Jurnal
Teknologi

Full Paper

AN APPLICATION OF SOFT SET IN DECISION MAKING

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Article history
Received
15 April 2015
Received in revised form
29 September 2015
Accepted
12 November 2015

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Abstract

In this paper, the definition of soft set and a detailed theoretical study of basic operations of soft sets such as *intersection, extended intersection, restricted intersection, union, restricted union, complement and relative complement, Null and universal soft set* are given. With the aid of definition of AND operation of soft sets and tabular representation of soft set, we are able to show that soft set has vital and real life application in decision making. The main aim of this paper is to use the concept of AND operation to sort out two best candidates out of five applicants in an interview conducted by a certain bank. Also the identification of Idempotent Property of **"AND"** and **"OR"** operation of soft sets is given and proved.

Keywords: Soft set

1.0 INTRODUCTION

Soft set theory was first proposed in (Molodtsov, 1999). It is a general mathematical tool for dealing with uncertainties and not clearly defined objects. Traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in nature. However, in most cases, complicated problems in economics, engineering, environment, social science, medical science, etc., that involved data which are not always all crisp cannot be successfully dealt with using classical methods because of various types of uncertainties present in these problems. Soft set is one of the various non-classical methods that can be considered as a mathematical tool for dealing with uncertainties.

Various potential applications of soft set in many areas like, in smoothness of functions, game theory, operations research, Riemann-integration, probability theory, theory of measurement and so on are highlighted in (Molodtsov, 1999).

2.0 CONCEPT OF SOFT SET AND BASIC DEFINITION

Definition 2.1 A pair (F, E) is called a soft set over a given universal set U, if and only if F is a mapping of a set of parameters E, into the power set of U [1]. That is,

$$F : E \longrightarrow P(U).$$

Clearly, a soft set over U is a parameterized family of subsets of a given universe U. Also, for any $p \in E$, $F(p)$ is considered as the set of p −approximate element of the soft set (F, E).

Example 1 [2]

(i) Let (X, τ) be a topological space, that is, X is a set and τ is a toplogy (a family of subsets of X called the open sets of X). Then, the family of open neighbourhoods $T(x)$ of point x, where $T(x) = \{V \in \tau \,|\, x \in V\}$ may be considered as the soft set $(T(x), \tau)$.

(ii) Also, fuzzy set is a special case of soft set; let A be a fuzzy set and μ_A be the membership function of the fuzzy set A, that is, μ_A is a mapping of U into $[0, 1]$, let $F(\alpha) = \{x \in U \,|\; \mu_A(x) \geq \alpha\}, \alpha \in [0, 1]$ be a family of α −level sets for function μ_A. If the family F is known $\mu_A(x)$ can be found by means of the definition: $\mu_A(x) =$

$Sup_{\alpha\in[0,1], x\in F(\alpha)}\ \alpha$. Hence every fuzzy set A may be considered as the soft set $(F,[0,1])$.

Example 2

Suppose $U = \{x_1, x_2, x_3\}$ be a set of cars under consideration and $A = \{x = \text{expensive}, y = \text{manual gear}, z = \text{Automatic gear}\}$ are the sets of parameters

Then (F,A) describes the attractiveness of the car which Mr. P (say) is going to buy. That is, to define a soft set, in this case, means to point out a car with some of the above parameters, i.e. expensive, manual gear cars or automatic

Suppose that $F(x) = \{x_1, x_2,\}$, $F(y) = \{x_2, x_3\}$, $F(z) = \{x_1, x_3\}$

Consider the mapping F which is "cars (.)" where dot (.) is to be filled up by a parameter $e_i \in E$. Thus we can view the soft set (F,E) as a collection of approximations as below:

$$(F,A) = \{F(x) = \{x_1, x_2,\}, F(y) = \{x_2, x_3\}, F(z) = \{x_1, x_3\}\}$$

For the purpose of storing a soft set in a computer for easy analysis in decision making, any soft set could be represented in a tabular form. For instance, Table 1, gives a tabular representation of the given example 2, where 1 means presence of attribute while 0 means absence of an attribute.

Table 1 Tabular representation of a soft set

	$x(expensive)$	$y(Manual\ gear)$	$z(Auto\ gear)$
x_1	1	0	1
x_2	1	1	0
x_3	0	1	1

Definition 2.2 A soft set (F,E) over a universe U is said to be null soft set denoted by $\tilde{\emptyset}$, if $\forall e \in E$, $F(e) = \emptyset$.

Definition 2.3 A soft set (F,A) over a universe U is called absolute soft set denoted by $\widetilde{(F,A)}$, if $\forall e \in E$, $F(e) = U$.

Definition 2.4 Let $E = \{e_1, e_2, e_3, \ . \ . \ ., e_n\}$ be a set of parameters. The not-set of E denoted by $\neg E$ is defined as $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \ . \ . \ ., \neg e_n\}$.

Definition 2.5 The complement of a soft set (F,E) denoted by $(F,E)^c$ is defined as $(F,E)^c = (F^c, \neg E)$.

Where: $F^c: \neg A \rightarrow P(U)$is a mapping given by $F^c(\alpha) = U - F(\neg\alpha)$, $\forall \alpha \in \neg A$

We call F^c the soft complement function of F.

Clearly, (i) $(F^c)^c = F$ and (ii) $((F,A)^c)^c = (F,A)$

Definition: 2.6 Let (F,A) and (G,B) be two soft set over a common universe U. The *restricted intersection* of (F,A) and (G,B) denoted by $(F,A) \Cap (G,B)$, and is define by

$(F,A) \Cap (G,B) = (H,C)$, where $C = A \cap B \neq \Phi$ and for all $c \epsilon C$,

$H(c) = F(e) \cap G(e)$. (see[3])

In addition to the above definition, Ali *et al.* [2] introduced a new definition for intersection called extended intersection as follows

Definition 2.7 The *extended intersection* of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = AUB$, and $\forall e \epsilon C$

$$H(e) = \begin{cases} F(e) & if\ e \epsilon A - B \\ F(e) \cap G(e) & if\ e \epsilon A \cap B \\ G(e) & if\ e \epsilon B - A \end{cases}$$

The relation is denoted by $(F,A) \cap_\varepsilon (G,B) = (H,C)$

Definition 2.8 Let (F,A) and (G,B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The *restricted union* of (F,A) and (G,B) is denoted by $(F,A) \cup_{\Re} (G,B) = (H,C)$ where $C = A \cap B$ and for all $c \epsilon C, H(c) = F(c) \cup G(c)$

Definition 2.9 Suppose (F,A) and (G,B) are two soft sets. The *extended* differences between (F,A) and (G,B) denoted by (F,A) - (G,B) is the soft set (H,C) where C = AUB, ∀e ϵ C.

$$H(e) = \begin{cases} F(e) & if\ e \epsilon A - B \\ F(e) - G(e) & if\ e \epsilon A \cap B \\ G(e) & if\ e \epsilon B - A \end{cases}$$

Definition 2.10 *(Conjunction of Soft sets)*

If (F,A) and (G,B) are two soft sets then "$(F,A)\ AND\ (G,B)$" denoted by $(F,A) \wedge (G,B)$ is defined by $(F,A) \wedge (G,B) = (H, A \times B)$,

Where $H(\alpha,\beta) = F(\alpha) \cap G(\beta)$, $\forall(\alpha,\beta) \in A \times B$.

Definition 2.11 *(Disjunction of soft sets)*

If (F,A) and (G,B) are two soft sets then "$(F,A)\ OR\ (G,B)$" denoted by $(F,A) \vee (G,B)$ is defined by $(F,A) \vee (G,B) = (P, A \times B)$.

Where, $P(\alpha,\beta) = F(\alpha) \cup G(\beta), \forall\ (\alpha,\beta) \epsilon A \times B$

3.0 ANALYSIS OF APPLICATION OF SOFT SET

Molodtsov [3] presented some applications of the soft set theory in several directions. In this section, we present an application of soft set theory in a decision making problem with the aid of AND/OR operation approach [4]. The following simple problem is considered.

A bank wants to employ two out of five people with certain ***qualities*** needed by the bank, they are not sure of the quality until after screening and scrutiny of the applicants. Among the ***qualifications*** needed by the bank are SSCE, B.Sc and M.Sc degree holders while for the ***quality*** of the applicants are Beautiful, Tall and brilliant and yet among the qualified applicants ugly are present with required qualification. In order to solve such problem, soft set is used to decide the best and right applicant as follows

Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ denotes the set of people in the interview for the job in the bank.
Let $A = \{a_1 = beautiful, a_2 = tall, a_3 = ugly, a_4 = brilliant\}$, and
$B = \{b_1 = SSCE, b_2 = B.Sc, b_3 = M.Sc\}$ be parameters representing the *quality* and *qualification* of the candidates respectively.
The soft set $(F, A) = \{F(a_1), F(a_2), F(a_3), F(a_4)\}$ where

$F(a_1) = \{x_1, x_2\}$, $F(a_2) = \{x_1, x_3, x_5\}$,
$F(a_3) = \{x_3, x_5\}$
$F(a_4) = \{x_1, x_2, x_3, x_4\}$

And $(G, B) = \{G(b_1), G(b_2), G(b_3)\}$

$G(b_1) = \{x_1, x_2, x_3, x_4, x_5\}$, $G(b_2) = \{x_1, x_2, x_5\}$,
$G(b_3) = \{x_1, x_5\}$

Then $(F, A) \wedge (G, B) = (H, A \times B)$,
Where
$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_2, b_3), (a_3, b_1), (a_3, b_2), (a_3, b_3), (a_4, b_1), (a_4, b_2), (a_4, b_3)\}$ and

$H(a_1, b_1) = F\{x_1, x_2\} \cap G\{x_1, x_2, x_3, x_4, x_5\} = \{x_1, x_2\}$ (1)

$H(a_1, b_2) = F\{x_1, x_2\} \cap G\{x_1, x_2, x_5\} = \{x_1, x_2\}$ (2)

$H(a_1, b_3) = F\{x_1, x_2\} \cap G\{x_1, x_2\} = \{x_1, x_2\}$ (3)

$H(a_2, b_1) = F\{x_1, x_3, x_5\} \cap G\{x_1, x_2, x_3, x_4, x_5\} = \{x_1, x_3, x_5\}$ (4)

$H(a_2, b_2) = F\{x_1, x_3, x_5\} \cap G\{x_1, x_2, x_5\} = \{x_1, x_5\}$ (5)

$H(a_2, b_3) = F\{x_1, x_3, x_5\} \cap G\{x_1, x_2\} = \{x_1\}$ (6)

$H(a_3, b_1) = F\{x_3, x_5\} \cap G\{x_1, x_2, x_3, x_4, x_5\} = \{x_3, x_5\}$ (7)

$H(a_3, b_2) = F\{x_3, x_5\} \cap G\{x_1, x_2, x_5\} = \{x_5\}$ (8)

$H(a_3, b_3) = F\{x_3, x_5\} \cap G\{x_1, x_2\} = \emptyset$ (9)

$H(a_4, b_1) = F\{x_1, x_2, x_3, x_4\} \cap G\{x_1, x_2, x_3, x_4, x_5\} = \{x_1, x_2, x_3, x_4\}$ (10)

$H(a_4, b_2) = F\{x_1, x_2, x_3, x_4\} \cap G\{x_1, x_2, x_5\} = \{x_1, x_2\}$ (11)

$H(a_4, b_3) = F\{x_1, x_2, x_3, x_4\} \cap G\{x_1, x_2\} = \{x_1, x_2\}$ (12)

Now
(1) Means beautiful applicant with SSCE are $\{x_1, x_2\}$
(2) Means beautiful applicant with B.Sc are $\{x_1, x_2\}$
……………………………………………………
(10) Means Brilliant applicant with SSCE are $\{x_1, x_2, x_3, x_4\}$
(11) Means Brilliant applicant with B.Sc are $\{x_1, x_2\}$
(12) Means Brilliant applicant with M.Sc are $\{x_1, x_2\}$

By representing (1) to (12) in a tabular form as in Table 2 below, the interviewer can easily sort out or decide what is needed as follows.

From the table the interviewer can deduce that $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ emerged the best for the fact that they have highest number of 1 (vertically) meaning they possessed the highest quality and qualification needed.

3.1 Idempotent Property of "AND" and "OR" operation of soft sets

Proposition The $\wedge$ and $\vee$ are not Idempotent i.e.
If (F, A) is a soft sets over a common universe U, then

$$(i)\ (F, A) \wedge (F, A) \neq (F, A)\ and\ (ii)\ (F, A) \vee (F, A) \neq (F, A)$$

Proof
(i) For soft set (F, A) over a common universe U, suppose $F(a) \in (F, A) \wedge (F, A)$, by the definition of AND operation on soft sets, the soft set $(F, A) \wedge (F, A)$ on the left side of (i) has the parameter set $A \times A$ and the soft set (F, A) on the right side of (i) has a set of parameters A, $\Longrightarrow F(a) \notin (F, A)$ therefore we cannot find any notion which ensure $A \times A = A$. Hence the proposition, the $(F, A) \wedge (F, A) \neq (F, A)$.
(ii) For soft set (F, A) over a common universe U, suppose $F(a) \in (F, A) \vee (F, A)$, by the definition of OR operation on soft sets, the soft set $(F, A) \vee (F, A)$ on the left side of (i) has the parameter set $A \times A$ and the soft set (F, A) on the right side of (i) has a set of parameters A, $\Longrightarrow F(a) \notin (F, A)$ therefore we cannot find any notion which ensure $A \times A = A$. Hence the proposition, the $(F, A) \vee (F, A) \neq (F, A)$.

Table 2 Tabular representations of a soft set

$(H, A \times B)$	a_1, b_1	a_1, b_2	a_1, b_3	a_2, b_1	a_2, b_2	a_2, b_3	a_3, b_1	a_3, b_2	a_3, b_3	a_4, b_1	a_4, b_2	a_4, b_3
x_1	1	1	1	1	1	1	0	0	0	1	1	1
x_2	1	1	1	0	0	0	0	0	0	1	1	1
x_3	0	0	0	1	0	0	1	0	0	1	0	0
x_4	0	0	0	0	0	0	0	0	0	1	0	0
x_5	0	0	0	1	1	0	1	1	0	0	0	0

4.0 CONCLUSION

With the aid of definition of AND operation of soft sets and tabular representation of soft set, we are able to show that soft set has vital and real life application in decision making problems. Successfully, two best candidates out of five applicants were sorted out in an interview conducted by a certain bank using the method. Also the identification of Idempotent Property of **"AND"** and **"OR"** operation of soft sets is given and proved.

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