

INFLUENCE OF HEAT TRANSFER ON THE MHD STAGNATION POINT FLOW OF A POWER LAW FLUID WITH CONVECTIVE BOUNDARY CONDITION

Article history

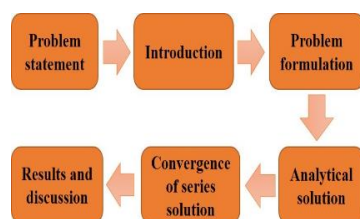
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Graphical abstract



Abstract

In this article, we examined the impact of heat transfer on the magnetohydrodynamic (MHD) stagnation point flow of a non-Newtonian power-law fluid with convective boundary condition. By using suitable similarity transformations, coupled nonlinear partial differential equations are transformed to ordinary differential equations. Then solved the resulting equations with Homotopy analysis method. Interesting flow parameters such as MHD M , stagnation parameter λ , convective parameter γ are discussed graphically. Convergence is checked at 20th order of approximation. Numerical values of physical interested parameter such as local Nusselt number are also tabulated.

Keywords: Convective boundary conditions, power-law fluid, heat transfer

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1.0 INTRODUCTION

A stagnation point flow in terms of fluid mechanics is a point in a flow field where the local velocity of fluid is zero. Effects of volumetric heat generation/absorption on mixed convection stagnation point flow on an isothermal vertical plate in a porous medium was investigated by Singh *et al.* [1]. Hiemenz [2] and Homann [3] studied the two dimensional and axisymmetric three dimensional stagnation point flows respectively. Different studies on stagnation point flows can be found in the works [4-6]. The problems of stagnation point flows can be studied in Newtonian as well as non-Newtonian fluids. Many researchers focused to the study of non-Newtonian fluids such as power-law fluid because its equation of motion have special relevance in industries such as molten plastic, polymer melts, extrusion process and many others. Andersson and Dandapat [7] first discussed the flow of a non-Newtonian fluid obeying power law model by extending the Newtonian model as considered by

Crane [8]. Again Anderson *et al.* [9] presented the MHD flow of a power law fluid over a stretching sheet. Numerical and series solution of magnetohydrodynamics stagnation point flow of a power law fluid towards a stretching surface are obtained by Mahapatra *et al.* [10, 11]. They compared both the results and found to be in good agreements. Again Mahapatra *et al.* [12] studied the above mentioned problems over a porous plate for suction or blowing case. They plotted the streamlines and observed that the velocity at a point increases with an increase in the magnetic parameter M . The convective boundary condition has been used by many researchers to revisit the problems studied with isothermal/isoflux boundary condition such as R. C. Bataller [13] discussed the Similarity solutions for flow and heat transfer of a quiescent fluid over a nonlinearly stretching surface. Then A. Ishak [14] explored the similarity solutions for flow and heat transfer over a permeable surface with convective boundary conditions. A convective boundary condition for MHD

mixed convection from a vertical plate embedded in a porous medium examined by O. D. Makinde and A. Aziz [15]. S. Yao, T. Fang and Y. Zhong [16] studied the heat transfer of a generalized stretching/shrinking wall problem with convective boundary condition. The aim of this paper is the study of heat transfer on stagnation-point flow of MHD power-law fluid with convective boundary condition. The steady two dimensional stagnation point flow of an ambient fluid which is subjected under the constant magnetic field is considered here: By taking the applied magnetic field uniform, the convergent series solutions are obtained by the HAM. Interesting flow parameters are discussed graphically Numerical values of local Nusselt number is also calculated in tabulated form. HAM is an efficient method proposed by Liao [18] This method has been successfully applied to obtained solution of non-linear problems [19 - 20]. As shown in Figure 8, this paper highlighted the problem in this field, formulate the problem, proposed the solution, analysis and discussing the result.

2.0 PROBLEM FORMULATION

We analyze the effects of heat transfer with a convective boundary condition instead of commonly used conditions of constant surface temperature or constant heat flux. Here, we consider the steady, two dimensional stagnation-point flow and heat transfer of an electrically conducting power-law fluid (cold fluid at temperature (T_∞) towards a flat stretching sheet coinciding with the plane $y=0$, the flow being confined to the region $y>0$. A uniform magnetic field of strength B_0 is imposed normal to the sheet (along the y -axis) when induced magnetic field is neglected under small magnetic Reynolds number assumption. It is assumed that the velocity of the external flow is $U_e(x)=ax$ and the velocity of the stretched sheet is $U_\infty(x)=cx$, where a, c are positive constants and x is the coordinate measured along the stretching sheet. It is also assumed that the bottom surface of the sheet is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f . In the present problem we have $\frac{\partial u}{\partial y} < 0$ when $\frac{a}{c} < 1$ and $\frac{\partial u}{\partial y} > 0$

when $\frac{a}{c} > 1$. Under these assumptions, the momentum and energy equations for the boundary layer flow of power-law fluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_e \frac{\partial U_e}{\partial x} - \frac{K}{\rho} \frac{\partial}{\partial y} \left(-\frac{\partial u}{\partial y} \right)^n - \frac{\sigma B_0^2}{\rho} (u - U_e), \quad (2)$$

when $\frac{a}{c} < 1$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_e \frac{\partial U_e}{\partial x} - \frac{K}{\rho} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)^n - \frac{\sigma B_0^2}{\rho} (u - U_e), \quad (3)$$

when $\frac{a}{c} > 1$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

where $K(>0)$ and n denote the consistency coefficient and flow behavior indices, respectively and σ , k and c_p are the electrical conductivity, the thermal conductivity and the specific heat, respectively.

The corresponding boundary conditions are

$$\left. \begin{aligned} u = U_\infty(x) = cx, v = 0, \quad -k \frac{\partial T}{\partial y} = h_f [T_f - T] \text{ at } y = 0, \\ u = U_e(x) = ax, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \right\} \quad (5)$$

Introducing the following similarity transformations

$$\left. \begin{aligned} \Psi = - \left(\frac{K/\rho}{c^{1-2n}} \right)^{1/(n+1)} x^{2n/1+n} f(\eta), \\ \eta = y \left(\frac{c^{2-n}}{K/\rho} \right)^{1/2} x^{(1-n)/(1+n)}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}. \end{aligned} \right\} \quad (6)$$

In the above equations, η denotes the similarity variables, $\Psi(x, y)$ the stream function, f and θ the dimensionless similarity functions related to the velocity and temperature respectively. In view of above transformations, we have

$$n[f'' \operatorname{sgn}(\lambda - 1)]^{n-1} f''' + \left(\frac{2n}{n+1} \right) f f'' - f'^2 - M f' + M \lambda + \lambda^2 = 0, \quad (7)$$

$$\frac{1}{\operatorname{Pr}} \theta'' + f \theta' = 0, \quad (8)$$

$$\left. \begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -\gamma [1 - \theta(0)], \\ f'(\eta) = \lambda, \quad \theta(\eta) = 0 \text{ as } \eta \rightarrow \infty. \end{aligned} \right\} \quad (9)$$

Here the other parameters are defined as follows

$$\left. \begin{aligned} M = \frac{\sigma B_0^2}{\rho c}, \quad \operatorname{Pr} = \frac{c^{3\left(\frac{k+1}{k-1}\right)}}{\alpha} \left(\frac{n}{\rho} \right)^{\frac{2}{1+n}} \left(\frac{2n}{1+n} \right) x^{-2\left(\frac{1-k}{1+k}\right)}, \\ \lambda = \frac{a}{c}, \quad \gamma = \frac{h}{n} \left(\frac{c^{2-n}}{n/\rho} \right)^{-1/2} x^{-\left(\frac{1-n}{1+n}\right)} \end{aligned} \right\} \quad (10)$$

In the above equation, M is the magnetic field parameter, Pr is the Prandtl number, λ is the stagnation parameter and γ is the convective heat transfer. The physical quantity of interest is the local Nusselt number Nu is defined by

$$Nu = \frac{xq_w}{k(T_f - T_\infty)}, \tag{11}$$

where $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{\eta=0}$ denoting the surface heat flux.

Using variables (7), we get

$$Re_x^{-1/2} Nu_x = -\theta' (0). \tag{12}$$

where $Re_x = \frac{(cx)^{2-n} x^n}{K/\rho}$ is the local Reynolds number.

3.0 THE HAM SOLUTIONS FOR INTEGER POWER-LAW INDEX $\lambda \neq 1$

We apply the Homotopy Analysis Method to solve the coupled system of equations (7) to (9). From the governing equations, we choose

$$L_f = \frac{d^3 f}{d\eta^3} + \beta|\lambda - 1| \frac{d^2 f}{d\eta^2}, \tag{13}$$

and

$$L_\theta = \frac{d^2 \theta}{d\eta^2} - \theta, \tag{14}$$

as auxiliary linear operators with the following properties

$$L_f [C_1 + C_2 \eta + C_3 \exp(-\beta|\lambda - 1|\eta)] = 0, \tag{15}$$

$$L_\theta [C_4 \exp(\eta) + C_5 \exp(-\eta)] = 0, \tag{16}$$

where C_i ($i = 1-5$) are arbitrary constants.

3.1 The zeroth-order Deformation Problems

We construct the zeroth order deformation problem as

$$(1 - p)L_f [\hat{f}(\eta; p) - f_0(\eta)] = p\hbar_f \mathbf{N}_f [\hat{f}(\eta; p)] \tag{17}$$

$$(1 - p)L_\theta [\hat{\theta}(\eta; p) - \theta_0(\eta)] = p\hbar_\theta \mathbf{N}_\theta [\hat{f}(\eta; p), \hat{\theta}(\eta; p)] \tag{18}$$

subject to the boundary conditions

$$\left. \begin{aligned} \hat{f}(\eta, p)\Big|_{\eta=0} &= 0, \quad \frac{\partial \hat{f}(\eta, p)}{\partial \eta}\Big|_{\eta=\infty} = 1, \quad \hat{\theta}(\eta, p)\Big|_{\eta=0} = -\gamma(1 - \hat{\theta})\Big|_{\eta=0}, \\ \frac{\partial \hat{f}(\eta, p)}{\partial \eta}\Big|_{\eta=\infty} &= \lambda, \quad \text{and} \quad \hat{\theta}(\eta, p)\Big|_{\eta=\infty} = 0, \end{aligned} \right\} \tag{19}$$

where

$$f_0(\eta) = \frac{\lambda\beta\eta - [1 - \exp(-|\lambda - 1|\beta\eta)]\text{sgn}(\lambda - 1)}{\beta}, \tag{20}$$

and

$$\theta_0(\eta) = \frac{\gamma}{1 + \gamma} e^{-\eta}, \tag{21}$$

as our initial approximation of f and θ . Here $p \in [0, 1]$ is an embedding parameter, \hbar_f and \hbar_θ are the auxiliary nonzero parameters for f and θ . For $n = 1$, the nonlinear operators are

$$\mathbf{N}_f [\hat{f}(\eta; p)] = \frac{\partial^3 \hat{f}(\eta; p)}{\partial \eta^3} + \hat{f}(\eta; p) \frac{\partial^2 \hat{f}(\eta; p)}{\partial \eta^2} - \left(\frac{\partial \hat{f}(\eta; p)}{\partial \eta} \right)^2 - M \frac{\partial \hat{f}(\eta; p)}{\partial \eta} + M\lambda + \lambda^2, \tag{22}$$

$$\mathbf{N}_\theta [\hat{\theta}(\eta; p)] = \frac{\partial^2 \hat{\theta}(\eta; p)}{\partial \eta^2} + \text{Pr} \hat{f}(\eta; p) \frac{\partial \hat{\theta}(\eta; p)}{\partial \eta}. \tag{23}$$

When $p = 0$ and $p = 1$ then it is easy to check that

$$\hat{f}(\eta; 0) = f_0(\eta), \quad \hat{f}(\eta; 1) = f(\eta), \tag{24}$$

$$\hat{\theta}(\eta; 0) = \theta_0(\eta), \quad \hat{\theta}(\eta; 1) = \theta(\eta), \tag{25}$$

So, as an embedding parameter p increases from 0 to 1, $\hat{f}(\eta; p)$ and $\hat{\theta}(\eta; p)$ vary from initial approximations $f_0(\eta)$ and $\theta_0(\eta)$ to the solution $f(\eta)$ and $\theta(\eta)$ of the original equations (7) to (8). Using Taylor's theorem and Eqs. (24) and (25), we expand $\hat{f}(\eta; p)$ and $\hat{\theta}(\eta; p)$ in the power series of an embedding parameter p as follows.

$$\hat{f}(\eta; p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m, \tag{26}$$

and

$$\hat{\theta}(\eta; p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m, \tag{27}$$

where

$$f_m(\eta) = \frac{1}{m!} \frac{\partial^m f(\eta; p)}{\partial p^m} \Big|_{p=0}, \quad \theta_m(\eta) = \frac{1}{m!} \frac{\partial^m \theta(\eta; p)}{\partial p^m} \Big|_{p=0}. \tag{28}$$

Observe that the zeroth-order deformation Eqs. (17) and (18) contain non-zero auxiliary parameters \hbar_f and \hbar_θ . Assume that these parameters are chosen so that the series (26) and (27) are convergent at $p = 1$. Hence, we have due to (28) that

$$f(\eta; p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{29}$$

$$\theta(\eta; p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta), \tag{30}$$

3.2 The mth-order Deformation Problem

Differentiating the zeroth-order deformation problems in eqs. (17) and (18) m -times with respect to p and then dividing by m . Finally letting $p = 0$, we obtain the following m th-order deformation problems for $m \geq 1$ the following problem

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f \mathbf{R}_m^f(\eta), \tag{31}$$

$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_\theta \mathbf{R}_m^\theta(\eta), \tag{32}$$

$$f_m(0) = 0, f'_m(0) = 0, f''_m(\infty) = 0, \theta'_m(0) = \theta_m(\infty) = 0, \tag{33}$$

where nonlinear operators for $n = 1, 2$ are as follows

$$\mathbf{R}_m^f(\eta) = f_{m-1}''' + \sum_{k=0}^{m-1} f_k f_{m-1-k}'' - \sum_{k=0}^{m-1} f'_k f'_{m-1-k} - M f'_{m-1} + M \lambda (1 + \lambda), \tag{34}$$

$$\mathbf{R}_m^f(\eta) = 2[\text{sgn}(\lambda - 1)] \sum_{k=0}^{m-1} [f_{m-1-k}'' f_{m-1}''' - M f'_{k-1}] + \frac{4}{3} \sum_{k=0}^{m-1} \begin{bmatrix} f_k f_{m-1-k}'' \\ -f'_k f'_{m-1-k} \end{bmatrix} + (1 - \chi_m) M \lambda (1 + \lambda), \tag{35}$$

$$\mathbf{R}_m^\theta(\eta) = \theta_{m-1}'' + Pr \sum_{k=0}^{m-1} f_k \theta'_{m-1-k}, \tag{36}$$

and χ_m is defined by Eq. (35)

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m \geq 2. \end{cases} \tag{37}$$

The general solutions of Eqs. (31)- (32) are

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 \exp(\eta) + C_3 \exp(-\eta), \tag{38}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 \exp(\eta) + C_5 \exp(-\eta), \tag{39}$$

in which $f_m^*(\eta)$ and $\theta_m^*(\eta)$ are the particular solutions of Eqs. (31)- (32). Note that Eqs. (31)- (32) can be solved by means of any symbolic computational software like Maple, Mathematica etc. one after the other in the order $m = 1, 2, 3, \dots$

4.0 CONVERGENCE OF THE SERIES SOLUTIONS

While using Homotopy Analysis method, the convergence depends upon the auxiliary parameter \hbar as it helps in adjusting and controlling the radius of convergence of the series solutions. Combine \hbar curves of $f''(0)$ and $\theta'(0)$ versus \hbar_f and \hbar_θ are sketched in Figure 1. Where \hbar_f is auxiliary parameter for $f''(0)$ and \hbar_θ is the auxiliary parameter for $\theta'(0)$. By keeping the values of other parameters fixed, we can obtain the convergence region of \hbar_f and \hbar_θ . The reasonable values are $-1.25 \leq \hbar_f \leq -0.25, -1.26 \leq \hbar_\theta \leq -0.26$. Table. 1 represents the convergent values of series solution. This shows the validity of our series solution.

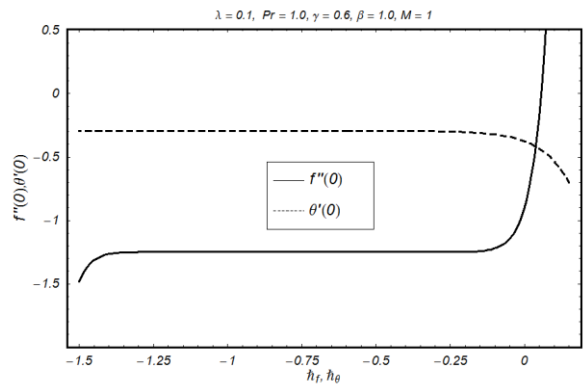


Figure 1 Combined \hbar curve for $f''(0)$ and $\theta'(0)$, at 20th order of approximation

Table 1 Convergence of the series solution at different order of approximations.

Order of approximations	$-f''(0)$	$-\theta'(0)$
1	0.339000	0.472500
5	0.380915	0.432714
10	0.385874	0.428559
15	0.386490	0.428388
20	0.386613	0.428427
25	0.386632	0.428860
30	0.386632	0.428860
40	0.386632	0.428860

Table 2 The numerical values of the local Nusselt number for various values of parameters

$[m, m]$	$n = 1, \gamma = 0.1, M = 1, \lambda = 1$	$n = 2, \gamma = 1, M = 2, \lambda = 2$
	$R^{-1/2}Nu$	$R^{-1/2}Nu$
1	0.446167	0.259200
3	0.380562	0.199372
5	0.342460	0.737020
7	0.317596	1.491360
10	0.309239	1.589980

5.0 RESULTS AND DISCUSSION

The effects of all other parameters and homotopy-Pade approximations results have been discussed in the paper by Mahapatra *et al.* [10]. Here we see the influence of following flow parameters γ convective heat transfer, Pr Prandtl number and λ stagnation parameter on the temperature distribution. Figures 2 and 3 are made to see the effect of parameter γ for Newtonian and non-Newtonian fluid. Here for $\gamma = 0$, we recover the case of constant surface temperature but for the convective heat transfer $\gamma = 0.3, 0.5, 0.7$ the temperature $\theta(\eta)$ increases as we increase the values of γ . Also curves for non-Newtonian power-law fluid fluctuates rapidly as compared to the Newtonian fluid. The influence of Prandtl number Pr on temperature for $n=1, 2$ is shown in Figures 4 and 5. From these figures we observe that by increasing the values of Pr the temperature distribution decreases for Newtonian fluid and hence the boundary layer thickness decreases; however, for non-Newtonian fluid it increases. The stagnation parameter λ effects on θ is sketched in Figure 6. It is noted that the boundary layer thickness and temperature profiles decreases as we increase the values of λ . Figure 7 shows that the velocity reaches to its peak values in the case of non-Newtonian fluids. Table 2 is made to see the numerical values of physical interested local Nusselt number and it is found to be in good agreement.

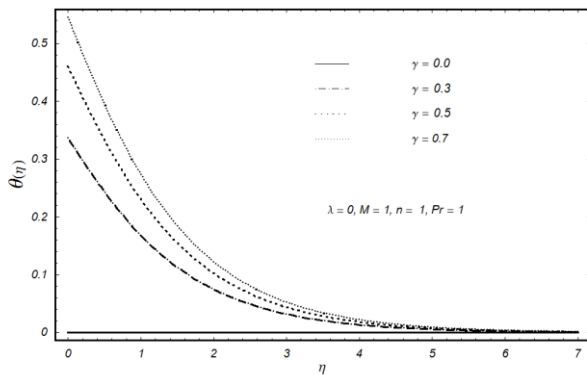


Figure 2 Temperature profiles for different values of γ by keeping the values of other parameters fixed

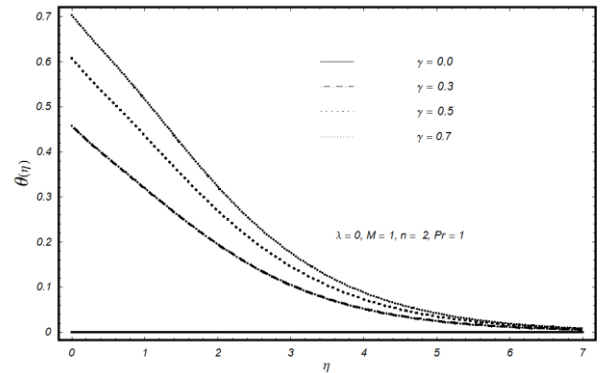


Figure 3 Temperature profiles for different values of γ by keeping the values of other parameters fixed at $n=2$

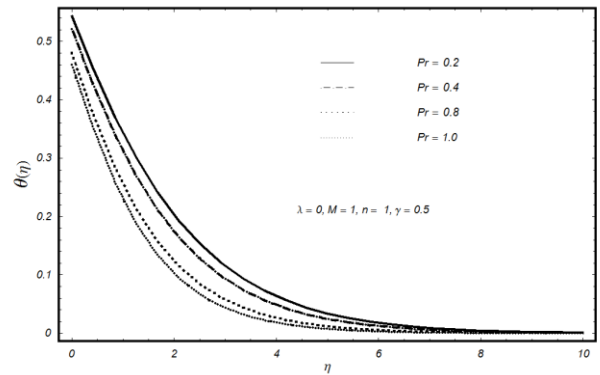


Figure 4 Temperature profiles for different values of Pr by keeping the values of other parameters fixed

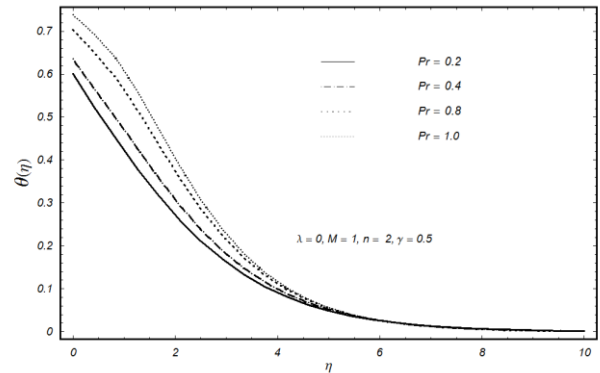


Figure 5 Temperature profiles for different values of Pr by keeping the values of other parameters fixed at $n=2$

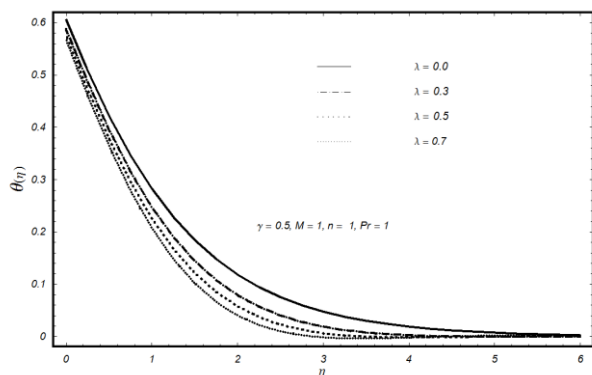


Figure 6 Temperature profiles for different values of λ by keeping the values of other parameters fixed

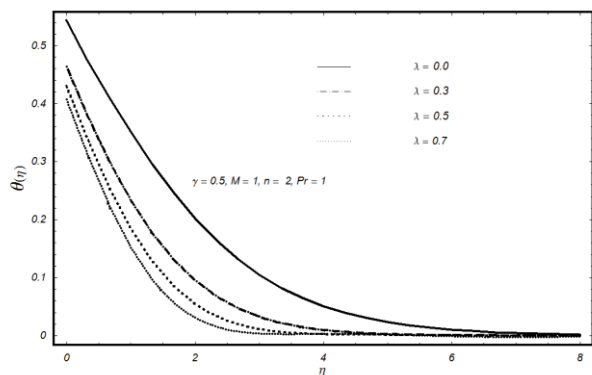


Figure 7 Temperature profiles for different values of λ by keeping the values of other parameters fixed at $n=2$

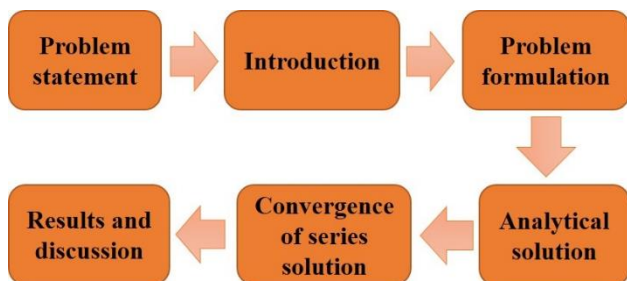


Figure 8 Block diagram of the processes of the system

6.0 CONCLUSION

In this paper we have discussed heat transfer analysis on stagnation-point flow of MHD power-law fluid with a convective boundary conditions in analytical way using Homotopy analysis method. This method is efficient to solve the boundary value problem in analytical way. From the numerical and graphical results, we conclude the following main observations:

- It is noted that the temperature increases for increasing values of γ for both Newtonian and power-law fluids. So it increases boundary layer thickness.

- The effects of Prandtl number Pr on Newtonian and power-law fluids is seen to be in opposite behavior.
- From the results, we observed a decrease in the temperature as well as the thermal boundary layer for increasing values of λ .

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