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CALCULATION OF ROD FORCE OF THE TRUSS CONSTRUCTION

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Graphical abstract

Scheme of Modeling
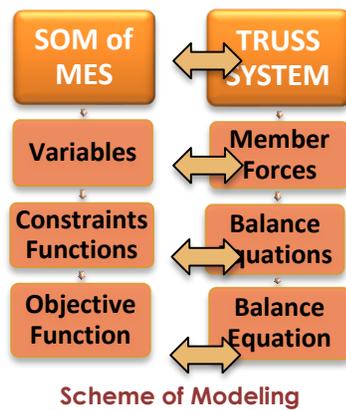
Abstract

The calculation of the forces of truss construction can be determined by graphical method, known as Cremona method and analytical method, known as Ritter method. Calculations in both ways can take much time and is dependent on the number of connecting points and considerable loading simulations. This paper describes an alternative analytical calculation method, which combines the two methods, namely the connection balance method and the optimization calculation method. And the last method is available in Microsoft Excel Solver (MES). The connection balance method is the fulfillment of the balance equation of $\Sigma Fx = 0$ and $\Sigma Fy = 0$ at each joint of the truss. The way of the balance fulfillment in this paper is different from the way of Ritter. In the way of Ritter, the balance fulfillment is applied to each connection point, one by one point. The connection point can be calculated if at the point there are maximum two unknown rod force and minimum one known force. While by using the MES program, the balance of connecting point is simultaneously computed for the entire construction. MES program works by simulating/finding the value of variables, and at the same time fulfilling the defined objective function and constraint functions. In this case, the variables are all the rod forces, while the constraint functions are the balance equation at all connecting point. An objective function can be taken from one of the $\Sigma Fx = 0$ or $\Sigma Fy = 0$ from one connecting point only. This paper has recorded 10 different types of geometric shapes of certain truss construction (statically determined) which can be considered to represent the ones in the field (including the form of Howe, Pratt, and Warren for the roof truss and bridge truss). Then the mathematical models that were adjusted to the MES program were created for each of the ten truss constructions. Based on the created mathematical models, the forces of the rods were then calculated using the MES program. For validation purposes, the calculation results are compared with the calculations result using the analytical Ritter method. It can be concluded that the presented calculation method is highly satisfactory, especially for the analysis of statically determined truss construction.

Keywords: Connections balance; truss construction; solver

1.0 INTRODUCTION

A truss is basically a structural frame formed by connecting various members at their ends to form a triangular system, arranged in pre-decided patterns depending upon the span, type of loading and functional requirements as in steel trusses which are commonly used in building and bridge industry [1].

The analysis of truss construction using the Ritter method could take a lot of time depending on the number of connections and variations of loading imposition of being simulated. By using this method, rod forces are gradually calculated from one connection

to another one by applying connection balancing law, ΣFx = 0 and ΣFy = 0. The connection that can be calculated is the one that has a maximum of two unknown rod forces and one minimum known rod force. If the stipulation is not fulfilled, then one or more of the rod forces must be previously calculated by applying the section balancing law [2].

Microsoft Excel Solver (abbreviated as MES) is one of the tools from the MS Excel that uses algorithms or solving methods to simulate a number of variables in such a way so that at the same time it meets a determined objective function and a number of determined-constraint function [3,5].

There are many literatures that discuss the MES programs in various sectors, especially in business and in technical sectors. The whole application could be classified into two categories, namely a category of linear programming and a category of non-linear programming. If the objective function or the obstacle function has the power of one, or both have the power of one, then it is called linear programming, and if they have the power of more than one, it is called non-linear programming [4].

Either in the linear programming or non-linear programming cases, there are several samples of the use of MES in solving problems such as: shortest-route, maximum flow, minimum-cost capacitated flow, quadratic and stochastic programs [5].

Some other examples of MES application in solving problems of linear programming are to determine the material percentage in planning of combined aggregate, to find a solution of transportation management problem in the determination of the optimum route of goods delivery and to determine solutions for the problem of network planning. In addition, some examples of the MES application in the case of non-linear programming include determining the equation of the rain intensity and assessing the discharge of the pipes in a system of closed pipeline network [6].

The MES program is also used to optimize the values of "weight associate" of an artificial neural network for modeling of river discharge flow into a reservoir, and it was accomplished with good results [7].

The researchers did not found any publication which specially discusses the application of MES for calculating the rod forces of truss construction using the joint method.

If both of the above methods (method of joint from Ritter and method of optimization from MS Excel) are combined, then it is very possible to develop a newly alternative method to calculate the member forces of a truss construction.

The objectives of the experiments are as follows:
a) To create an optimizing model which simultaneously meets the needs on input data for MES program and connection-equilibrium law, and hereinafter referred as Solver Optimization Model (abbreviated as SOM)
b) To calculate member forces of the truss construction using MES program based on the above SOM

c) To calculate the same rod forces, but uses the Ritter method for the purpose of validation
d) To apply the developing method in calculating the influence line

2.0 EXPERIMENTAL

2.1 Types of Trusses and FBD

Prior to modeling, the researchers firstly determined a free body diagram (FBD) of the truss construction which was completed with such outward forces. Based on literature studies, this writing has recorded 10 type of construction of a truss (3 types represent roof truss, and 7 types represent bridge order/framework) to be analyzed using the method developed in this paper. The configuration of the ten types is presented in Table 1, where the smallest number of the rod is 21 and the largest one is 49, while the smallest number of the node is 12 points and the largest one is 26 points. The left half portions of the ten FBD are presented in Figure 1 until Figure10.

Data in Table 1 shows that the ten types of truss construction that have been analyzed are certainly static, because every type meets the following conditions:

$$Nm = 2 * Nc - Nr$$

Where:
   Nm = the number of the truss member(column 5)
   Nc= the number of connection or node (column 6)
   Nr = the amount of pedestal reaction (= 3, because they are located on hinge and roll pedestal)

2.2 Application of Loads

For every type of bridge trusses which is counted, the burden of 1 kN is assumed to work vertically down at the point of connection that is located at the bottom in the midst of span. The reason of choosing the location of such burden is to be symmetrical only, so that only a half of the calculation result will be presented.

While for all types of roof truss, it is applied prevalent configuration, in this matter is assumed the 2 kN load works vertically down at any connection between the two sides of a footstool. On both of the footstool connections (or connection above footstool) the researchers assumed the 1 kN load works in the same direction. Based on FBD and loading application, the placement reaction has been calculated. The configuration of the burden and the imposition reaction of all types of truss is presented in Table 2.

Table 1 Configuration of 10 types of calculated trusses

Initial of Truss	Type of truss	Span in meter	Height in meter	Nr. of members	Nr. of joints
Type-1	Warren	18	3	21	12
Type-2	Howe	21	3	29	16
Type-3	Pratt	24	6	29	16
Type-4	Howe-1	24	6	29	16
Type-5	Howe-2	24	5	33	18
Type-6	Deck Warren	30	5	41	22
Type-7	Deck Warren	18	3	45	24
Type-8	Trough Warren	18	3	45	24
Type-9	Bowstring	48	5	45	24
Type-10	K-Truss	32	6	49	26

Table 2 Configuration of Loading

Initial of Truss	Loading assuming	Left pedestal reaction R_A in kN	Right pedestal reaction, R_B in kN
Type-1	1 kN at symmetrical bottom joint	0.500	0.500
Type-2	As above with type-1	0.500	0.500
Type-3	2 kN at each middle top joint and 1 kN at each pedestal joint	8.000	8.000
Type-4	As above with type-3	8.000	8.000
Type-5	2 kN at each middle top joint and 1 kN at each joint above the pedestal	8.000	8.000
Type-6	1 kN at symetical top joint	0.500	0.500
Type-7	As above with type-6	0.500	0.500
Type-8	As above with type-1	0.500	0.500
Type-9	As above with type-1	0.500	0.500
Type-10	As above with type-1	0.500	0.500
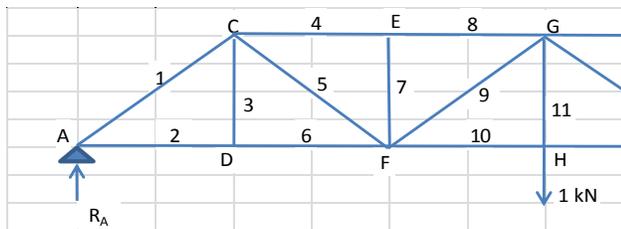
Figure1 FBD of truss of Type-1 (left half portion)
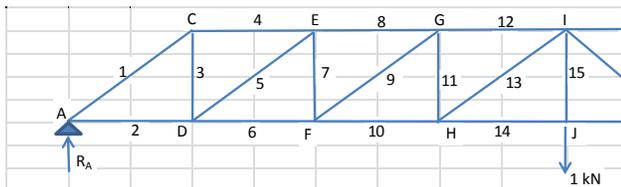
Figure 2 FBD of truss of Type-2 (left half portion)
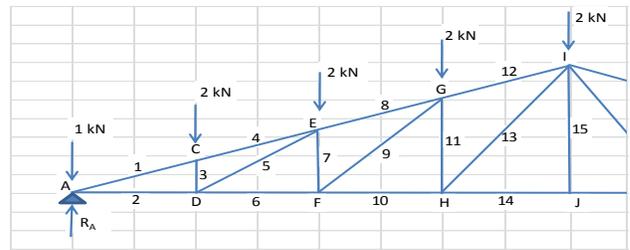
Figure 3 FBD of truss of Type-3 (left half portion)
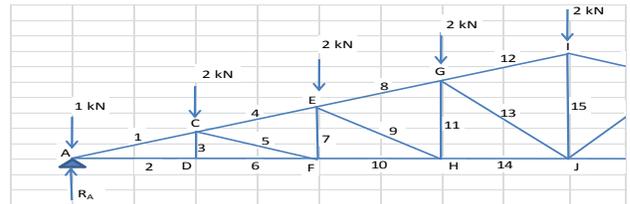
Figure 4 FBD of truss of Type-4 (left half portion)
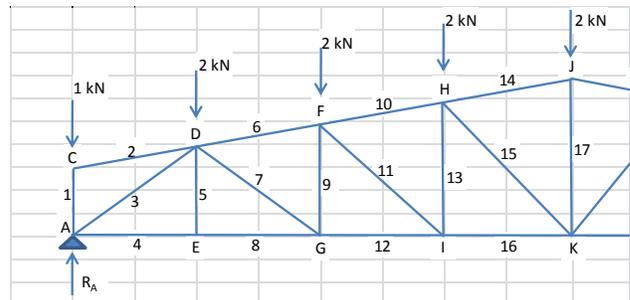
Figure 5 FBD of truss of Type-5 (left half portion)
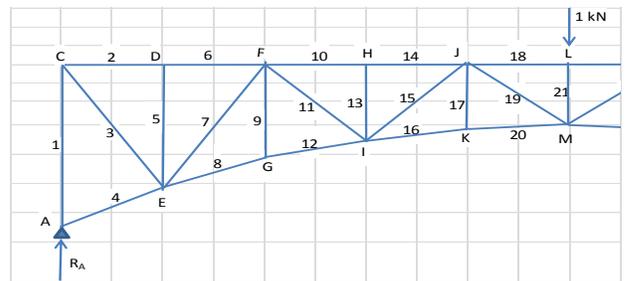
Figure 6 FBD of truss of Type-6 (left half portion)
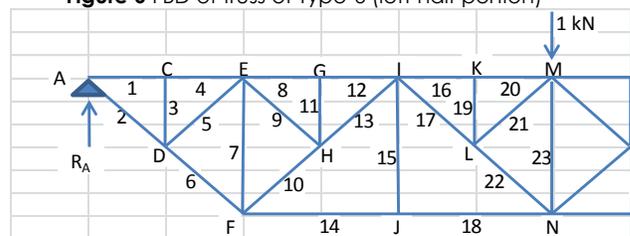
Figure 7 FBD of truss of Type-7 (left half portion)
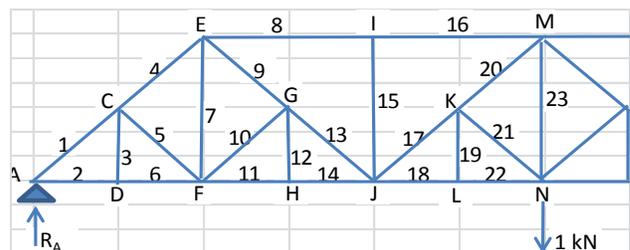
Figure 8 FBD of truss of Type-8 (left half portion)
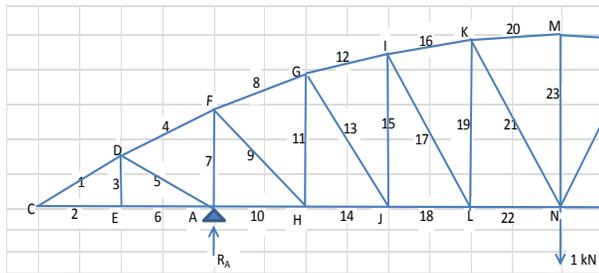
Figure 9 FBD of truss of Type-9 (left half portion)
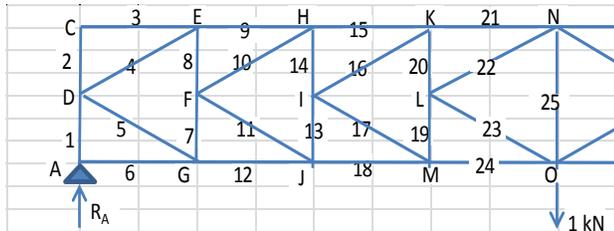
Figure 10 FBD of truss of Type-10 (left half portion)

2.3 The Development of Solver Optimization Model

Solver Optimization Model (abbreviated as SOM) is a mathematical model which is arranged on worksheets of MS Excel, as well as connecting the three elements, i.e. one element of the variables and two elements of the functions. The first function of this model is an objective function which is a function of part or all of variables, depending on the purpose of modeling.

There are three goals of optimizing, and they are; to maximize the objective function, to minimize the objective function, and to make the objective function to have a certain value as well as the objectivity of modeling. Especially, for the purpose of this paper which is to calculate the forces of the truss, therefore, the chosen purpose function is the third, namely the one that makes the objective function having a certain value (in this case one of the balance condition = 0)
While the second function is the constraint functions, that were conceived to meet all the required stipulation so that the purpose of modeling is achieved. The scheme of the model development is presented in Fig. 11.

Due to the modeling purpose is to determine the rod forces of the truss structure, then SOM is arranged with the following three criteria:
1. The variables of this model are all the rod forces that will be counted, therefore, the number of variables is equal to the number of the shafts.
   Fi = rod force *i*, where *i* is the rod number from *1* to *n*, where *n* is the number of the rods
2. The constraint functions are all of the balance equations at any connection of all truss construction. Because every connection must meet two balancing requirements, therefore, the number of the constraints functions are twice the number of connections. All connections are marked with serial numbers in alphabetical manners, and special footstool connection in left and right side is given notation of A and B.

Every connection is then applied with the following equations:

$$\sum H = \sum_{i=l}^{h} Fi_x = 0 \quad \text{and} \quad \sum V = \sum_{i=l}^{h} Fi_y = 0$$

Where l = smallest rod number and h = largest rod number at every connection.
Fi_x = horizontal component of member force Fi
Fi_y = vertical component of rod force Fi
3. The objective function in this case is taken from one equilibrium equation, namely $\sum V = 0$ which is located in footstool connection of B.
Based on the above 3 criteria, it has been prepared 10 SOM on MS Excel worksheets, however, for time saving purpose, in this paper will be presented one framework type 1 as well as presented in Table 3. In Table 3 it can be seen the three modeling elements. The first one is "Variables" in the left side, consists of 21 cells, where each of them has the value of rod forces namely F1 until F21. The second element is "Constraints Function" which is located on the right side that contains of fulfillment of all the balance condition (third column). Every cell in this column contains data that is a function of cells is on the second column of "Variables", in accordance with rod forces on the point of connection and the required stipulation. The third element is called as "Objective function" that is located at the bottom right of worksheets, where value is taken by the stipulation of $\sum V = 0$ in the connection B. From Table 3 it can be seen that "constraints function" cannot be completed because not all cells display the value "0". This is common since the calculation was not done by using MES, and the variable value is assumed uniformly at the value of 1 kN.
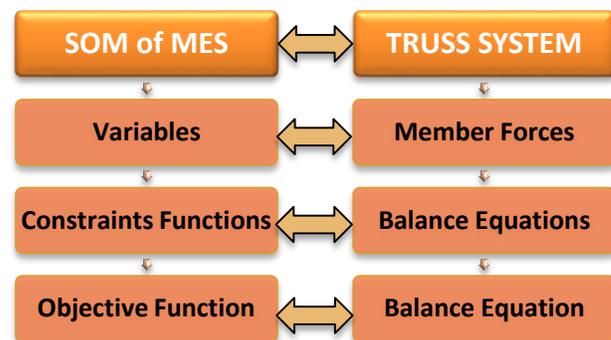
Figure 11 Scheme of Modeling

2.4 The Calculation of Rod Force Using MES

MES program is activated from the main menu of the MS Excel in worksheet containing SOM as it is presented in Table 3. MES can usually be loaded by clicking on a sequence: "Data", "What if Analysis", "Solver Analysis". After the dialog window "Solver Parameters" appears, then input data from SOM to MES program can be executed through the dialog window. The order of data input is done by inserting the objective

function cell, the variable cells, the cells of the obstacle function cells, and the choice of Solving Method "Simplex LP". MES will start counting as soon as the button of "Solve" is clicked.

Table 3 Modeling worksheet before calculating by Microsoft Excel Solver

Variables			Constraints functions			
Initial of member forces	Member forces in kN	Joint	Initial of Formula	Formulation of □H and □V at each joint	Equilibrium condition
F1	1.000	A	SH =	1.707	= 0
F2	1.000		SV =	1.207	= 0
F3	1.000	C	SH =	1.000	= 0
F4	1.000		SV =	-2.414	= 0
F5	1.000	D	SH =	0.000	= 0
F6	1.000		SV =	1.000	= 0
F7	1.000	E	SH =	0.000	= 0
F8	1.000		SV =	-1.000	= 0
F9	1.000	F	SH =	0.000	= 0
F10	1.000		SV =	2.414	= 0
F11	1.000	G	SH =	0.000	= 0
F12	1.000		SV =	-2.414	= 0
F13	1.000	H	SH =	0.000	= 0
F14	1.000		SV =	0.000	= 0
F15	1.000	I	SH =	0.000	= 0
F16	1.000		SV =	-1.000	= 0
F17	1.000	J	SH =	0.000	= 0
F18	1.000		SV =	2.414	= 0
F19	1.000	K	SH =	-1.000	= 0
F20	1.000		SV =	-2.414	= 0
F21	1.000	L	SH =	0.000	= 0
			SV =	1.000	= 0
		B	SH =	-1.707	= 0
			SV =	1.207	= 0

Objective function			
Joint	Initial of Formula	Formulation of □V	Equilibrium condition
B	SV =	1.207	= 0

Table 4 Modeling worksheet after calculating by Microsoft Excel Solver

Variables			Constraints functions			
Initial of member forces	Member forces in kN	Joint	Initial of Formula	Formulation of □H and □V at each joint	Equilibrium condition
F1	-0.707	A	SH =	0.000	= 0
F2	0.500		SV =	0.000	= 0
F3	0.000	C	SH =	0.000	= 0
F4	-1.000		SV =	0.000	= 0
F5	0.707	D	SH =	0.000	= 0
F6	0.500		SV =	0.000	= 0
F7	0.000	E	SH =	0.000	= 0
F8	-1.000		SV =	0.000	= 0
F9	-0.707	F	SH =	0.000	= 0
F10	1.500		SV =	0.000	= 0
F11	1.000	G	SH =	0.000	= 0
F12	-1.000		SV =	0.000	= 0
F13	-0.707	H	SH =	0.000	= 0
F14	1.500		SV =	0.000	= 0
F15	0.000	I	SH =	0.000	= 0
F16	-1.000		SV =	0.000	= 0
F17	0.707	J	SH =	0.000	= 0
F18	0.500		SV =	0.000	= 0
F19	0.000	K	SH =	0.000	= 0
F20	-0.707		SV =	0.000	= 0
F21	0.500	L	SH =	0.000	= 0
			SV =	0.000	= 0
		B	SH =	0.000	= 0
			SV =	0.000	= 0

Objective function			
Joint	Initial of Formula	Formulation of □V	Equilibrium condition
B	SV =	0.000	= 0

The MES program works to optimize the function of the purpose, and in this research, to create the value of the cell = 0 (thus meeting one of the balance equation), by varying the value of variables (the value of rod forces), in such a way until all the obstacle functions are fulfilled (the value of any cell that contains requirements of balance connection = 0). MES is capable to simulate dozens or hundreds of defined variables simultaneously.

Table 5 Recapitulation of Calculation result using MES (left half portion**)**

Initial of member forces	Member forces of each truss in kN									
	Type-1	Type-2	Type-3	Type-4	Type-5	Type-6	Type-7	Type-8	Type-9	Type-10
F1	-0.707	-0.707	-15.652	-15.652	-1.000	-0.500	-0.500	-0.707	0.000	-0.500
F2	0.500	0.500	14.000	14.000	0.000	-0.385	0.707	0.500	0.000	0.000
F3	0.000	0.500	-2.000	0.000	-10.359	0.631	0.000	0.000	0.000	0.000
F4	-1.000	-0.500	-15.652	-13.416	7.636	0.000	-0.500	-0.707	0.000	-0.417
F5	0.707	-0.707	2.828	-2.236	0.000	0.000	0.000	0.000	0.000	0.417
F6	0.500	1.000	12.000	14.000	-10.602	-0.385	0.707	0.500	0.000	0.000
F7	0.000	0.500	-3.000	1.000	3.594	-0.978	0.000	0.000	-0.500	-0.250
F8	-1.000	-1.000	-13.416	-11.180	7.636	1.020	-0.500	-1.000	-0.543	0.250
F9	-0.707	-0.707	3.606	-2.828	-2.429	0.082	0.000	0.707	0.647	-0.333
F10	1.500	1.500	10.000	12.000	-10.914	-1.826	-0.707	0.000	0.000	-0.417
F11	1.000	0.500	-4.000	2.000	0.465	1.091	0.000	0.500	-0.374	0.417
F12		-1.500	-11.180	-8.944	10.286	1.001	-0.500	0.000	-0.909	0.333
F13		-0.707	4.472	-3.606	-0.353	0.000	-0.707	0.707	0.507	-0.250
F14		2.000	8.000	10.000	-9.895	-1.826	1.000	0.500	0.528	0.250
F15		1.000	0.000	6.000	-1.714	-1.307	0.000	0.000	-0.349	-0.667
F16					10.588	2.855	-1.500	-1.000	-1.239	-0.417
F17					2.800	0.220	0.707	-0.707	0.505	0.417
F18						-3.750	1.000	1.500	0.896	0.667
F19						1.098	0.000	0.000	-0.376	-0.250
F20						2.838	-1.500	-0.707	-1.601	0.250
F21						-1.000	0.000	0.000	0.578	-1.000
F22							0.707	1.500	1.233	-0.417
F23							-1.000	1.000	0.107	0.417
F24										1.000
F25										0.500

Table 6 Recapitulation of Calculation result using Ritter (left half portion)

Initial of member forces	Member forces of each truss in kN									
	Type-1	Type-2	Type-3	Type-4	Type-5	Type-6	Type-7	Type-8	Type-9	Type-10
F1	-0.707	-0.707	-15.652	-15.652	-1.000	-0.500	-0.500	-0.707	0.000	-0.500
F2	0.500	0.500	14.000	14.000	0.000	-0.385	0.707	0.500	0.000	0.000
F3	0.000	0.500	-2.000	0.000	-10.359	0.631	0.000	0.000	0.000	0.000
F4	-1.000	-0.500	-15.652	-13.416	7.636	0.000	-0.500	-0.707	0.000	-0.417
F5	0.707	-0.707	2.828	-2.236	0.000	0.000	0.000	0.000	0.000	0.417
F6	0.500	1.000	12.000	14.000	-10.602	-0.385	0.707	0.500	0.000	0.000
F7	0.000	0.500	-3.000	1.000	3.594	-0.978	0.000	0.000	-0.500	-0.250
F8	-1.000	-1.000	-13.416	-11.180	7.636	1.020	-0.500	-1.000	-0.543	0.250
F9	-0.707	-0.707	3.606	-2.828	-2.429	0.082	0.000	0.707	0.647	-0.333
F10	1.500	1.500	10.000	12.000	-10.914	-1.826	-0.707	0.000	0.000	-0.417
F11	1.000	0.500	-4.000	2.000	0.465	1.091	0.000	0.500	-0.374	0.417
F12		-1.500	-11.180	-8.944	10.286	1.001	-0.500	0.000	-0.909	0.333
F13		-0.707	4.472	-3.606	-0.353	0.000	-0.707	0.707	0.507	-0.250
F14		2.000	8.000	10.000	-9.895	-1.826	1.000	0.500	0.528	0.250
F15		1.000	0.000	6.000	-1.714	-1.307	0.000	0.000	-0.349	-0.667
F16					10.588	2.855	-1.500	-1.000	-1.239	-0.417
F17					2.800	0.220	0.707	-0.707	0.505	0.417
F18						-3.750	1.000	1.500	0.896	0.667
F19						1.098	0.000	0.000	-0.376	-0.250
F20						2.838	-1.500	-0.707	-1.601	0.250
F21						-1.000	0.000	0.000	0.578	-1.000
F22							0.707	1.500	1.233	-0.417
F23							-1.000	1.000	0.107	0.417
F24										1.000
F25										0.500

3.0 RESULTS AND DISCUSSION

3.1 Result and Discussion of the Calculation Using MES

Based on the ten SOM, the researchers had successfully analyzed the ten types of truss frameworks which had been previously selected. However, the results presented only data from rod truss type 1. Table 4 is a display of the modeling worksheet of truss tipe-1 which had been calculated by MES, where it can be seen the elements of "Constraints Function" and all the cells in column 3 contain the value of "0.000". It means that all the conditions of equilibrium connection are met. As a consequence, the numbers on the element of "Variables" on the second column, namely rod forces have been determined correctly. The recapitulation of the rod forces calculation result (left half for) of all types of the trusses is presented in Table 5. The rod forces with positive sign are the tensile force, and ones with negative sign are compressive forces.

3.2 The Results of Calculation by Ritter Method

In order to be more reliable, the rod forces are also calculated by Ritter's analytical method. Table 6 displays t result recapitulation of the rod forces calculation (left half portion) using Ritter's method.

3.3 The Comparison of Calculation Results

After executing the calculation, it was found the calculation result by Ritter's method using is (as might be expected) the same with the result of the method used in this experiment. Thus, it might be concluded that the calculation by the combination method (the combination between method of knot balance and the Solver optimizing method) is valid and is accountable.

3.4 Determination of the Influence Lines by MES

The calculation of influence lines of the truss becomes relatively easier and faster if it is calculated by the method which is developed in this report. The rod forces

of truss type 1 due to the imposition of one unit load (vertically downward) at knot D is already calculated. The same calculation is also consecutively executed at knot F, H, J, L. As a result, it was obtained 5 lists of the rod forces where each of them consists of rod force 1 (F1) until rod force 21 (F21) as it has been presented on table 7.

Based on the Table 7, if it is desired, the influence lines of the whole truss can be easily described. As an example, through Figure 12, the featuring images of the influence lines (IL) for 3 rod forces are displayed, such as rod force F8, F9, and F10.

In addition to the last column of Table 7, the maximum value of the rod force which is taken from the variation of the location of the burden unit has been applied. In this column, the largest compressive member force, namely -1.333 can be read and it will be experienced by the rod number 4, 8, 12, and 16. While the maximum tensile rod force is 1,500 and it will occur in the rod number 10 and 14.

12 a. IL for member force F 8

12 b. IL for member force F 9

12 c. IL for member force F 10
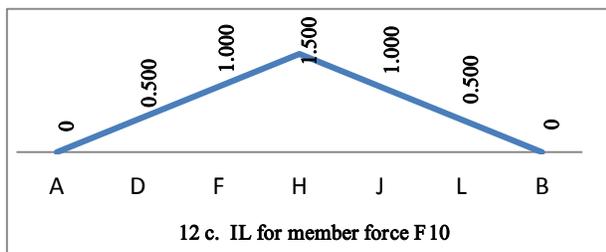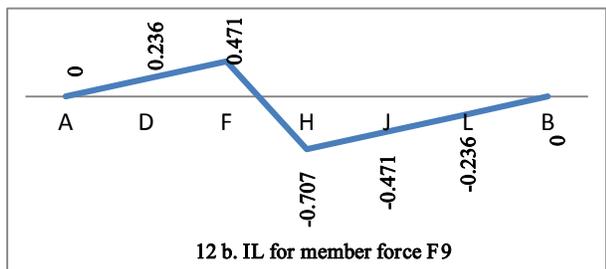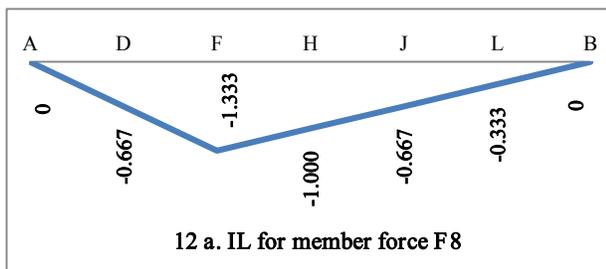
Figure 12 Influence lines for three member forces of type-1 truss

4.0 CONCLUSION

Based on the calculation result and discussion in this paper, the researchers make the following conclusions:

a. Calculation by optimization method of Microsoft Excel Solver (MES) can be applied to simultaneously accomplish all of the connection balance equation at truss system, so that the member forces can be determined.

b. The main step in using MES is how to create the optimization model of the truss system (mathematical model) that will be analyzed by MES. It means how to determine the variables, the constraint functions, and the objective function in the Solver Optimization Model.

c. The calculation method developed in this paper has been used to calculate ten type of truss, from the simplest one (12 connection and 21 members) until the most complex one (26 joints and 49 rods).

d. The result of calculation by MES in this paper is very convincing, because it has been validated by comparing it with the result of calculation that use a very popular method which is well known as Ritter method.

e. The calculation method presented in this paper can also be used to quickly determine the influence lines in the construction of truss.

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