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STABILITY ANALYSIS AND VIBRATION CONTROL OF A CLASS OF NEGATIVE IMAGINARY SYSTEMS

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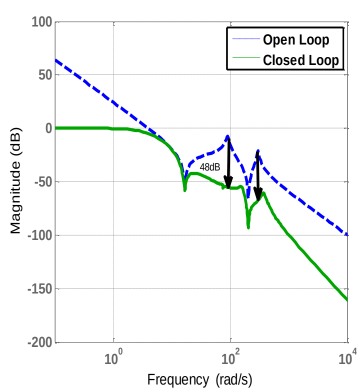
Graphical abstract



Abstract

This paper presents stability analysis and vibration control of a class of negative imaginary systems. A flexible manipulator that moves in a horizontal plane is considered and is modelled using the finite element method. The system with two poles at the origin is shown to possess negative imaginary properties. Subsequently, an integral resonant controller (IRC) which is a strictly negative imaginary controller is designed for the position and vibration control of the system. Using the IRC, the closed-loop system is observed to be internally stable and simuation results show that satisfactory hub angle response is achieved. Furthermore, vibration magnitudes at the resonance modes are suppressed by 48 dB.

Keywords: Negative imaginary systems, Flexible manipulator Integral resonant control

1.0 INTRODUCTION

Flexible manipulator offer several advantages over rigid robots such as they require less material, light in weight, consume less power, require small actuators, more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio. These types of robots are used in a wide spectrum of applications starting from simple pick and place operations of an industrial robot to micro-surgery, maintenance of nuclear plants and space robotics [1]. However, this type of flexible structures is highly resonant system, and consequently, subject to high amplitude oscillations in the presence of weak disturbances. The oscillations may result in significant loss of precision. Therefore there is a need to damp or control the oscillations that arise in flexible structures, however, control of flexible manipulators to maintain accurate positioning is extremely challenging. It has been reported that flexible structures with collocated actuators and sensors result in Negative Imaginary (NI) transfer function matrix [2, 3].

The theory of NI systems was first introduced by Lanzon and Petersen [4, 5]. NI systems are class of systems that belongs to the real rational stable system. The transfer function matrix $G(j\omega) = D + C(j\omega I - A)^{-1}B$ is negative imaginary if it satisfies the condition for negative imaginariness $j[G(j\omega) - G^{*}(j\omega)] \geq 0$ for all $\omega \, \epsilon \, (0, \infty)$ [5-8]. The Bode plot of NI systems has phase lag between 0 and $-\pi$ for all $\omega > 0$ and their Nyquist plot lies below the real axis as the frequency varies from 0 to ∞ [9, 10]. Examples of NI systems are DC machines, electrical active filter circuits and lightly damped flexible structures [11].

Dynamic model of flexible structures is highly resonant with high amplitude oscillation. The vibration and oscillation control problems for flexible structures are found in many systems such as in large space structures and nano-positioning of atomic force microscopes.

NI systems are defined by authors with different points of views. Initially, a system is NI system if and only if all the poles of $G(j\omega)$ lie in the open left half of the complex plane [2, 8]. This theory has been extended by [12] which allows for the NI systems to have poles in the closed left half of the complex plane excluding poles at origin. In [5, 8] systems that has poles on the imaginary axis are not regarded as NI systems. This definition was later extended in [6, 7] where the systems are allowed to have a simple pole on the imaginary axis except at the origin. Another modification in the definition of NI systems was presented by [13] in which the system is allowed to have a pole at origin. Furthermore, in [11, 14] the definition was extended to allowed for the systems with double poles at origin. The definition was extended to includes systems with non-rational transfer function in [15]. Another category known as lossless negative imaginary system are introduced by [16-18].

Stability analyses for interconnected NI systems with different approaches are presented in [5, 11, 13, 14, 19-26]. Spectral conditions for NI systems with application to nano-positioning were presented in [27-29]. A feedback control utilizing NI controller for vibration control was introduced by [8, 30]. Robust control of NI systems was introduced in [3, 9, 10]. In [31, 32] a methods of enforcing NI properties into the systems that are known to be NI systems, but loss some of their NI properties especially when the system is modeled using system identification methods was presented. Strictly Negative Imaginary (SNI) lemmas and theorems were presented in [33-34]. A finite frequency and infinite dimensional NI systems were introduced in [7, 35].

This paper presents analysis of the stability and negative imaginariness property of a single link flexible manipulator which is a class of NI systems. It is desirable to analysis the system, as it has double poles at origin. The negative imaginariness property test is based on the theorems and lemmas presented in [11, 14]. Subsequently, a SNI controller is designed in this paper to add damping to the flexible manipulator and control the unwanted oscillations of the system during its operations. The control objective is to move the flexible manipulator to a desired location faster with low end-point vibration. The paper is organized as follows: Section 2 introduces the theorem use for negative imaginary systems. Section 3 describes the flexible manipulator used in this work and testing of NI properties. Section 4 discusses on the controller design and simulation result and discussion are presented in section 5. Finally the paper is concluded in section 6.

2.0 BASIC THEOREMS

In this paper the NI property test was based on the theorems and lemmas presented in [11, 14]. These theorems and lemmas give rooms for systems with double poles at origin to be considered as NI systems. The following theorems and lemmas, discuss definitions of NI and SNI.

Theorem 1 [14]: A square transfer function matrix $G(s)$ is NI if the following conditions are satisfied:

1) $G(s)$ has no pole in $Re[s] > 0$.
2) For all $\omega \geq 0$ such that $j\omega$ is not a pole of $G(s), j[G(j\omega) - G(j\omega)^*] \geq 0$
3) If $s = j\omega_0, \omega_0 > 0$ is a pole of $G(s)$ then it is a simple pole.

Additionally, if $s = j\omega_0, \omega_0 > 0$ is a pole of $G(s)$, then residual matrix $K = \lim_{s \to j\omega_0} (s - j\omega_0) jG(s)$ is positive semidefinite Hemitian. If $s = 0$ is a pole of $G(s)$, then it is a simple pole or a double poles. If it is a double poles then $\lim_{s \to 0} s^2 G(s) \geq 0$.

Theorem 2 [14]: A square transfer function matrix $G(s)$ is SNI if the following conditions are satisfied:

1) $G(s)$ has no pole in $Re[s] \geq 0$.
2) For all $\omega > 0$, $j[G(j\omega) - G(j\omega)^*] > 0$.

Lemma 1 [14]: Consider a square real rational proper transfer function matrix $G(s)$ with the state space realization $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ such that $D = D^T$ and the transfer function matrix $\tilde{G}(s) = G(s) - D$. Then the transfer function matrix $G(s)$ is NI if and only if the transfer matrix $H(s) = s\tilde{G}(s)$ is positive real assuming all pole-zero cancellation in $s\tilde{G}(s)$ is taking care to obtained $H(s)$.

Explanation: Suppose that $H(s)$ is a positive real, then $H(j\omega) + H(j\omega)^* \geq 0$ for all $\omega \, \epsilon \, (-\infty, \infty)$ such that $j\omega$ is not a pole of $H(s)$. This shows that $j\omega(\tilde{G}(j\omega) - \tilde{G}(j\omega)^*) \geq 0$ for all $\omega \geq 0$ such that jω is not a pole of $G(s)$. Then $(\tilde{G}(j\omega) - \tilde{G}(j\omega)^*) \geq 0$ for all such that $\omega \, \epsilon \, (0, \infty)$.
Similarly,

$$\lim_{s \to j\omega_0} (s - j\omega_0) jH(s) = \lim_{s \to j\omega_0} (s - j\omega_0) s\tilde{G}(s)$$
$$= \omega_0 \lim_{s \to j\omega_0} (j\omega_0) j\tilde{G}(s)$$

Thus, it can be seen from Theorem 1, $\tilde{G}(s)$ is NI and hence $G(s)$ is NI [14].

A generalize lemma is provided which allows for a simple pole or double pole at the origin. Consider a linear time invariant system as

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$
$$y(t) = Cx(t) + Du(t) \quad (2)$$

Where, $A \in \Re^{m \times m}, B \in \Re^{m \times n}, C \in \Re^{n \times m}$, and $D \in \Re^{n \times n}$ and $m \times m$ and $m \times n$ are the dimensions of the state space.

Lemma 2 [14]: Let $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ be a minimal realization of the transfer function matrix $G(s)$ for the system in Equations (1) and (2). Then $G(s)$ is NI if and only if $D = D^T$ and there exist a matrix $P = P^T \geq 0$ such that the Linear Matrix Inequality (LMI) below satisfy the condition

$$P = P^T \geq 0 \quad (3)$$

$$\begin{bmatrix} PA + A^T P & PB - A^T C^T \\ B^T P - CA & -(CB + B^T C^T) \end{bmatrix} \leq 0 \quad (4)$$

3.0 THE FLEXIBLE MANIPULATOR AND NI PROPERTIES TEST

This section describes the flexible manipulator considered in this study. Figure 1 shows the schematic diagram of a single link flexible manipulator system. w,ere $\{O, X_0, Y_0\}$ and $\{O, X, Y\}$ represent the stationary and moving frames respectively. τ is the torque applied at the hub of the manipulator. The rotation of frame $\{O, X, Y\}$ relative to frame $\{O, X_0, Y_0\}$ is described by the angle, θ. The displacement of the link from the axis OX at a distance x is designated as $v(x, t)$.

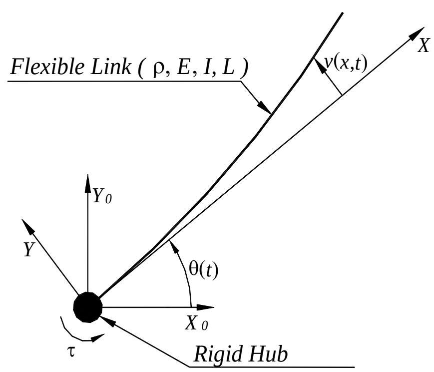


Figure1 Schematic diagram of flexible link

The flexible link used in this study is made up of a piece of thin aluminum alloy. The parameters of the system are; length of the flexible link $L = 1.0$ m, Young Modulus $E = 207.87363 \times 10^6$ N/m^2, width of the link 12.50 mm, thickness of 1.440 mm, second moment of inertia $I = 5.1924$ m^4, and mass density per unit volume $\rho = 2710$ kg/m^3. The model of the flexible manipulator used in this paper is obtained using finite element method as presented in [36].

The transfer function that relates the hub angle $\theta(s)$ to the input torque $\tau(s)$ can be obtained as

$$G(s) = \frac{1014s^4 + 4553s^3 + 4.235\times10^7 s^2 + 2.865\times10^7 s + 1.178\times10^{10}}{s^2(s^4 + 33.37s^3 + 97260s^2 + 1.164\times10^6 s + 7.257\times10^8)} \quad (5)$$

It is noted in Equation (5) that the system is a type two system with double poles at the origin. The transfer function can be represented in a state space form as

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 58209 & -27441 & 0 & -33 & -6 \\ 0 & -38548 & 16329 & 0 & -27 & 4 \\ 0 & 93918 & -58611 & 0 & 16 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1013.6 \\ -821.0 \\ 304.1 \end{bmatrix}; \quad C = [1\ 0\ 0\ 0\ 0\ 0]; \quad D = [0]$$

Subsequently, the investigation is conducted to test whether the flexible manipulator system satisfies the condition for NI. To test for the negative imaginariness of the system based on the conditions in Lemma 2, the value of *P* should be obtained such that

$P = P^T \geq 0$ and

$\begin{bmatrix} PA + A^T P & PB - A^T C^T \\ B^T P - CA & -(CB + B^T C^T) \end{bmatrix} \leq 0$ as in Equations (3) and (4).

By using the LMI Matlab Toolbox the value of *P* can be obtained as

$$P = \begin{bmatrix} 0.0149 & -0.6630 & -0.0061 & -0.0101 & -0.0120 & 0.0014 \\ -0.6630 & 300.9641 & -25.4759 & 3.5785 & 4.2026 & -0.5465 \\ -0.0061 & -25.4759 & 16.1161 & 0.0310 & 0.0452 & 0.0074 \\ -0.0101 & 3.5785 & 0.0310 & 0.0543 & 0.0628 & -0.0079 \\ -0.0120 & 4.2026 & 0.0452 & 0.0628 & 0.0741 & -0.0093 \\ 0.0014 & -0.5465 & 0.0074 & -0.0079 & -0.0093 & 0.0014 \end{bmatrix} \quad (6)$$

The eigenvalues of *P* are obtained as

[0.0002, 0.0006, 0.0070, 0.0134, 13.8772, and 303.3266]

As all the eigenvalues of *P* are positive, $P = P^T \geq 0$ and the conditions in Equations (3) and (4) are satisfied. In addition, $D = D^T = 0$. Thus, the system can be proved as a NI system.

Figure 2 shows the open loop frequency response plot of the system. The frequency response plot shows that the phase angle lags between 0 to $-\pi$ for all $\omega > 0$ which implies that the system is a NI system. Figure 3 shows the root locus plot and open loop poles location with two poles at the origin.

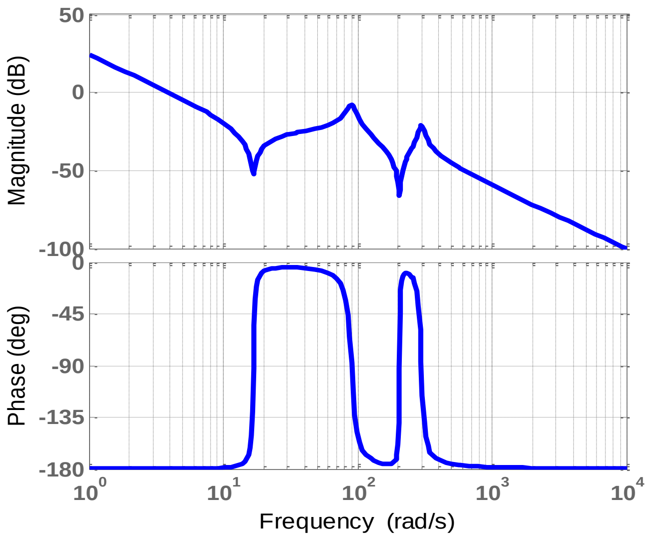


Figure 2 Open Loop Bode Plot

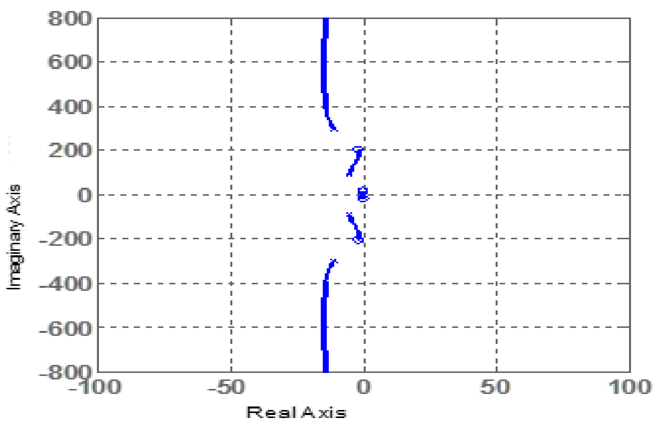


Figure 3 Root Locus Plot

4 0 CONTROLLER DESIGN

As the flexible manipulator has been shown to be a NI system, the SNI controller can be designed for position control and vibration suppression. In this paper an Integral Resonant Controller (IRC) as shown in Figure 4 is considered. The IRC consists of two blocks, namely a resonant controller (RC) block designed using the resonant frequencies and damping ratios of the system to add damping and suppressed the vibration and an integral control block, designed to cancel steady state error.

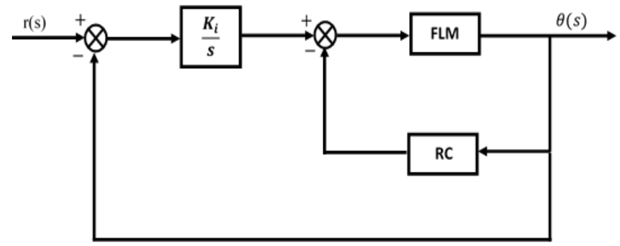


Figure 4 IRC Block Diagram

A Resonant Controller has a general transfer function as

$$C_i^{\alpha} = \sum_{i=1}^{N} \frac{\alpha_i s^2}{s^2 + 2\delta_i w_i + w_i^2} \qquad (6)$$

where N is the number of resonant modes of the flexible manipulator, δ_i and w_i are the damping ratios and resonant frequencies respectively. The variable α_i lies between
$0 \leq \alpha_i \leq 1$.

In this work, experimental results of resonant frequency and damping ratio reported in [35] are used for controller design. Table 1 shows resonance frequencies and their corresponding damping ratios obtained experimentally.

Table 1 Resonance frequency and damping ratio

N	$w_i(rad/sec)$	δ_i
1	79.3354	0.007
2	202.4442	0.015
3	309.6637	0.314

5.0 RESULTS AND DISCUSSIONS

This section presents the simulation results obtained using Matlab and discusses on the controller performance. The flexible manipulator is required to move to a desired angle of 1 rad. Hub angle response in time and frequency domains are monitored for assessment of the controller performance. Figure 5 shows the Nyquist plot of the controller function, which only touches the real axis at 0 and ∞. This implies that the controller is SNI controller. The Nyquist plot of the closed loop system is shown in Figure 6 which indicates that the overall closed loop system is NI.

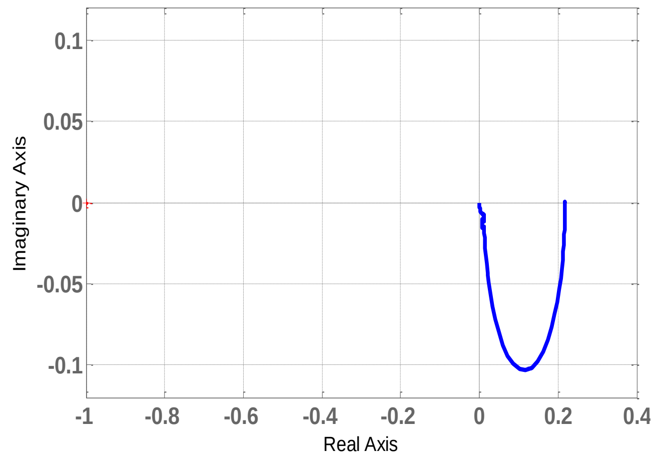


Figure 5 Nyquist Plot for the Controller

Figure 7 shows the hub angle response of the system with IRC. It is noted that a satisfactory hub angle tracking of 1 rad with settling time of 1.36 seconds is observed. Moreover, the manipulator moves to the desired location without overshoot. The controller has successfully added damping to the system and suppressed vibration to achieve precise hub angle positioning. The controller performance is further demonstrated in Figure 8 that shows the frequency response of the open loop and closed loop systems. At the resonance frequencies, a reduction of 48 dB of magnitude can be observed which implies lower system vibration.

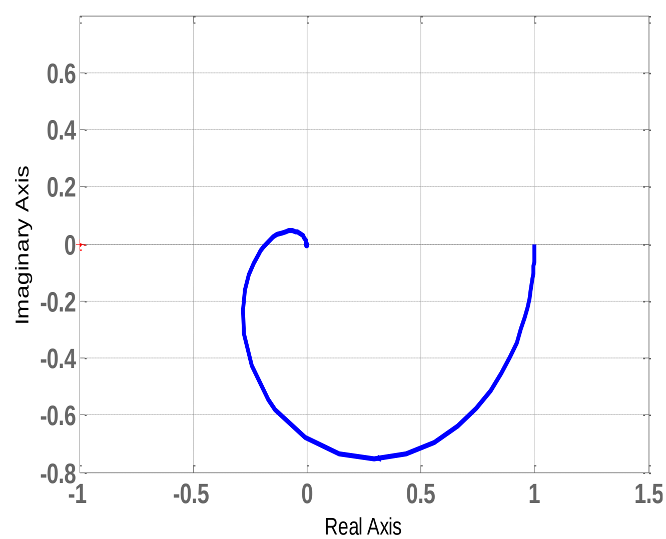


Figure 6 Nyquist Plot for the Closed loop System

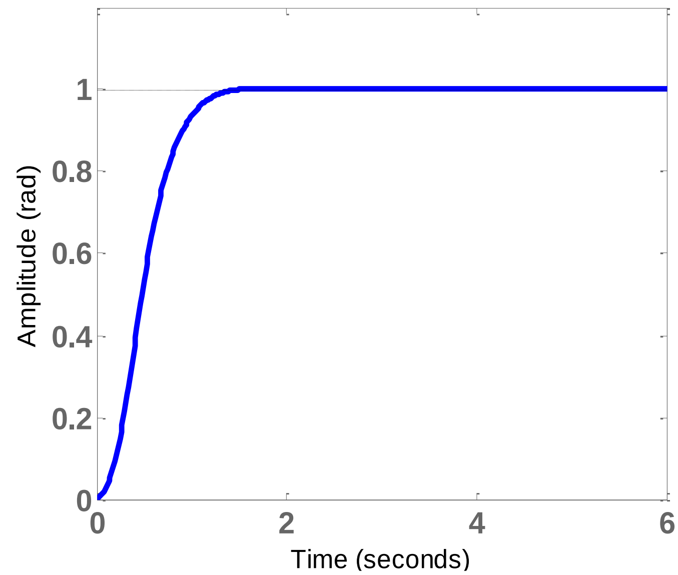


Figure 7 Closed Loop Step Response

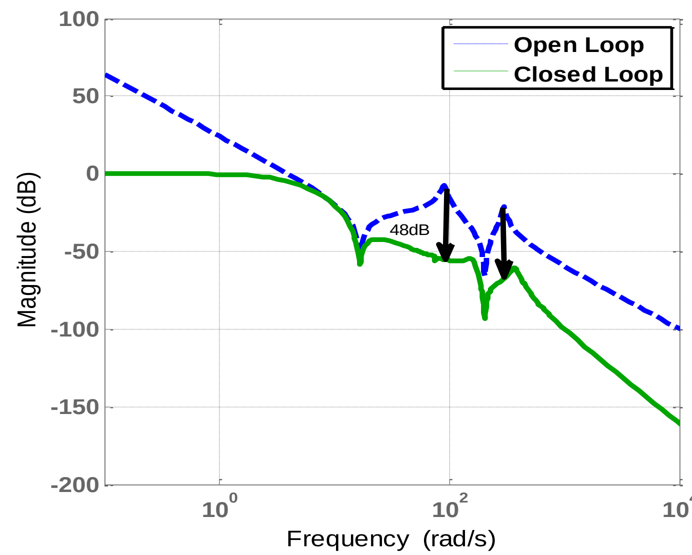


Figure 8 Closed-loop and Open-loop Bode Plots

6.0 CONCLUSION

A stability analysis and negative imaginariness property test has been successfully conducted on a single link flexible manipulator system. It is found that the flexible manipulator satisfied all the conditions of negative imaginariness. An IRC which has also been tested to be an SNI controller has been designed to control vibration and precise positioning of the system. The results show that the controller successfully adds damping to the system. Satisfactory hub angle response has been achieved with vibration reduction of 48 dB. In future this control will be implemented on a real system.

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