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# TRANSFORMATION MATRICES IN GENERATION OF CIRCULAR PATTERNS

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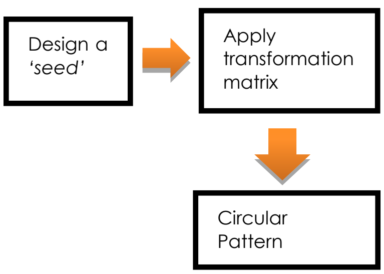
## Graphical abstract




## Abstract

This paper aims to explore the generation of circular patterns based on transformation matrix. The idea proposed here is to first design a fundamental pattern, known as '*seed*'. This is then followed by applying the transformation matrix to arrange the fundamental pattern to form a circular pattern. The transformation matrix can be applied once or applied iteratively. By assigning different values to the parameters involved, the patterns produced act in an interesting and appealing way. The proposed method contributes to a simple and efficient framework in computational generation of endless circular patterns.

*Keywords*: Transformation matrix, patterns generation, geometry



## 1.0 INTRODUCTION

Nature provides the best source of inspiration for pattern creation. People have long been attempting to find the intelligence behind all these fascinating patterns, and then further apply the explored idea to enhance and design the new outputs. Mathematical community has carried out various studies to investigate the underlying behavior and set up the corresponding mathematical principles to duplicate, create or analyze the patterns inspired in nature [1].

Some of the well-established models related to patterns in nature are as follows: Formation of animal skin pigment/coat patterns can be described by Reaction diffusion (RD) system [2]; fractal with infinite self-similar patterns have been shown exist in a lot of nature phenomena such as tree, river network, waves, clouds, blood vessel branching, snowflakes etc. [3], and partial differential equations built upon physically based models have been used to describe the surface crack patterns [4]. Whereas, there are quite a number of research groups that have developed the mathematical modelling to generate and duplicate the nature element such as tree, flowers, coastal geomorphology etc. [5, 6, 7].

Ancient civilization evidently is another source of inspiration in patterns generation. Work on symmetry and antisymmetry analysis have been long developed to study the Palcolithic and Neolithic ornamental art [8]. Various models have been developed to construct the Islamic patterns [9], such as technique for blending the 1-point and 2-point applications of the 'polygons in contact' [10]; axioms of absolute geometry for parameterized family of tiling in Islamic star patterns [11]; grid method based on the Minimum Number of Grids (MNG) and Lowest Geometric Shape (LGS) in the construction of Islamic geometric patterns [12] etc.

The invention of digital computer provides a new means of encoding and simulating the mechanism to execute more ideas in pattern generation. Moire´ patterns which are constructed by overlapping the intersection of two or more sets of lines can be generated by the product-delay algorithm utilizing the sine and square waves [13, 14]. Interesting geometric patterns can also be created based on recursive relative motions. In [15], trajectories of the waltz dancer are traced and a specific palette is used to color the trajectories to form the patterns. In the aspect of pattern in hyperbolic geometry, fast FR algorithm with flexible construction of invariant mappings has been developed [16]. Thus, the study related to pattern generation is a non-closed problem.

Motivated by the arrangement of flower petals of blooming flowers in nature, this project aims to explore the generation of circular patterns utilizing the transformation matrix. Transformation matrix is commonly used in computer graphics, mainly for the purpose to alter or manipulate an object, for example in changing the size or orientation, repositioning the object. Here we demonstrate that transformation matrix can be used to create visually appealing circular patterns by exploring various ways to arrange a fundamental design with the assistance of computer program.

The core idea here can be described in two phases. Phase one is to design a fundamental pattern which we refer as '*seed*'. In phase two, the '*seed*' will then be scaled, rotated, translated etc. by applying various transformation matrices to fill up the layout in circular form. The final generated pattern is somehow different with the well-known pattern such as epicycloid, epitrochoid, hypocycloid, hypotrochoid which can be produced by spirograph drawing tool, or even the rhodonea curve or farris wheel. These well-known patterns are generated based on mathematical relations such as trajectory equation or parametric equations [17, 18] which are different with ours where the patterns are obtained based on applying the transformation matrices to arrange a prior designed fundamental pattern.

The transformation matrices set up here are simple and direct. They can be applied once or applied iteratively. With some variations to the parameters in the chosen transformation matrices, the patterns generated though are deterministic but acts in an interesting yet appealing way. These patterns offer a high potential values to be used by those design researchers for artistic purposes rather than just only as illumination of mathematics.

The proposed methodology utilizes two-phase construction process. The first phase is to create a fundamental pattern, known as '*seed*'. Next in the second phase, the transformation matrix specifies the movement of the initial point $(x, y)$ in the '*seed*' to a new location $(x_0, y_0)$ are defined to fill the '*seed*' in the overall layout of the $xy$-plane. Each '*seed*' is a fundamental style, which is to be duplicated, translated, rotated etc. to fill up the plane.

The developed transformation matrices are discussed in Section 2.0 and Section 3.0. Section 2.0 mainly focuses on the basic transformation matrices with the respective generated patterns. Whereas, Section 3.0 showed the variations involved in the transformation matrices if the transformation are to be applied iteratively. Finally, conclusion is wrapped up in Section 4.0.

## 2.0 BASIC TRANSFORMATION AND RESULTS

In the first phase of pattern generation, we have to choose a fundamental style to act as the 'seed'. The '*seed*' can be of any design. Motivated from the shape of the flower petals in nature, thus the '*seed*' is then chosen to take the shape of a flower petal for illustration purpose here. For ease of discussion, one specific example of '*seed*' will be used throughout subsequent discussion. Here, the petal of the flower '*Plumbago auriculata*' has been chosen in creating the '*seed*'. Figure 1 depicts the original photo of the flower, its traced outline and the corresponding outline of one petal chosen as the 'seed' for the subsequent pattern generation.

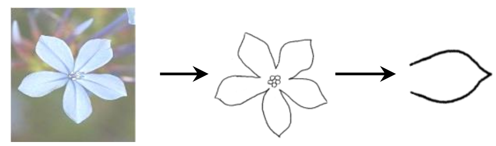


**Figure 1** The original flower '*Plumbago auriculata*', the traced outline and the '*seed*'

In generating the pattern, the '*seed*' will first be placed on the $xy$-plane with $O_1$ as the origin and the coordinates for point $A$ and $B$ are $(0, a_0)$ and $(0, -a_0)$, respectively (see Figure 2). Of course, for variation in pattern creation, the '*seed*' does not necessarily possess symmetrical pattern and the origin can be set to any point on the '*seed*'.

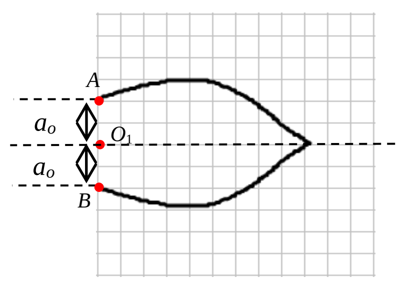



**Figure 2** Example of '*seed*' with petal style in $xy$-plane

### 2.1 Rotation and Translation

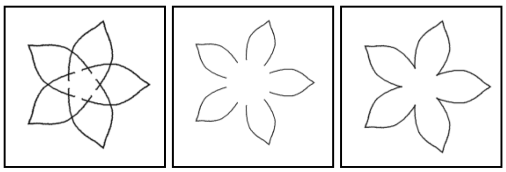
Motivated by the circular arrangement of petals for blooming flower, combination of rotation and translation as follows are used to rotate the '*seed*' about the angle $\theta$ and then further translate the rotated '*seed*' to a circle of radius $r$ with origin as the center:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(i\theta) & -\sin(i\theta) \\ \sin(i\theta) & \cos(i\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} r\cos(i\theta) \\ r\sin(i\theta) \end{bmatrix}, \quad i = 0,1,..,n-1, \quad (1)$$

where $\theta = 2\pi / n$ and $n$ is a predefined positive integer specifies the number of '*seed*' required to complete the arrangement in one circle.

For $n$ '*seeds*' to be arranged around the circle, the '*seed*' may intersect, separate with some space in between or joined nicely depend on the value of $r$

used in transformation matrix specified by equation (1). If the *'seed'* follows the pattern of a flower petal with symmetry property as shown in Figure 2, then there exist a value of the radius of circle, $r_0$ such that the ends of the petals can join nicely with its neighboring petal without intersection. We refer to this pattern as *'joined seed'* in the discussion hereafter, otherwise it is known as *'non-joined seed'*. Figure 3 shows the pattern of *'joined seed'* and *'non-joined seed'* situations generated from transformation matrix (1) for various $r$ values and $n=5$ using the *'seed'* as depicted in Figure 2.



(a) (b) (c)

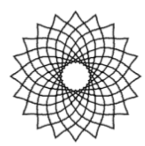
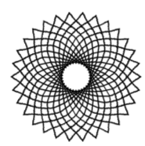
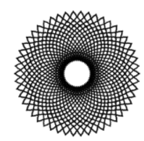
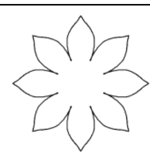
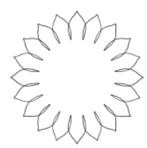
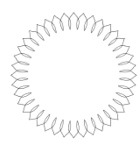
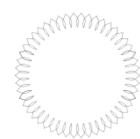
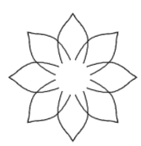
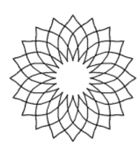
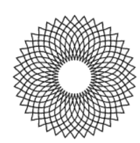
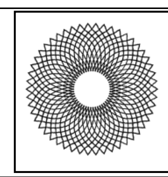
**Figure 3** Patterns of *'non-joined seed'*; (a) $r=0$ (b) $r>r_0$ and *'joined seed'*: (c) $r=r_0$

Obviously, $r_0$ varies according to $n$ for fixed size of *'seed'*. Given a preset $n$ value, it is not difficult to see that radius $r_0$ can be determined by the equation

$$r_0 = a_0 \cot\left(\frac{\theta}{2}\right) = a_0 \cot\left(\frac{\pi}{n}\right) \quad (2)$$

Table 1 shows some circular patterns generated by transformation matrix (1) for different parameters of $n$ and $r$ using the same *'seed'* from Figure 2.

**Table 1** Patterns generated based on transformation matrix (1) for various $r$ and $n$

| | $n = 8$ | $n = 20$ | $n = 30$ | $n = 50$ |
|---|---|---|---|---|
| $r = 0$ |  |  |  |  |
| $r = r_0$ |  |  |  |  |
| $r = a_0 \cot\left(\frac{\pi}{5}\right)$ |  |  |  |  |

## 2.2 Scaling

Observations of flowers in nature show that flower petals are generally unequal in size, this motivates us to use the scaling property to arrange the *'seed'* of different sizes instead of one standard size. In resizing the *'seed'*, width and height of the *'seed'* can be scaled according to the scaling factor, $k_x$ along $x$ direction and $k_y$ along $y$ direction. If the proportion of the original *'seed'* is to be remained, uniform scaling has to be performed by setting $k_x = k_y$. By adding the scaling property to transformation matrix (1) leads to

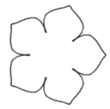
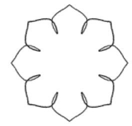
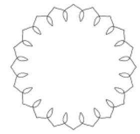
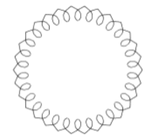
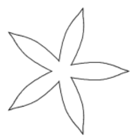
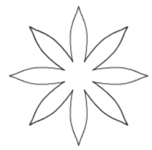
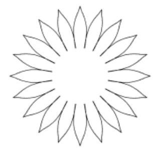
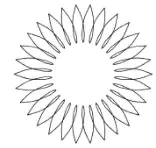
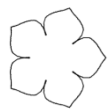
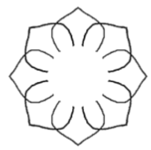
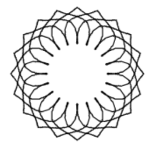
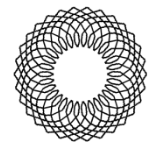
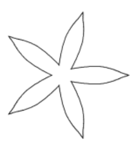
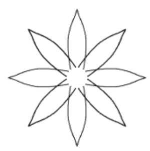
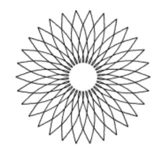
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(i\theta) & -\sin(i\theta) \\ \sin(i\theta) & \cos(i\theta) \end{bmatrix} \begin{bmatrix} k_x x \\ k_y y \end{bmatrix} + \begin{bmatrix} r\cos(i\theta) \\ r\sin(i\theta) \end{bmatrix}, \; i = 0,1,..,n-1. \quad (3)$$

Of course, if no scaling is required then $k_x = k_y = 1$, the above equation will be reduced to equation (1) exactly. For the *'joined seed'* pattern, $r$ in equation (3) will be replaced by $r_0$ which can be calculated by modifying equation (2) to

$$r_o = k_y a_0 \cot\left(\frac{\theta}{2}\right) = k_y a_0 \cot\left(\frac{\pi}{n}\right). \quad (4)$$

Table 2 lists various patterns generated by transformation matrix (3) with different values of parameters $k_x$, $k_y$, $n$ and $r$, with the same *'seed'* in Figure 2.

**Table 2** Patterns generated based on transformation matrix (3) with different combination of $k_x$, $k_y$, $n$ and $r$

| | $n = 5$ | $n = 8$ | $n = 20$ | $n = 30$ |
|---|---|---|---|---|
| $r = r_0$ $k_x = 0.5$ $k_y = 1$ |  |  |  |  |
| $r = r_0$ $k_x = 1$ $k_y = 0.5$ |  |  |  |  |
| $r = a_0 \cot\left(\frac{\pi}{5}\right)$ $k_x = 0.5$ $k_y = 1$ |  |  |  |  |
| $r = a_0 \cot\left(\frac{\pi}{5}\right)$ $k_x = 1$ $k_y = 0.5$ |  |  |  |  |

### 2.3 Alternate '*Seed*' Size Arrangement

For more variations in pattern generation, arrangement of '*seed*' can be in alternate size: one is in the original '*seed*' size and another one is the scaled '*seed*' with scaling factor $k_x$ and $k_y$. If the '*seed*' of two sizes are to be arranged in a circle with alternate '*seed*' size, then given a prefix $n$ and $r$ values, a pattern can be generated based on the following transformation matrices:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(i\theta) & -\sin(i\theta) \\ \sin(i\theta) & \cos(i\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} r\cos(i\theta) \\ r\sin(i\theta) \end{bmatrix}, \quad i = 0,2,4,\ldots,2m,$$

and

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(i\theta) & -\sin(i\theta) \\ \sin(i\theta) & \cos(i\theta) \end{bmatrix} \begin{bmatrix} k_x x \\ k_y y \end{bmatrix} + \begin{bmatrix} r\cos(i\theta) \\ r\sin(i\theta) \end{bmatrix}, \quad i = 1,3,\ldots,2m-1, \tag{5}$$

with $m = \lfloor n/2 \rfloor$. For the '*joined seed*' pattern, the values of $r_{o1}$ and $r_{o2}$ have to be calculated. $r_{o1}$ and $r_{o2}$ are the distances from the origin to the reference point $O_1$ of the original '*seed*' size and the scaled '*seed*', respectively. From equation (2), we observe that

$$r_{o1} = a_0 \cot\left(\frac{\theta_1}{2}\right) \text{ and } r_{o2} = k_y a_0 \cot\left(\frac{\theta_2}{2}\right). \tag{6}$$

Here $\theta_1$ is the angle for the original '*seed*' size and $\theta_2$ is the angle for the scaled '*seed*'. For fixed number of $n$ '*seed*' in one circle, $\theta_1$ and $\theta_2$ can be calculated by employing the following argument:

Since $n$ '*seed*' of alternate size must be arranged in a circular path, so we have $\frac{n}{2}(\theta_1 + \theta_2) = 2\pi$ which gives

$$\theta_2 = \frac{4\pi - n\theta_1}{n}. \tag{7}$$

The distance from the origin to the ends of the petals for both the original '*seed*' and the scaled '*seed*' must be the same. Based on the trigonometry property, we have

$$\frac{a_0}{\sin\frac{\theta_1}{2}} = \frac{k_y a_0}{\sin\frac{\theta_2}{2}}. \tag{8}$$

Substituting equation (7) to the above equation and solve for $\theta_1$ gives

$$\theta_1 = 2\tan^{-1}\left(\frac{\sin^2\left(\frac{2\pi}{n}\right)}{k_y + \cos\left(\frac{2\pi}{n}\right)}\right), \tag{9}$$

with values of $\theta_1$ and $\theta_2$ calculated from (9) and (7), respectively, we could obtain $r_{o1}$ and $r_{o2}$ as given by (6). Hence, transformation matrix (5) can be modified as follows to generate the alternate '*seed*' size arrangement for '*joined seed*' pattern:

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} \cos(i\omega) & -\sin(i\omega) \\ \sin(i\omega) & \cos(i\omega) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} r_{o1}\cos(i\omega) \\ r_{o1}\sin(i\omega) \end{bmatrix}, \quad i = 0,2,\ldots,2m,$$

and

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} \cos(i\omega) & -\sin(i\omega) \\ \sin(i\omega) & \cos(i\omega) \end{bmatrix} \begin{bmatrix} k_x x_i \\ k_y y_i \end{bmatrix} + \begin{bmatrix} r_{o2}\cos(i\omega) \\ r_{o2}\sin(i\omega) \end{bmatrix}, i = 1,3,\ldots,2m-1, \tag{10}$$

where $\omega = (\theta_1 + \theta_2)/2$ and $m = n/2$. Table 3 lists various patterns generated by equation (5) and (10) with different combinations of $k_x$, $k_y$, $n$ and $r$.

### 2.4 'Seed' Size Changing Continuously

Different with the idea discussed in previous subsection, '*seed*' size can also be specified to increase or decrease gradually as the '*seed*' rotates along the circular path in the arrangement. Let $c$ be a constant which monitors the change of '*seed*' size. The angle for each '*seed*' varies according to the arithmetic sequence $\theta_1 + (n-i)c\theta_1$, $i = 1,\ldots,n$. If $c < 0$, the '*seed*' will reduce in size. When $c = 0$ the '*seed*' size remain the same. While $c > 0$, the '*seed*' size increases. For any given fixed $n$ and $c$, the sum of all the angles for each '*seed*' must fulfil

$$\sum_{i=1}^{n} (\theta_1 + [n-i]c\theta_1) = 2\pi .$$

Solving the above gives the value of $\theta_1$. The angles for the subsequent *i*th-'*seed*' can then be obtained based on

$$\theta_{n-i} = \theta_1 + [n-i]c\theta_1, \quad i = 2,3,\ldots n .$$

To determine the scale factor for the *i*th-'*seed*', we utilize the fact that all the ends of the petals must be joined, applying the same argument in getting equation (8), we have

$$\frac{a_0}{\sin\left(\frac{\theta_1}{2}\right)} = \frac{k_i a_0}{\sin\left(\frac{\theta_i}{2}\right)}, \quad i = 2,3,\ldots,n.$$

This gives the scaling factor for *i*th-'*seed*' as

$$k_i = \frac{\sin\left(\frac{\theta_i}{2}\right)}{\sin\left(\frac{\theta_1}{2}\right)}, \qquad i = 2,3,\ldots,n.$$

Hence, the transformation matrix used can be modified from equation (3) as

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\Omega_i) & -\sin(\Omega_i) \\ \sin(\Omega_i) & \cos(\Omega_i) \end{bmatrix} \begin{bmatrix} k_i x \\ k_i y \end{bmatrix} + \begin{bmatrix} r\cos(\Omega_i) \\ r\sin(\Omega_i) \end{bmatrix}, \; i = 1,2,\ldots,n, \tag{11}$$

with $k_1 = 1$, $\Omega_i = \frac{\theta_i}{2} + \sum_{j=2}^{i-1}\theta_j + \frac{\theta_1}{2}$, $i = 2,3,\ldots,n$ and $\Omega_1 = 0$.

For the '*joined seed*' pattern, $r$ in equation (11) should be substituted by $r = r_{0i}, i = 1,\ldots,n$ can be obtained from equation (3) as:

$$r_{oi} = k_i a_0 \cot\left(\frac{\theta_i}{2}\right), \tag{12}$$

with $k_i$ and $\theta_i$ are the scaling factor and angle for *i*th-petal. Observe that the transformation matrix (11) can be easily reduced to the transformation matrices specified before by fixing the parameters to be tally with the conditions taken in every case. Table 3 gives the pattern generated by transformation matrix (11) with different combination of $c$, $n$ and $r$.

## 3.0 ITERATIVE TRANSFORMATION AND RESULTS

Motivated by the blooming flowers in nature that are arranged with layers of petals around pistil, the transformation matrices defined can be applied iteratively to arrange the '*seed*' in more than one circular path with different values of radius *r*. Furthermore, parameters used in the transformation matrix for each circle can be varied in order to create more living and fascinating patterns. Let $J$ be the number of circles required in one pattern. The following list gives some variations, which can be added to the transformation matrices derived before:

### 3.1 Change in the Number of Repeated 'Seed' for Different Circle

Let $\Delta n$ be defined as the difference in the number of repeated '*seed*' for two consecutive circles. For a given $n$ and $\Delta n$, the number of '*seed*' in each circle-*j* is defined by

$$n^{(j)} = n^{(1)} + (j-1)\Delta n , \quad j = 1,2,\ldots,J \text{ and } n^{(1)} = n ,$$

with $j = 1$ refer to the arrangement of '*seed*' for the smallest circle.

### 3.2 Change in the Radius Values for Different Circle

Two situations to be considered here, either the '*joined seed*' pattern or '*non-joined seed*' pattern.

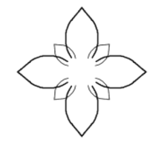
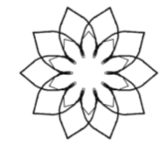
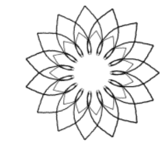
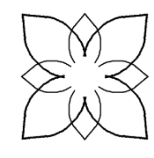
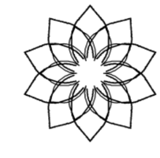
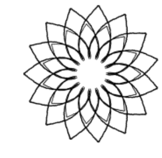
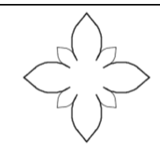
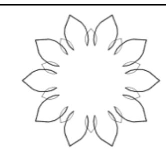
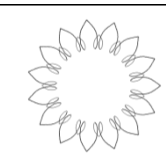
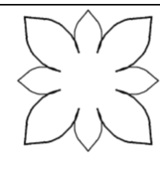
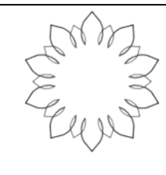
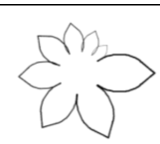
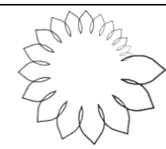
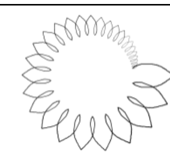
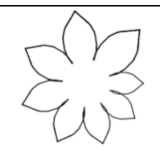
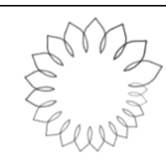
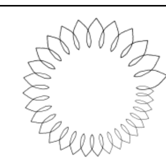
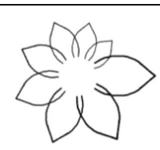
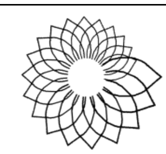
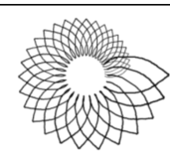
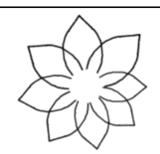
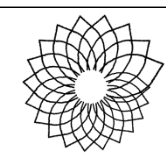
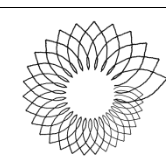
(a) For '*non-joined seed*' pattern, the radius for each circle-*j* is given as

$$r^{(j)} = r^{(1)} + (j-1)\Delta r , \quad j = 1,2,\ldots,J , \tag{13}$$

where $\Delta r$ is difference in the radius value of two consecutive circles.

(b) For '*joined seed*' pattern, the radius for each round of circle-*j*, $r_0^{(j)}$ must be calculated as given by equation (2), (4), (6) or (12) depend on the preference of the transformation matrices used.

**Table 3** Patterns generated based on transformation matrix (5), (10) and (11)

| | $n = 8$ | $n = 20$ | $n = 30$ |
|---|---|---|---|
| *rule* (5) $k_x = k_y = 0.5$ $r = a_0 \cot\left(\frac{\pi}{3}\right)$ |  |  |  |
| *rule* (5) $k_x = k_y = 1.5$ $r = a_0 \cot\left(\frac{\pi}{3}\right)$ |  |  |  |
| *rule* (10) $k_x = k_y = 0.5$ $r = r_0$ |  |  |  |
| *rule* (10) $k_x = k_y = 1.5$ $r = r_0$ |  |  |  |
| *rule* (11) $c < 0$ $r = r_{0i}, i = 1,\ldots,n$ |  |  |  |
| *rule* (11) $c > 0$ $r = r_{0i}, i = 1,\ldots,n$ |  |  |  |
| *rule* (11) $c < 0$ $r = a_0 \cot\left(\frac{\pi}{3}\right)$ |  |  |  |
| *rule* (11) $c > 0$ $r = a_0 \cot\left(\frac{\pi}{3}\right)$ |  |  |  |

### 3.3 Change in the Initial Angle for the First 'Seed' in Each Circle

If $\Delta\theta$ is difference of angle for two consecutive circles, then two conditions can be considered here:

(a) Change in the initial angle for each circle:
The initial angle for circle-*j* change linearly following the rule

$$\hat{\theta}^{(j)} = (j-1)\Delta\theta, \quad j = 1,2,\ldots,J. \tag{14}$$

(b) Alternating initial angle:
The initial angle for each *j*-circle can be alternated at

$$\hat{\theta}^{(j)} = \begin{cases} 0 & j = 1,3,..,2m-1 \\ \Delta\theta & j = 2,4,..,2m \end{cases} \tag{15}$$

where $\hat{\theta}^{(1)} = 0$ and $m = \lfloor J/2 \rfloor$.

With the above definition of $\hat{\theta}^{(j)}$. Thus, $\theta^{(j)} = \theta^{(j)} + \hat{\theta}^{(j)}$ is to be used for the transformation matrix in (1), (3) and (5), $\omega^{(j)} = \omega^{(j)} + \hat{\theta}^{(j)}$ to be applied in (10), or

$\Omega_i^{(j)} = \theta_i^{(j)}/2 + \sum_{k=2}^{i-1}\theta_k^{(j)} + \theta_1^{(j)}/2 + \hat{\theta}^{(j)}$ for the transformation matrix in (11).

### 3.4 Change of Scaling Factor in Each Circle

If $\Delta k$ is the difference in scaling factor for two subsequent circles, the size of *'seed'* in circle-*j* can be changed according to the following:

(a) Recursive scaling factor as described by

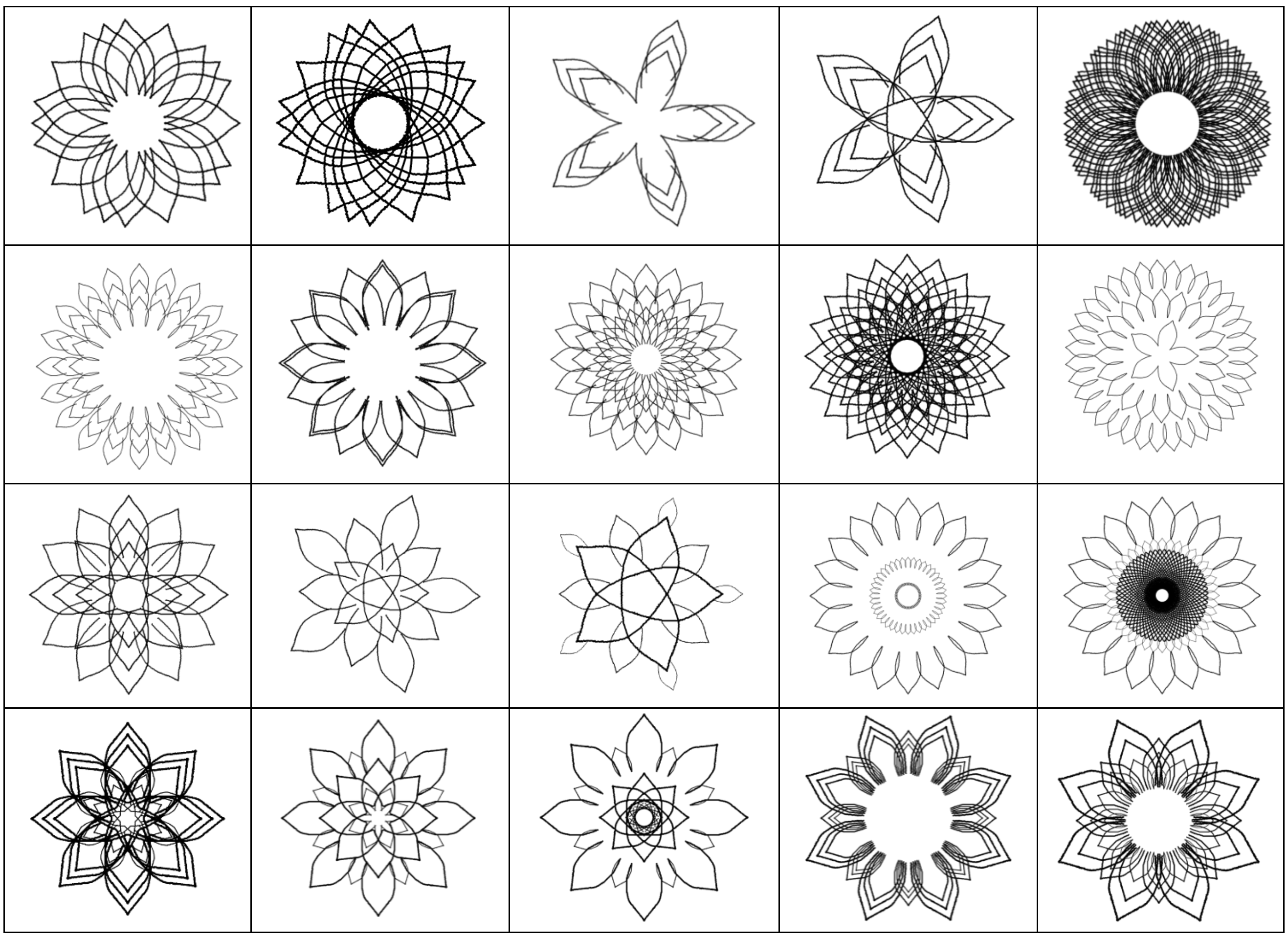
$$k^{(j)} = k^{(j-1)}(1+\Delta k),\quad \text{with } k^{(1)} = k_x = k_y .$$

(b) Increment/ decrement in the scale factor

$$k^{(j)} = k + ((j-1)\Delta k)k,\ \text{with } k = k_x = k_y .$$

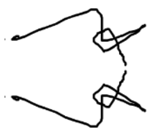
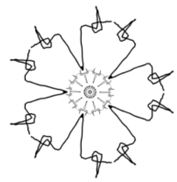
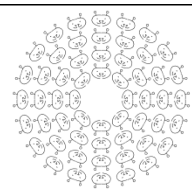
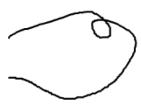
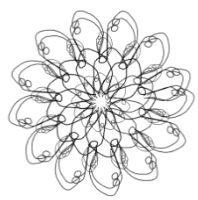
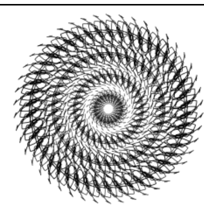
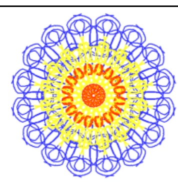
The transformation matrices defined the arrangement of *'seed'* is deterministic. In spite of this determinism, the parameters used in the transformation matrices add the element of complexity to the creation of pattern. With some variations in the parameters used, the model seems to act in an interesting manner and generates the patterns beyond initial expectation. Table 4 lists the patterns based on the *'seed'* from Figure 2 and generated under iterative transformation matrices with $J = 3$ and various combinations of parameters.

**Table 4** Patterns generated based on iterative transformation with different combination of parameters



With the proposed approach in generating circular patterns, one single *'seed'* can be used to generate vast number of circular patterns which are similar in a way but yet each pattern is unique enough with its own features. Complex element can be further added into the pattern generation if arbitrarily styles are used as the *'seed'*. Table 5 depicts some patterns when random patterns are used as *'seed'*.

**Table 5** Patterns generated based on iterative transformation with different *'seed'*

| Transformation matrix used | *'seed'* | pattern |
|---|---|---|
| (3), (14) and (15) |  |  |
| (3) and (13) |  |  |
| (5) and (15) |  |  |
| (5),(13) and (15) |  |  |
| (3),(13) and (15) |  |  |

## 4.0 CONCLUSION

In this work, we have proposed an approach to generate circular patterns by applying the transformation matrices to a predesigned *'seed'*. Starting with one single *'seed'*, the proposed approach is capable to generate a lot of interesting patterns by just varying the parameters used in the transformation matrices. Although the transformation matrices developed are relatively simple, but it is difficult to conceive the simple rules underlined in producing these appealing and fascinating images. We have also shown that if some simple random fundamental piece is use as *'seed'*, by applying the proposed transformation matrices, somehow we can eventually generate some visually appealing patterns. The possibilities are endless here. These computer generated patterns can be handful in the multimedia area and to be used in the designing work such as designing the wall paper, wrapping paper and etc. For future work, the transformation matrices can be further extended to incorporate 3-dimensional visualization pattern generation.

## Acknowledgement

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