

NUMERICAL MODELLING OF HOMOGENEOUS TWO-PHASE GAS-LIQUID FLOW IN A PIPE

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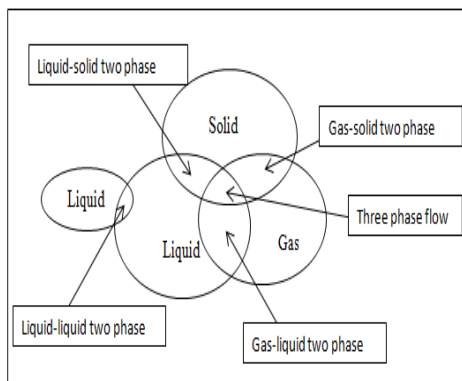
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Graphical abstract



Two-Phase Flow

Abstract

A one-dimensional model which represent a system of partial differential equations that describe mathematically the two-phase flow has been considered for the gas-liquid mixture flow in a pipeline. The Implicit Steger -Warming flux vector splitting method is used for the numerical computation on air-water compressible flow problems. The results for pressure wave propagation, celerity or speed of sound and mass flow rate for different values of mass ratio were obtained. It was observed that the propagation of pressure along the pipeline and the mass flow rate there decreases along the pipe and maintained near a steady flow until it reaches the downstream of the pipe signifying the effect of gas build up during the pump control in pipeline.

Keywords: Two-Phase Flow, Mixture, Steger-Warming, Transient Flow

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1.0 INTRODUCTION

In many industries, it is known that industrial liquid mostly water contains a small amount of free gaseous phase. The presence of small amount of free gas in the liquid reduces the pressure wave speed in the mixture compared to that of liquid only. It can be mentioned that two-phase flow involves fluid mixtures of different phases such as gas-liquid flow, gas-solid, liquid-liquid and liquid-solid. Two-phase flow phenomena are currently focused on the transient flow occurring in the piping system in many industries. Studies on transient flow for the propagation of pressure waves in two phase flow are often considered in hydraulic installation systems like nuclear, geothermal power plants, chemical and petroleum industries [1]. The mathematical modelling of transient flow phenomenon has gained attention due to the occurrence of unsteadiness or effects of flow properties such as pressure waves in the gas industry. Considering the compressibility of any free gas on transient two-phase gas-liquid mixture flow,

the wave propagation speed changes with pressure and making a system of equations describing the transient two-phase flow to be difficult. As a result, the study of two-phase becomes more complex than in single phase flow. A mathematical model for three differential equations written to represent the two-phase transient flow was developed by Martin [2, 3]. The finite difference Lax-Wendroff Scheme was used to solve the differential equations. They considered a greater void fraction in determining the pressure surges in the slug flow. Chaudry *et al.* [4], assumed the gas-liquid mixture as a pseudo-fluid for a small void fraction. A second order explicit finite difference method with a characteristics equation at the pipe boundary was employed to numerically solve the equations. Study on the propagation of pressure waves in two phase gas-liquid mixture flow was carried out by Mori *et al.* [5]. In their work, the void fraction was varied in relation to the pressure rise with the effect of pipe elasticity on the velocity propagation of the pressure waves.

The effect of gas mass fraction on the pressure evolution on homogeneous gas-liquid mixture transient flow has been investigated. A numerically solution for the transient two-phase flow using the method of characteristics and finite difference conservation method was studied by Hadj-Taieb and Lili [6]. In their work, the gas-fluid mass ratio was considered to be constant. A mathematical model for the transient homogeneous two phase flow by taking into account the pipe elasticity effect on pressure wave propagation using the method of characteristics was studied by Hadj-Taieb and Lili, [7]. The gas-fluid mass ratio was used and assumed constant on the determination of pressure surges. Study on the transient flow of gas-liquid mixtures pipelines using the method of characteristics was carried out to observe the influence of different values of gas mass fraction and Young's modulus on the pressure evolution [8]. The study of transient phenomena in a two-phase homogeneous flow accounting for both geometrical parameters of the pipe and mass fraction of the gas in the two-phase mixture flow was investigated by Zohra *et al.* [9]. The method of characteristics was employed for the numerical solution for the governing equations. In most of these studies liquid flows only are often considered. Most methods used in the transient two-phase mixture flow were the characteristics method, Lax-Wendroff, explicit and implicit methods. However, the mass fraction was varied for elastic pipes there by the compressibility of the fluid were neglected.

Although, the Godunov Type Scheme (GTS) have also been applied for the numerical solution of two-phase homogeneous of air and water flow mixture [10, 11]. Leon *et al.*, [12] studied the modelling of two-phase bubbly homogeneous air-water mixture using the single equivalent approximation. They used a second order Finite Volume (FV) shock-capturing scheme for the numerical solution of the two-phase flow.

In this paper, the finite difference techniques of Implicit Steger Warming flux splitting scheme was used for the numerical solution of the transient two-phase mixture flow. The mass fraction for the gas-fluid was varied with the pressure evolution for pipeline system. The mass flow of the flow mixture is investigated with the effect of mass fraction.

2.0 MATHEMATICAL FORMULATION

Transient one-dimensional flow for the homogeneous gas-liquid mixture flow is considered. By applying the mass conservation and momentum law to an element of fluid between two sections of abscissa x and dx of the pipe, the following equations of continuity and motion were obtained [13].

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho AV)}{\partial x} = 0 \quad (2.1)$$

$$\frac{\partial(V)}{\partial t} + V \frac{\partial(V)}{\partial x} + \left(\frac{1}{\rho}\right) \frac{\partial p}{\partial x} + \frac{\lambda V |V|}{2D} = 0 \quad (2.2)$$

Where ρ is the mixture density of the fluid, A is the cross section area of the pipe, V is the velocity, P is the pressure, λ is the friction coefficient which is consider to be constant, t is the time and x is the distance along the pipe.

Equations (2.1) and (2.2) are taken to be two linear partial differential equations of hyperbolic in which the pressure P and the velocity V are considered the main variables of the flow. In order to solve these equations numerically, it is important to express the density of the mixture ρ according to the fluid pressure. The elasticity of the wall is assumed to have no effect and is therefore neglected.

The average mixture density ρ defined in terms of the gas mass ratio θ is expressed as [6].

$$\rho(p) = \left[\frac{\theta}{\rho_g} \left(\frac{P}{P_0} \right) + (1-\theta) / \rho_l \right]^{-1} \quad (2.3)$$

The gas mass ratio is given as:

$$\theta = \left[\frac{M_g}{M_g + M_l} \right] \quad (2.4)$$

where M_g and M_l are the masses of the gas and liquid respectively.

Under the polytrophic law the gas density can be expressed as:

$$\rho_k = \rho_{k0} \left[\frac{P}{P_0} \right]^{1/n} \quad (2.5)$$

where ρ_{k0} represents the density at the initial conditions, kg/m³; P_0 is the permanent regime pressure, N/m²; k stands for gas or liquid.

The celerity of sound speed in the fluid can be represented by the expression:

$$c = \left(\frac{\partial \rho}{\partial P} \right)^{-1/2} = \left(\frac{g n p}{\rho_g} \right)^{1/2} \left(\frac{P}{P_0} \right)^{(n-1)/(2n)} \left(1 + \left(\frac{P_0}{P} \right)^{1/n} \frac{(1-\theta)\rho_g}{g \rho_l} \right) \quad (2.6)$$

3.0 NUMERICAL SCHEME

The governing equations in the partial differential equation formed for the one dimensional model are solved numerically which require numerical scheme by the finite difference method of discretization approach which has been widely used in computational fluid dynamics (CFD) and has been described in several numerical textbooks [14, 15].

The transient flow problem has a particularly analytical solution which is obtained as follows; By subjecting the sound speed C , the continuity equation can be expressed as;

$$\frac{1}{\rho C^2} \left(\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x} \right) + \frac{\partial V}{\partial t} = 0 \quad (2.7)$$

Following the characteristics theory [16], the following ordinary differential equations are given as;

$$C^+ \begin{cases} dV + dU = [\lambda V |V| / 2D] dt \\ dx = (V + C) dt \end{cases} \quad (2.8)$$

and

$$C^- \begin{cases} dV - dU = [\lambda V |V| / 2D] dt \\ dx = (V - C) dt \end{cases} \quad (2.9)$$

whereby U is the Riemann invariant defined by

$$dU = \frac{dP}{\rho C} \quad (2.10)$$

Considering equations (2.3) and (2.6), equation (2.10) becomes;

$$U = \left\{ (2 / (n-1)) (n \vartheta P_0 / \rho_{g0})^{1/2} (P / P_0)^{(n-1)/2} \right\} \quad (2.11)$$

$$P = P_0 \left\{ (n-1) / 2 (\rho_{g0} / n \vartheta P_0)^{1/2} U \right\}^{2n/(n-1)} \quad (2.12)$$

3.1 Implicit Steger-Warming Splitting Flux Vector Scheme

The finite difference technique method (Implicit Steger-Warming Flux Splitting) which is used for the transient solution is faster than other methods. The process of discretising the continuous derivatives in the governing partial differential equations is to replace equations with finite difference expressions and rearranging the resulting algebraic equation into an algorithm for the dependent variables at each grid or mesh cell. The linearized form of the equations is derived to find the corresponding eigensystem. The eigenvalues obtained represent the characteristic of the hyperbolic system and therefore, described the direction of the propagation of flow.

Equations (2.1) and (2.2) can be rewritten to a more compacted matrix form as;

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = H \frac{\partial Q}{\partial x} \quad (3.1)$$

where Q is a vector of unknowns, F is a physical flux vector, H contains non-conservative terms that exit in the model given respectively as follows:

It is also possible to transform (3.1) into the quasi-linear form as;

$$\frac{\partial Q}{\partial t} + \frac{\partial E(Q)}{\partial x} - S(Q) = 0 \quad (3.2)$$

The implicit algorithm of finite difference expressions for equation (3.2), for the time derivatives is approximated by a first order backward difference approximation which is given as;

$$\frac{Q^{n+1} - Q^n}{\Delta t} + \left(\frac{\partial E}{\partial x} \right)^{n+1} - S^{n+1} = 0 \quad (3.3)$$

Considering that, the change in flow properties for time step as:

$$\Delta Q = Q^{n+1} - Q^n \quad (3.4)$$

Expressing the second and third term of (3.3) into Taylor series expansion, we obtain:

$$E^{n+1} = E^n + \frac{\partial E}{\partial t} \Delta t + O(\Delta t)^2 \quad (3.5)$$

$$S^{n+1} = S^n + \frac{\partial S}{\partial t} \Delta t + O(\Delta t)^2 \quad (3.6)$$

respectively.

Substituting (3.4), (3.5) and (3.6) into (3.3), we get:

$$\frac{\Delta Q}{\Delta t} + \frac{\partial}{\partial x} \left(E^n + \frac{\partial E}{\partial t} \Delta t \right) - \left(S^n + \frac{\partial S}{\partial t} \Delta t \right) = 0 \quad (3.7)$$

(3.7) is written in terms of Jacobian matrices A and B as:

$$\frac{\Delta Q}{\Delta t} + \frac{\partial}{\partial x} (A \Delta Q) - B \Delta Q = - \frac{\partial E^n}{\partial x} + S^n \quad (3.8)$$

Factorizing $\left(\frac{\Delta Q}{\Delta t} \right)$ from (3.8), we obtain

$$\left[I + \Delta t \left(\frac{\partial A}{\partial x} \right) - B \Delta t \right] \Delta Q = - \Delta t \left(\frac{\partial E^n}{\partial x} + S^n \right) \quad (3.9)$$

where I is the identity matrix

$$\left(\frac{\partial A}{\partial x} \right) \Delta Q \Rightarrow \frac{\partial}{\partial x} (A \Delta Q)$$

Expressing (3.9) as an implicit Steger-Warming flux vector splitting method (SWFVSM) for the numerical scheme in the spatial term, we have;

$$\left[I + \left\{ \Delta t \frac{\partial}{\partial x} (A^+ + A^-) - B \right\} \right] \Delta Q = - \Delta t \left[\frac{\partial}{\partial x} (E^+ - E^-) + S \right] \quad (3.10)$$

The flux vector E and the flux Jacobian Matrix A are splitted as E^+ , E^- and A^+ , A^- respectively.

Taking the backward difference approximation for the positive terms and a forward difference approximation for the negative terms, we have;

$$\left[I + \frac{\Delta t}{\Delta x} \left\{ (A_i^+ + A_{i-1}^+ + A_{i+1}^- - A_i^-) - \Delta t B_i \right\} \right] \Delta Q = - \Delta t \left[\frac{1}{\Delta x} (E_i^+ - E_{i-1}^+ + E_{i+1}^- - E_i^-) + S_i \right] \quad (3.11)$$

Rearranging the above equation in terms of the grid point i for the Jacobian matrix A (or say for the right hand side of equation 3.11) we get;

$$-\left(\frac{\Delta t}{\Delta x} A_{i+1}^- \right) \Delta Q_{i+1} + \left[I + \frac{\Delta t}{\Delta x} (A_i^+ - A_i^-) - \Delta t B_i \right] \Delta Q_i + \left(\frac{\Delta t}{\Delta x} A_{i-1}^+ \right) \Delta Q_{i-1} = - \frac{\Delta t}{\Delta x} (E_i^+ - E_{i-1}^+ + E_{i+1}^- - E_i^-) + \Delta t S_i \quad (3.12)$$

The linearized equation of (3.12) is expressed as;

$$-\left(\frac{\Delta t}{\Delta x} A_{i-1}^0 \right) \hat{Q}_{i-1}^{n+1} + \left(1 + \frac{\Delta t}{\Delta x} (A_i^{0+} - A_i^{0-}) - \Delta t B_i^0 \right) \hat{Q}_i^{n+1} + \left(\frac{\Delta t}{\Delta x} A_{i+1}^0 \right) \hat{Q}_{i+1}^{n+1} = \hat{Q}_i^n \quad (3.13)$$

3.2 Initial Condition

The flow initially is assumed to be at steady state condition, therefore, the initial condition ($P_0(x)$, $V_0(x)$) of the fluid mixture can be obtained by computing the solution of the following system of ordinary differential equations:

$$d(\rho V) / dx = 0 \text{ and } dF(P, V) / dx = -gJ \quad (3.14)$$

with the values for $x=0$ and the head loss given as $J = \lambda V |V| / (2Dg)$

$$P_0(0) = \rho [P_0(0)] g H_0 + P_0 \text{ and } V_0(0) = 4Q_0 / (\pi D^2) \quad (3.15)$$

The required solution of the differential equation (3.14) may be obtained by the Runge-Kutta method [17]. The results are presented in Figure 2 for mass ratio value $\theta = 10^{-4}$.

3.3 Boundary Conditions

In the transient flow physical boundary conditions at the inlet and outlet of pipelines will be imposed to allow the consideration of a wide variety of field situations. The transient flow occurred by a rapid pump failure at the upstream end $x=0$, meaning that $V(0, t) = 0$. At the downstream end $x=L$ and $t > 0$, the condition is expressed by the reservoir at the fixed level: $P(L, t) = P_0(L)$.

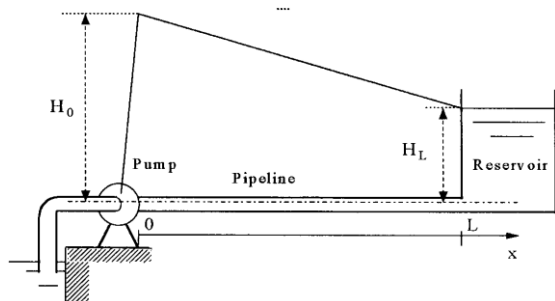


Figure 1 Systematic Hydraulic system

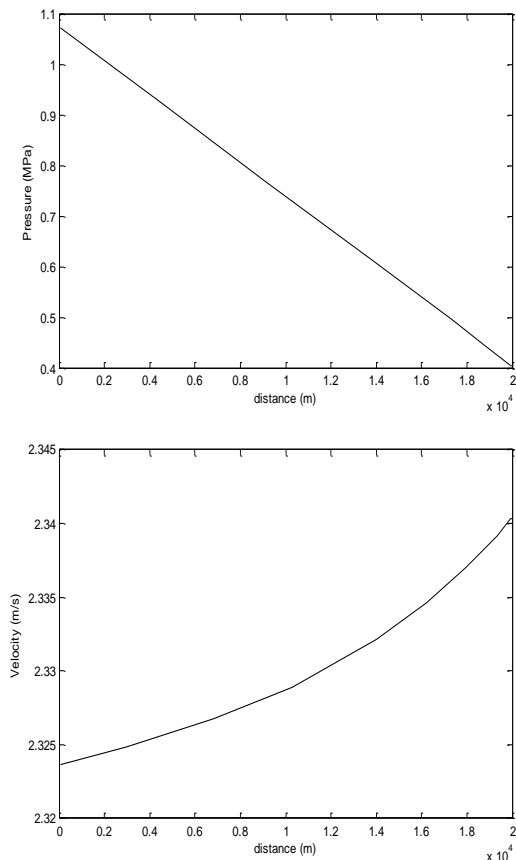


Figure 2 Initial Steady state flow profiles at $\theta = 10^{-4}$

4.0 RESULTS AND DISCUSSION

The Implicit Steger-Warming flux splitting method has been employed to simulate the pressure wave propagation for a two-phase gas-liquid mixture of transient flow.

In view of the hydraulic system represented by Figure 1, the problem is studied for a pipeline with the length $L=20,000\text{m}$, diameter $D=2\text{m}$, $P_0=10^5 \text{ MPa}$ (initial pressure), mass flow rate $Q_0=730 \text{ m}^3/\text{s}$, liquid density is $\rho_l = 1000 \text{ kg/m}^3$, gas density $\rho_g = 1.29 \text{ kg/m}^3$ and the friction factor $\lambda = 0.025$.

The result in Figure 3 present the comparison of the Implicit Steger-Warming flux splitting method with the characteristics methods at the upstream end of the pipe $x=0$ with mass ratio $\theta = 10^{-5}$. It can be observed that the present method show a good agreement with the characteristics methods by Hadj-Taieb and Lili [7].

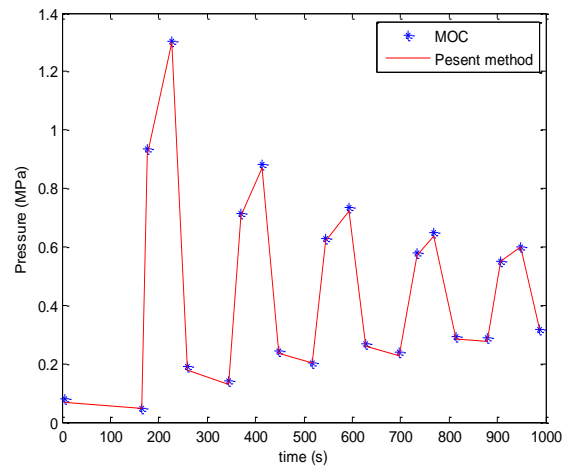


Figure 3 Comparison of Steger-Warming flux splitting method (present method) and method of characteristics (MOC) [7] at the upstream end of the pipe ($x=0$) with mass ratio $\theta = 10^{-5}$

Figure 3-7 presents the pressure wave propagation as a function of time at for different values of mass ratio as $\theta = 10^{-5}$, $\theta = 5.10^{-5}$, $\theta = 2.510^{-5}$ and $\theta = 10^{-4}$. Due to a sudden pump failure, a decreasing pressure wave was observed along a steady flow pressure gradient until it reaches the downstream end of the pipe.

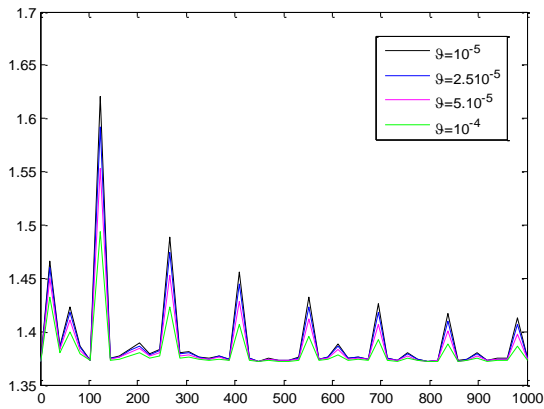


Figure 4 Pressure wave distributions against time at the pipe length $x = L$

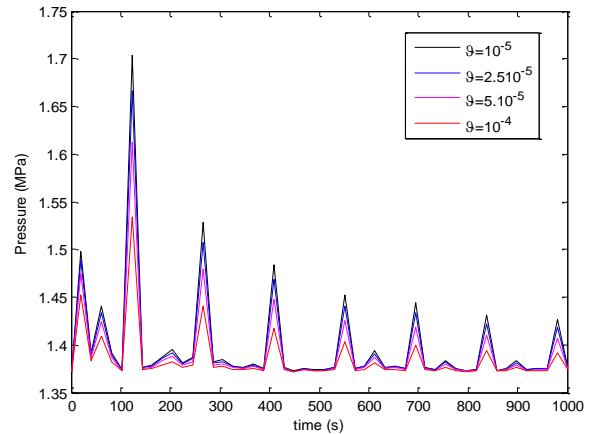


Figure 7 Pressure wave distributions against time at the pipe length $x = 3L/4$

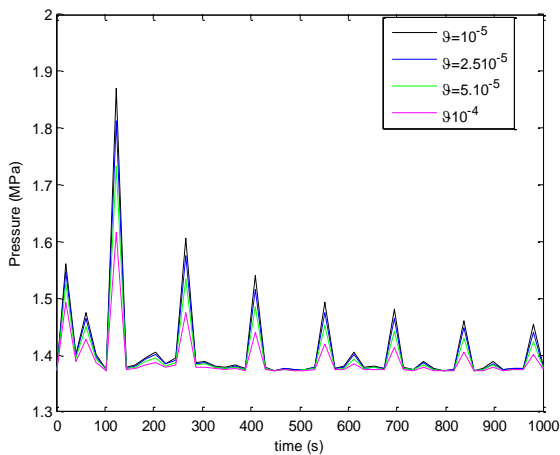


Figure 5 Pressure wave distributions against time at the pipe length $x = L/2$

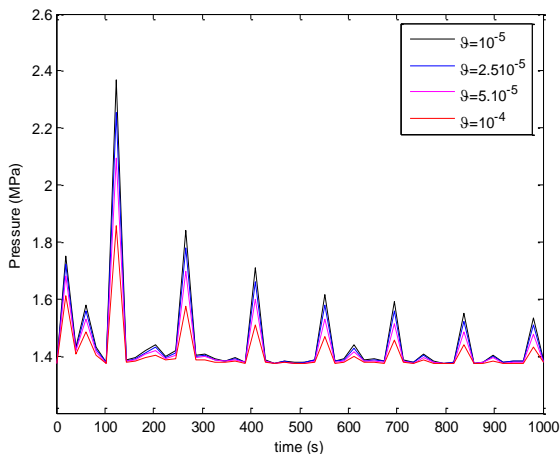


Figure 6 Pressure wave distributions against time at the pipe length $x = L/4$

Figure 8 represents the effect mass ratio against celerity as a function of time, for mass ratio $\varrho = 10^{-4}$ and $\varrho = 10^{-5}$. The upward increase in the direction of celerity was observed at a steady flow continues until it reaches the downstream end of the pipe.

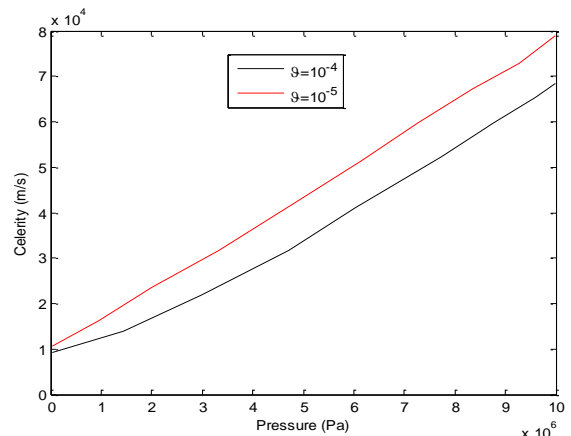


Figure 8 Celerity against time for value of mass ration $\varrho = 10^{-5}$ and $\varrho = 10^{-4}$

In Figure 9 the mass flow rate for values of mass ration of gas-liquid mixture was obtained as against time. It can be observed that the mass flow rate tends to zero at about 50s until the end of the pipe at a time of 1000s. This shows the effect the gas builds up during the pump control.

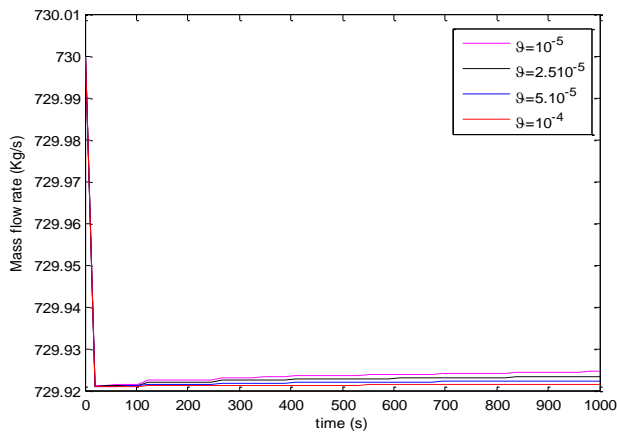


Figure 9 Mass flow rate against time for different mass ration

5.0 CONCLUSION

The one dimensional transient equation for homogeneous gas-liquid two phase flow in a pipe has been presented. The numerical scheme of Implicit Steger-Warming flux splitting method was used to solve numerically the governing equation. Pressure wave propagation for different values of mass ratio given as $\vartheta = 10^{-5}$, $\vartheta = 5.10^{-5}$, $\vartheta = 2.510^{-5}$ and $\vartheta = 10^{-4}$ was analysed. The celerity wave and mass flow rate with the effect of mass ration was also investigated. The pressure wave decreases along the pipe due to a sudden pump failure. It was also observed that the mass flow rate decrease which shows the effect the gas build up during the pump control.

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