + d^{mp} + \lambda N + e \qquad (1)$$

$$P = \rho + c\left(dt^s - dt_R\right) + d^{ion} + d^{trop} +$$
$$\left(dH^s + dH_R\right) + d^{mp} + e \qquad (2)$$

where *P* is the code range observation in metres, *L* is the carrier phase observation in metres, ρ is the geometric range between satellite and receiver in metres, *c* is the speed of electromagnetic waves in metre per second, dt^s and dt_R are the satellite and

receiver clock biases respectively, d^{ion} and d^{trop} are the ionospheric and tropospheric delays respectively, dH^s and dH_R are the satellite and receiver hardware delay respectively, d^{mp} is the multipath effect, λ is the wavelength of corresponding carrier phase in metres, *N* is the unknown "integer carrier phase ambiguity" and *e* is the measurement noise.

The effects of satellite and receiver clock errors known as the GNSS common errors are cancelled using differential concept.

In order to fix user's position, the solution of the system of either Equation (1) or (2) is obtained by linearization using a linear model of the form:

$$y = Ax + e \qquad (3)$$

where *y* is a matrix containing the observables, *A* is a coefficient matrix which represents the geometry between user receiver and the satellites used while *x* is the matrix of the unknown user position and *e* the measurement noise.

Based on generalized least square theory, the solution to the system of equation (3) is obtained as [10]:

$$x = \left(A^T A\right)^{-1} A^T y \qquad (4)$$

Since different ranges are measured from user's position to the satellites at different epochs, observation errors are uncorrelated but having equal variances (σ^2). Hence the covariance of the errors in positional solution (x) is given as [2, 3, and 10]:

$$\sigma_{pos} = \sigma^2 \left(A^T A\right)^{-1} \qquad (5)$$

Therefore, $\left(A^T A\right)^{-1}$ contains information (i.e. DOP) about amplification of the variance on to the positional solutions and the term is defined as [1, 3]:

$$\left(A^T A\right)^{-1} = \begin{bmatrix} EDOP^2 & \bullet & \bullet & \bullet \\ \bullet & NDOP^2 & & \\ \bullet & \bullet & VDOP^2 & \bullet \\ \bullet & \bullet & \bullet & TDOP^2 \end{bmatrix} \qquad (6)$$

The DOP could generally be described in terms of positional (PDOP), time (TDOP), or geometry (GDOP) dilution of precision [8]. The PDOP consists of the horizontal (EDOP and NDOP) and the vertical (VDOP) dilution of precisions while GDOP is the combined effects of PDOP and TDOP. Their relationships are given by [3] as:

$$HDOP = \sqrt{EDOP^2 + NDOP^2} \qquad (7)$$

$$PDOP = \sqrt{EDOP^2 + NDOP^2 + VDOP^2} \qquad (8)$$

$$GDOP = \sqrt{HDOP^2 + TDOP^2} = \sqrt{trace\left(A^T A\right)^{-1}} \qquad (9)$$

Equations (6) through (9) present the quantification of the impact of GPS satellite geometry on the derived relative position.

3.1 Materials and Methods

This study was carried out on four different control points spread across Minna in Niger State (Figure 3); Nigeria. Point L40 (Primary Control Point) was used as the base station for observations of other three points (secondary control points). The study area (Minna) lies within latitude 9° 25' 00" and 9° 40' 00" North of the equator and longitude 6° 24' 20" and 6° 36' 40" East of the Meridian.
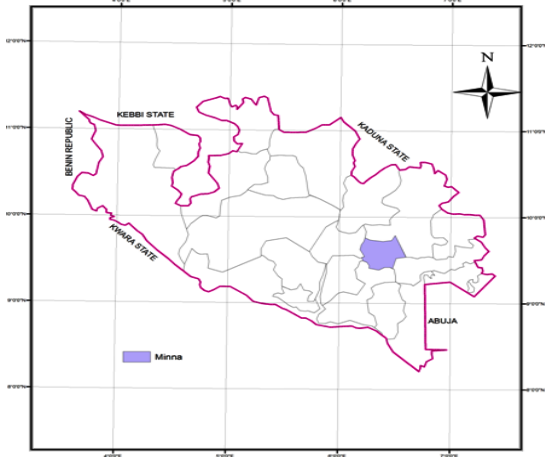

Figure 3 Map of Niger State showing Minna

Some of the information on the selected controls was obtained from the Department of Surveying and Geoinformatics, Federal University of Technology Minna, while the coordinates of L40 and CSN168S were obtained from the office of Surveyor General of Niger State. These are as shown in the Table 1 below.

Table 1 Locations of Control Points used for DGPS observations in the Study Area

Station ID	Control Category	Coordinates (m)	
		Easting	**Northing**
L40 (Base)	Primary	227423.232	1066041.870
FUT09/055	Secondary	233413.820	1060188.295
CSN168S	Secondary	227914.615	1069224.788
SVG/GPS 01	Secondary	220563.650	1055093.618

The data for the study was obtained using Differential Global Positioning System (DGPS) at three (3) epochs in a day. The study points were evenly

selected covering Minna and its environs. The DGPS Rovers were not referenced to any CORS station, rather they were referenced to a base station (i.e. L40: Nigeria's Datum) and its known coordinate was used in post processing the DGPS observations. Figure 4 shows the network design for the study.
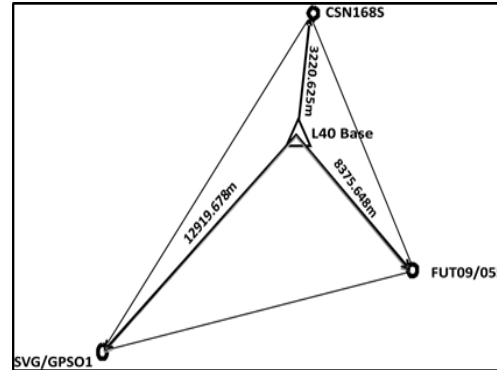

Figure 4 Network Design of the Control Points

3.2 Data Collection

The equipment set-up instructions are the same for both the base and the rover stations. Setting up of the instruments was first carried out at the base station (Figure 5) before the rovers were subsequently set up. In all cases the observation points offered a clear view of the sky in all directions. The DGPS equipment was set up in static mode and data were continuously acquired by the receivers for minimum time duration of an hour at each epoch of observations.

The entire field procedure lasted between the hours of 8.00am to 11.00pm local time (i.e 7.00hr to 22.00 hr UTC), covering three epoch of a day i.e. morning, afternoon and evening.

Figure 5 DGPS Set-Up at L40 Acquiring Satellite Data

3.3   Data Processing and Adjustment

The GNSS solution software provides the tools required for downloading and processing GPS satellite data from ProMark3 receiver to produce relative positions of all logged points. In the post processing of the raw data, the original coordinate of **L40** was entered as the control station and the rover stations were processed relative to the coordinate of **L40** relying on the system of Equations (1-2)

In order to achieve the desired accuracy in relative positioning, a network adjustment is usually performed to test for blunders and errors in the observations. This was realized based on Equations (3-5) using the GNSS solution software. This process removes errors due to atmospheric influence, satellite orbit & clock drifts, multipath effect, etc. thus leaving the influence of geometry on the obtained position. To handle this effect, the knowledge of the standard error in positioning and the DOP is essential.

The relationship between Standard error in positioning and DOP can be obtained by explicitly rewriting Equation (5) as:

$$\sigma_{pos} = DOP \times \sigma_r \qquad (10)$$

Where, σ_{pos} is Standard Error in Positioning (m), and

σ_r is Standard Error in Range measurement (m).

In GPS data analysis using the GNSS solution, the Land Survey Overview of the Processed data gave precisional indicators in terms of temporal variation in the values of **PDOP** and σ_r. These values were extracted and used in Equation (10) to obtain the temporal variations in standard error in relative positioning (see Table 6).

One way ANOVA procedure was used in testing for the significant difference in the means of the PDOP and values obtained at the three epochs of observations. The test was carried out using Statext V1.0 software and was conducted at 0.05 significant levels.

4.0 RESULTS AND DISCUSSION

From the foregoing procedures, the mean of the adjusted coordinates of the entire test points were obtained as shown in Table 2. Also, the coordinates difference, standard error in processed vectors, number of visible satellites and PDOP values are presented in Tables 3, 4 and 5 respectively.

Table 2 DGPS Positional Data at Three Epochs

STATIONS		EASTINGS (m)	NORTHINGS (m)	HEIGHTS(m)
L40	Morning	227423.232	1066041.870	
	Afternoon	227423.232	1066041.870	
	Evening	227423.232	1066041.870	
	MEAN	**227423.232**	**1066041.870**	
FUT 09/055	Morning	233414.008	1060188.287	
	Afternoon	233414.033	1060188.268	
	Evening	233414.041	1060188.285	
	MEAN	**233414.027**	**1060188.280**	
CSN 168S	Morning	227913.918	1069224.603	
	Afternoon	227913.920	1069224.600	
	Evening	227913.903	1069224.573	
	MEAN	**227913.914**	**1069224.592**	
SVG/GPS 01	Morning	220563.575	1055093.171	
	Afternoon	220563.549	1055093.162	
	Evening	220563.725	1055093.109	
	MEAN	**220563.616**	**1055093.147**	

Table 3 Difference between mean of each station with their known values

L40

Coordinates	Eastings (m)	Northings (m)	Height (m)
Existing Coordinate	227423.23	1066042	279.603
Mean of Obs. Coordinate	227423.23	1066042	279.603
Difference	0.000	0.000	0.000

FUT 09/055

Coordinates	Eastings (m)	Northings (m)	Height (m)
Existing Coordinate	233413.82	1060188	268.39
Mean of Obs. Coordinate	233414.03	1060188	268.826
Difference	0.207	0.015	0.436

CSN 168S

Coordinates	Eastings (m)	Northings (m)	Height (m)
Existing Coordinate	227914.62	1069225	306.646
Mean of Obs. Coordinate	227913.91	1069225	307.891
Difference	0.701	0.196	1.245

SVG/GPS 01

Coordinates	Eastings (m)	Northings (m)	Height (m)
Existing Coordinate	220563.65	1055094	234.138
Mean of Obs. Coordinate	220563.62	1055093	238.524
Difference	0.034	0.471	4.386

Table 4 Temporal variations in the number of visible satellite and PDOP

VECTOR IDENTIFIER	VECTOR LENGTH (m)	TIME (h)	PDOP	$\sigma_{r\,(m)}$	*σ_{POS} (m)
L40 – CSN168S	3218.942	9.35	1.6	0.016	0.0256
FUT09/055 – CSN168S	10573.851	9.35	2.1	0.054	0.1134
L40 – FUT09/055	8371.974	10.02	1.6	0.041	0.0656
L40 – SVG/GPS01	12913.677	10.43	1.6	0.063	0.1008
FUT09/055 – SVG/GPS01	13817.029	10.43	1.6	0.067	0.1072
L40 – CSN168S	3218.935	14.52	1.5	0.016	0.0240
L40 – FUT09/055	8372.006	14.90	1.5	0.041	0.0615
FUT09/055 – CSN168S	10573.868	14.90	1.5	0.051	0.0765
L40 – SVG/GPS01	12913.709	15.30	1.7	0.063	0.1071
FUT09/055 – SVG/GPS01	13817.069	15.30	1.7	0.067	0.1139
L40 – CSN168S	3218.911	19.08	1.9	0.016	0.0304
L40 – FUT09/055	8370.000	19.52	1.5	0.041	0.0615
FUT09/055 – CSN168S	10573.852	19.52	2.0	0.052	0.1040
L40 – SVG/GPS01	12913.650	19.87	1.6	0.063	0.1008
FUT09/055 – SVG/GPS01	13816.943	19.87	2.1	0.067	0.1407

Table 5 Temporal Variations in Standard Errors of Relative Positioning

VECTOR IDENTIFIER	TIME (h)	NO. OF VISIBLE SAT.	PDOP
L40 – CSN168S	9.35	9	1.6
FUT09/055 – CSN168S	9.35	8	2.1
L40 – FUT09/055	10.02	9	1.6
L40 – SVG/GPS01	10.43	9	1.6
FUT09/055 – SVG/GPS01	10.43	9	1.6
L40 – CSN168S	14.52	9	1.5
L40 – FUT09/055	14.90	10	1.5
FUT09/055 – CSN168S	14.90	9	1.5
L40 – SVG/GPS01	15.30	10	1.7
FUT09/055 – SVG/GPS01	15.30	10	1.7
L40 – CSN168S	19.08	10	1.9
L40 – FUT09/055	19.52	10	1.5
FUT09/055 – CSN168S	19.52	10	2.0
L40 – SVG/GPS01	19.87	10	1.6
FUT09/055 – SVG/GPS01	19.87	9	2.1

*σ_{POS} = PDOP × σ_r

The temporal variation in satellite visibility at the various times of observation is shown in Figure 6.
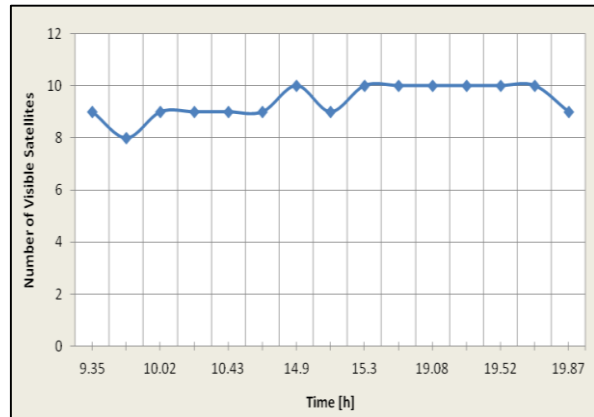

Figure 6 The Temporal Variations in Satellite Visibility at the three epochs

While, Figure 7 shows the temporal variation in Positional Dilution of Precision (PDOP) at the various times of observation.

Figure 7 The Temporal Variations in PDOP at the three epochs

Finally, the Temporal Variations in Standard Errors in Relative Positioning at the various times of observation is presented in Figure 8.
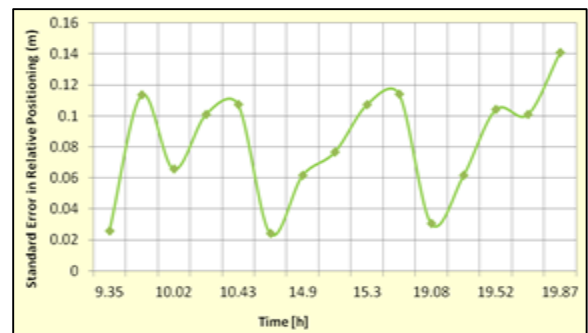

Figure 8 The Temporal Variations in Standard Errors in Relative Positioning at the three epochs

From the results shown in Table 4, the variations in relative positions arising from the variations in the satellite geometry and standard error in range measurement could be discerned.

For the purpose of analysis, the baseline measurement was grouped into short and medium ranges. The short range falls within 3 to 8km while the medium range is within 10-14km. Over the short range baselines, positional errors of 0.026m to 0.066m, 0.024m to 0.062m and 0.030m to 0.062m with PDOP of 1.6, 1.5 and 1.5 to 1.9 were observed for morning, afternoon and evening observations respectively. Over the medium range, standard errors of 0.110m to 0.113m, 0.077m to 0.114m and 0.104m to 0.141m were respectively obtained in observed positions for morning, afternoon and evening observation; while having the PDOP range to be 1.6 to 2.1, 1.5 to 1.7 and 2.0 to 2.1 respectively for the three epochs.

Furthermore, the graphical results as shown in Figures 6, 7 and 8 above clearly indicate series of variations in satellite visibility, PDOP and Standard error in relative positioning respectively. The statistical tests (one way ANOVA) conducted on the PDOP and standard errors in range measurement obtained at three epochs at 0.05 significant levels showed no significant differences in their respective means. The results of the statistical test therefore indicate a measure of consistency during the periods of observations. For the short ranges, an average standard positional error of 0.046m, 0.043m and 0.092m were obtained at the three epochs respectively; while for the medium ranges an average standard positional error of 0.112m, 0.096m, and 0.123m were obtained at the three epochs respectively. From the foregoing, it could be ascertained that locations with high PDOP value have high positional error and vice versa.

5.0 CONCLUSION

This study investigated the effect of satellite geometry on DGPS positioning in Minna, Nigeria and it has shown that, the precision of DGPS positioning is dependent on the geometry of the number of visible satellites and their geometrical arrangement in space with respect to the receivers' station. In as much as a poor satellite visibility results in low precision in positioning, a good satellite visibility with poor geometry would also results in low precision. For example in Tables 5 and 6, at the first epoch 8 visible satellite have 2.1 PDOP and in the third epoch 9 visible satellite have 2.1 PDOP, both cases resulted in

high standard errors of 0.113m and 0.141m respectively in relative positioning. These are due to poor satellite geometry and partly accounts for the phenomena of floating stations in satellite positioning. Thus, the requirements for high precision in DGPS positioning after removal of other error sources during data processing are: a good satellite visibility and good satellite geometry, resulting in low PDOP. This research has justified the impact of satellite geometry in the precision of DGPS positioning. The statistical test conducted on the temporal variations in PDOP and Standard error in relative positioning indicated the precision in the final coordinates of the occupied stations at three epochs. It can be finally concluded that the greater the occupation time, the better the satellite geometry and thus the higher the precision in DGPS positioning.

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