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# IDENTIFICATION OF ON-AND OFF-LINE LINEAR STATE SPACE MODELS USING SUBSPACE METHODS

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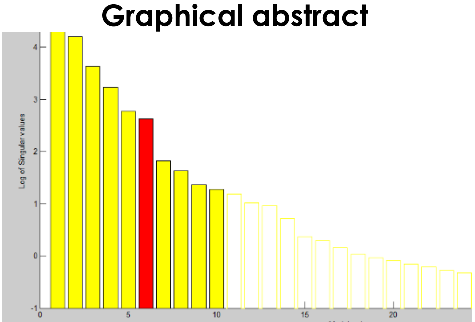
**Graphical abstract**




## Abstract

In this paper, subspace identification methods are proposed to analyze the differences between On-And Off-Line Linear State Space Models Using Subspace Methods. There are several ways that can estimate the order of the system. For this paper, Singular Value Decomposition (SVD) is used to estimate the order of the system. Comparing with the others methods, this method only need a limited number of input and output data for the determination of the system matrices. Two methods of the subspace algorithm are used which is N4SID (Numerical algorithm for Subspace State Space System Identification) and MOESP (Multivariable Output-Error State-Space model identification).

*Keywords*: Identification, Subspace, Singular Value Decomposition (SVD)

## Abstrak

Dalam kertas ini, kaedah pengenalan sub ruang di perkenalkan untuk menganalisis perbezaan antara atas-talian dan luar-talian model lurus state space. Terdapat beberapa cara yang boleh menganggarkan tertib sesebuah sistem. Dalam kertas ini, Penguraian Nilai Singular (SVD) digunakan sebagai untuk menganggarkan tertib sistem ini. Jika di bandingkan dengan kaedah lain, kaedah ini hanya memerlukan bilangan data masukan dan keluaran yang terhad bagi menentukan sistem matriks. Dua kaedah sub ruang yang di gunakan iaitu N4SID (Algoritma Berangka untuk Sistem Identifikasi Sub ruang State Space) dan MOESP (Pemboleh Ubah Pelbagai Ralat Identifikasi Model Keluaran State Space).

*Kata kunci*: Identifikasi, Sub ruang, Penguraian Nilai Singular (SVD)



## 1.0 INTRODUCTION

Identification aims to finding a mathematical model from the measurement record of data inputs and outputs of a system. A state space model is a most obvious choice for a mathematical representation. It is because of its widespread use in system theory and control. Still, reliable general purpose state space identification schemes have not become standard tools so far, mostly due to the computational complexity involved [1]. The systems of large dimensions are usually found in industry, so that the application of subspace identification algorithms in this field is very promising. Each subspace identification method is different from the other ones in concept, interpretation and computational implementation. Subspace identification methods have lately obtained a certain level of perfection. Some of the subspace identification algorithms are Numerical algorithms for Subspace State Space System Identification (N4SID), Multivariable Output-Error State-Space model identification (MOESP) and Canonical Variate Analysis (CVA). A glass tube drawing process is chosen to simulate the Off-and On-line identification for this paper.

## 2.0 BLOCK HANKEL MATRICES AND INPUT-OUTPUT EQUATIONS

Block Hankel matrices play an important role in subspace identification algorithms [1]. These matrices can be easily constructed from a given input and output data. Input and output matrices are defined in equation 1 and 2:

$$U_{0|i-1}=\begin{pmatrix} u_0 & u_1 & u_2 & \dots & u_{j-1} \\ u_1 & u_2 & u_3 & \dots & u_j \\ \dots & \dots & \dots & \dots & \dots \\ u_{i-1} & u_i & u_{i+1} & \dots & u_{i+j-2} \end{pmatrix} \tag{1}$$

$$Y_{0|i-1}=\begin{pmatrix} y_0 & y_1 & y_2 & \dots & y_{j-1} \\ y_1 & y_2 & y_3 & \dots & y_j \\ \dots & \dots & \dots & \dots & \dots \\ y_{i-1} & y_i & y_{i+1} & \dots & y_{i+j-2} \end{pmatrix} \tag{2}$$

## 3.0 OFF-AND ON-LINE ALGORITHM

### 3.1 Off-Line Algorithm

$H_1$ and $H_2$, defined by equations 3 and 4 below.

$$H_1=\begin{bmatrix} u[k] & u[k+1] & \dots & \dots & \dots & u[k+j-1] \\ y[k] & y[k+1] & \dots & \dots & \dots & y[k+j-1] \\ u[k+1] & u[k+1] & \dots & \dots & \dots & u[k+j] \\ y[k+1] & y[k+1] & \dots & \dots & \dots & y[k+j] \\ \dots & \dots & \dots & \dots & \dots & \\ u[k+i-1] & u[k+i] & \dots & \dots & \dots & u[k+j+i-2] \\ y[k+i-1] & y[k+i] & \dots & \dots & .. & y[k+j+i-2] \end{bmatrix} \tag{3}$$

$$H_2=\begin{bmatrix} u[k+i] & u[k+i+1] & \dots & \dots & \dots & u[k+j-1] \\ y[k+i] & y[k+i+1] & \dots & \dots & \dots & y[k+j-1] \\ u[k+i+1] & u[k+i+2] & \dots & \dots & \dots & u[k+j] \\ y[k+i+1] & y[k+i+2] & \dots & \dots & \dots & y[k+j] \\ \dots & \dots & \dots & \dots & \dots & \\ u[k+2i-1] & u[k+2i] & \dots & \dots & \dots & u[k+j+i-2] \\ y[k+2i-1] & y[k+2i] & \dots & \dots & .. & y[k+j+i-2] \end{bmatrix} \tag{4}$$

The system matrices A, B, C and D matrices can be obtained as following steps. Step 1 is to find U and S in the SVD of H as shown in equation 5. SVD produces a diagonal matrix S.

$$H=U.S.V^t=\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}.\begin{bmatrix} S_{11} & 0 \\ 0 & 0 \end{bmatrix}.V^t \tag{5}$$

Next, calculate the SVD of $U_{12.}{}^{t.}U_{11}.S_{11}$ as shown in equation 6. The matrices have the dimension as shown in equation 7 [1]

$$U_{12}^t.U_{11}.S_{11}=\begin{bmatrix} U_q & U_q^{\perp} \end{bmatrix}\begin{bmatrix} S_q & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_q^t \\ V_q^{\perp t} \end{bmatrix} \tag{6}$$

$$\begin{aligned} \dim(U_{11}) &= (mi+li)\times(2mi+n) \\ \dim(U_{12}) &= (mi+li)\times(2li-n) \\ \dim(U_{21}) &= (mi+li)\times(2mi+n) \\ \dim(U_{22}) &= (mi+li)\times(2mi-n) \\ \dim(S_{11}) &= (2mi+li)\times(2mi+n) \end{aligned} \tag{7}$$

Then, solve the following set of linear equation to get A, B, C and D matrix as shown in equation 8 [1].

$$\begin{bmatrix} U_q^t.U_{12}^t.U(m+l+1:(i+1)(m+l),;).S \\ U(mi+li+m+1:(m+l)(i+1),:).S \end{bmatrix}=\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} U_q^t.U_{12}^t.U(1:(1:mi+li),;).S \\ U(mi+li+1:mi+li+m,:).S \end{bmatrix} \tag{8}$$

As a summary, the main step in the identification procedure consists in SVD of a block Hankel matrix. Then, the system matrices identified by solving an overdetermined set of linear equation.

### 3.2 On-Line Algorithm

Off-line algorithm can converted into an adaptive version [1]. Every time step a new input and output measurement becomes available, a new column must be added to the matrix *H*. That's mean the older measurements should be discarded by successively deleting columns from matrix *H*. The off-line algorithm of the previous section is then applied to the updated *H* matrix. Instead of using this moving window technique, it can also apply exponential weighting method where weighting factor (a) with condition a is less than 1 or equal to 1[1]. For step 1, construct a new column to be added to *H*, using the 2*i* latest.Input-output measurements. Next, calculates the SVD as shown equation 9 and its partition is shown in equation 10 [1]

$$U_k.S_k.V_k^t=\left[\alpha.U_{k-1}.S_{k-1}\ \ column\right] \tag{9}$$

$$U_k.S_k=\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}.\begin{bmatrix} S_{11} & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

Then, next step calculate the SVD of $U_{12}{}^{t}.U_{11}.S_{11}$ as shown in equation 11. This step is the same as step 2 that shown in Off-line algorithm part. The matrices have the following dimension as shown in equation (12).

$$U_{12}^t.U_{11}.S_{11} = \begin{bmatrix} U_q & U_q^{\perp} \end{bmatrix} \begin{bmatrix} S_q & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_q^t \\ V_q^{\perp t} \end{bmatrix} \tag{11}$$

$$\begin{aligned} \dim(U_{11}) &= (mi+li)\times(2mi+n) \\ \dim(U_{12}) &= (mi+li)\times(2li-n) \\ \dim(U_{21}) &= (mi+li)\times(2mi+n) \\ \dim(U_{22}) &= (mi+li)\times(2mi-n) \\ \dim(S_{11}) &= (2mi+li)\times(2mi+n) \end{aligned} \tag{12}$$

Then, solve the following set of linear equation to get A, B, C and D matrix as shown in equation 13 below.

$$\begin{bmatrix} U_q^t.U_{12}^t.U(m+l+1:(i+1)(m+l),:).S \\ U(mi+li+m+1:(m+l)(i+1),:).S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_q^t.U_{12}^t.U(1:(1:mi+li),:).S \\ U(mi+li+1:mi+li+m,:).S \end{bmatrix} \tag{13}$$

## 4.0 RESULTS AND ANALYSIS

### 4.1 Off-Line Identification

Figure 1 shows the SVD based on N4SID and Figure 2 shows the SVD based on MOESP method.

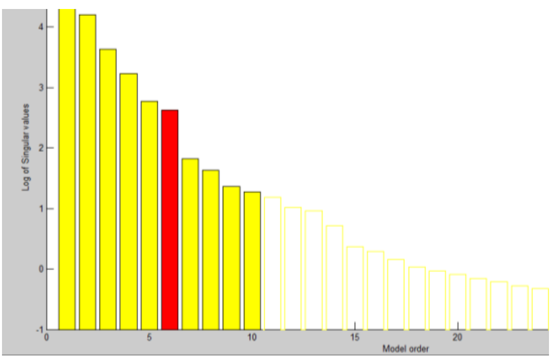



**Figure 1** Singular Value Decomposition (SVD) based on N4SID

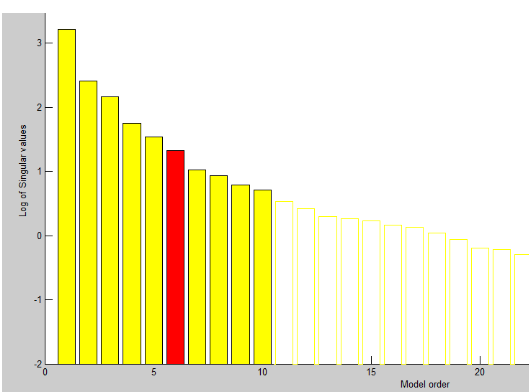



**Figure 2** Singular Value Decomposition (SVD) based on MOESP

From Figure 1 and Figure 2 its shows that the best order is six for both N4SID and MOESP. Figure 3 show the result of the validation based on N4SID and Figure 4 shows the results validation based on MOESP.

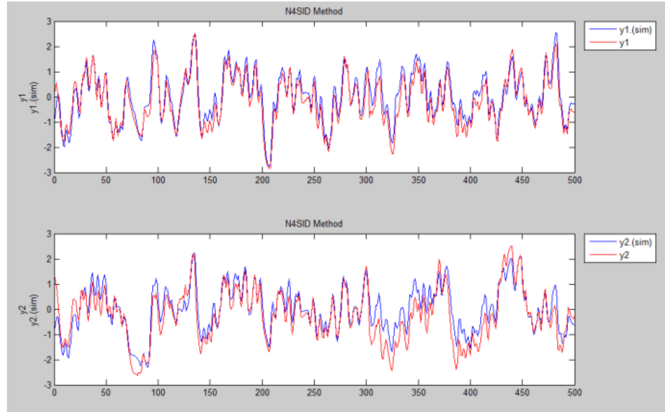



**Figure 3** Validation based on N4SID

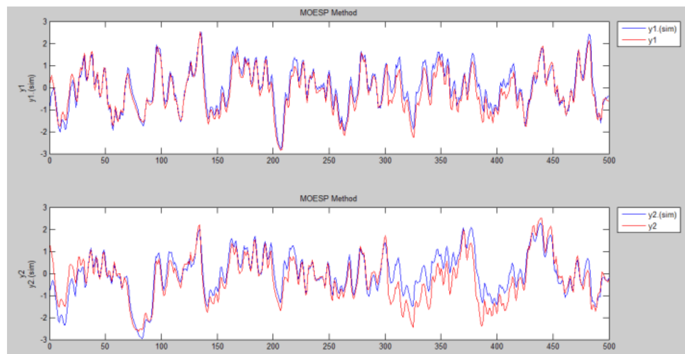



**Figure 4** Validation based on MOESP

For Figure 3, the best fitness for output 1 is 96.71% and output 2 is 84.79%. For Figure 4 the best fitness for output 1 is 93.07% and for output 2 is 80.23%. Figure 5 and Figure 6 show poles of the obtained system based on N4SID and MOESP respectively. The stability is checked by checking the poles of the system or eigenvalues of the A matrices.

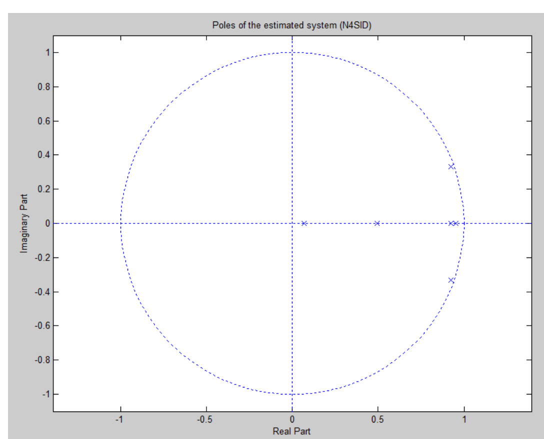



**Figure 5** Poles of the estimated system based on N4SID

Below is the eigenvalues of the A matrices. Below is the eigenvalues of the A matrices for N4SID and MOESP respectively.

$$\begin{pmatrix} 0.0692 \\ 0.4960 \\ 0.9236 + 0.3311i \\ 0.9236 - 0.3311i \\ 0.9515 \\ 0.9257 \end{pmatrix}$$

$$\begin{pmatrix} 0.8848 + 0.3346i \\ 0.8848 - 0.3346i \\ 0.9712 \\ 0.8520 + 0.0810i \\ 0.8520 - 0.0810i \\ 0.7791 \end{pmatrix}$$

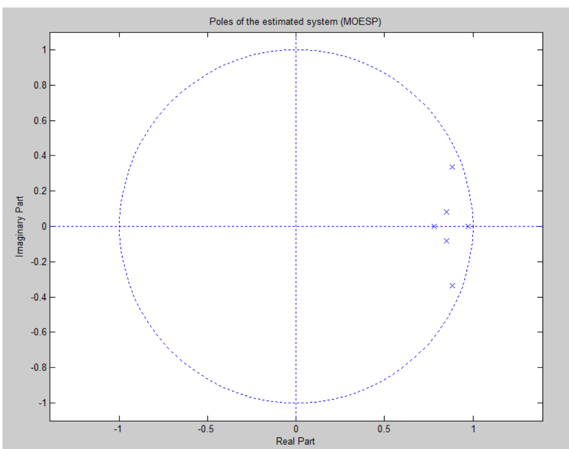



**Figure 6** Poles of the estimated system based on MOESP

## 4.2 On-Line Identification

For the on-line identification, the result is taken by apply manual value of weightings factor (a).

Figure 7 below show the SVD based on N4SID and Figure 8 shows the SVD based on MOESP when weighting factor is 1.

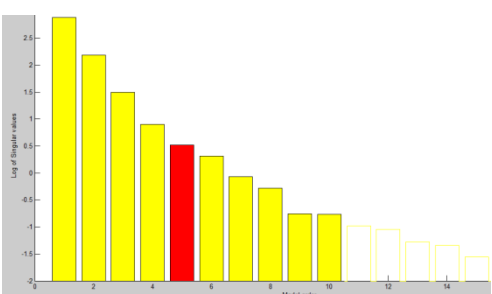


**Figure 7** Singular Value Decomposition (SVD) based on N4SID

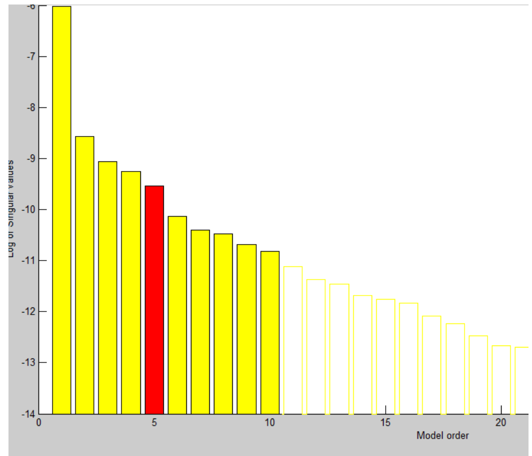



**Figure 8** Singular Value Decomposition (SVD) based on MOESP

From Figure 7 and Figure 8 its shows that the best order is five for both N4SID and MOESP. Figure 9 show the result of the validation based on N4SID and Figure 10 show the results of the validation based on MOESP.

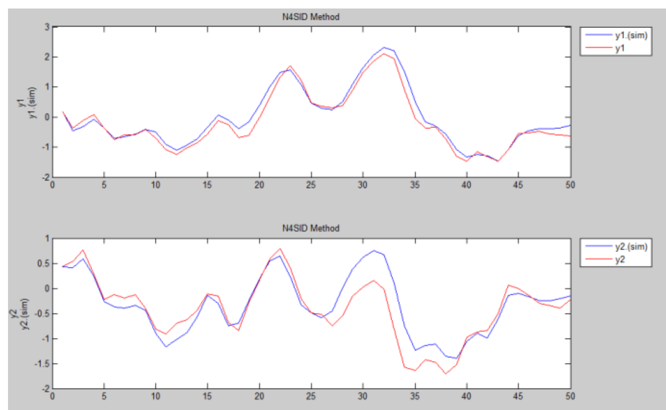



**Figure 9** Validation based on N4SID

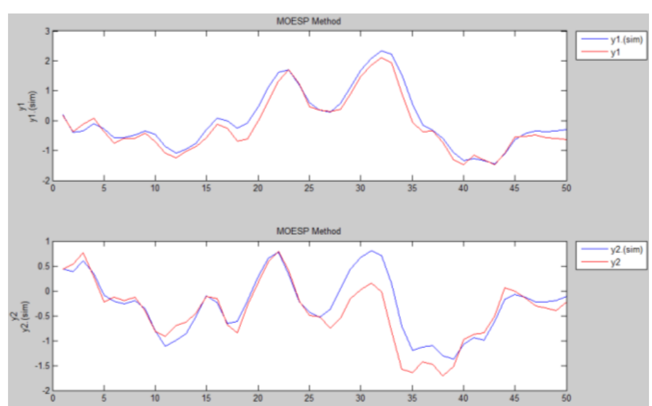



**Figure 10** Validation based on MOESP

For Figure 9, the best fitness for output 1 is 78.92% and output 2 is 74.03%. However, for Figure 10 the best fitness for output 1 is 76.38% and for output 2 is 71.56%. Figure 11 and figure 12 below show poles of the obtained system based on N4SID and MOESP respectively.

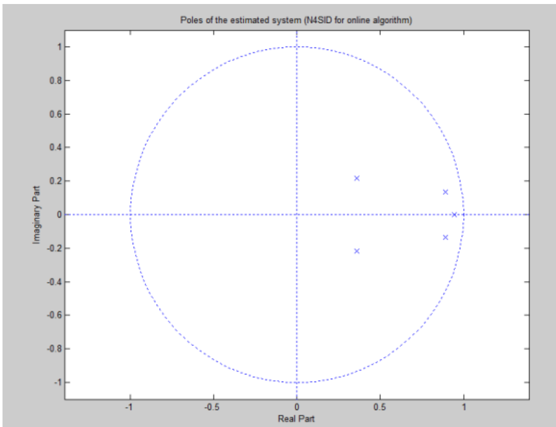



**Figure 11** Poles of the estimated system based on N4SID

Below is the eigenvalues of the A matrices for N4SID and MOESP respectively.

$$\begin{bmatrix} 0.3592 + 0.2178i \\ 0.3592 - 0.2178i \\ 0.8929 + 0.1365i \\ 0.8929 - 0.1365i \\ 0.9435 \end{bmatrix}$$

$$\begin{bmatrix} -0.1943 + 0.4704i \\ -0.1943 - 0.4704i \\ 0.8900 \\ 0.5716 \\ 0.7115 \end{bmatrix}$$

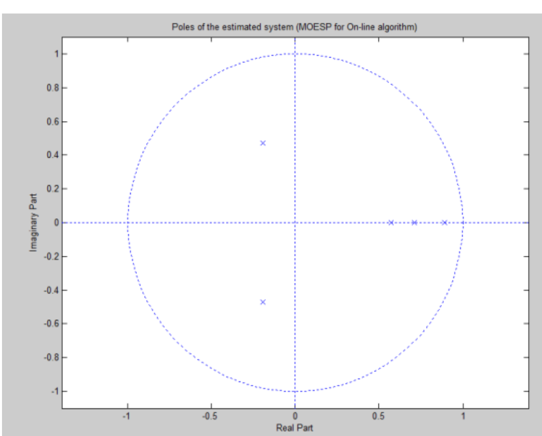



**Figure 12** Poles of the estimated system based on MOESP

On-line identification when weighting factor (a) is 0.8

Figure 13 below show the SVD based on N4SID and Figure 14 shows the SVD based on MOESP when weighting factor is 0.8.

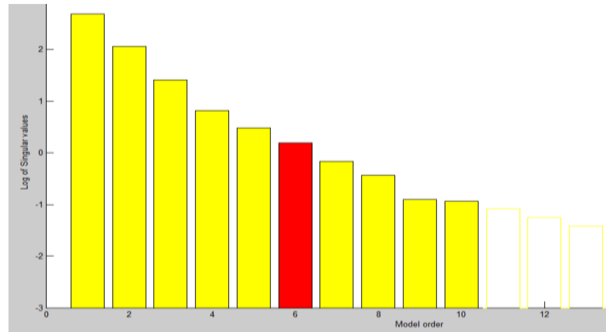



**Figure 13** Singular Value Decomposition (SVD) based on N4SID

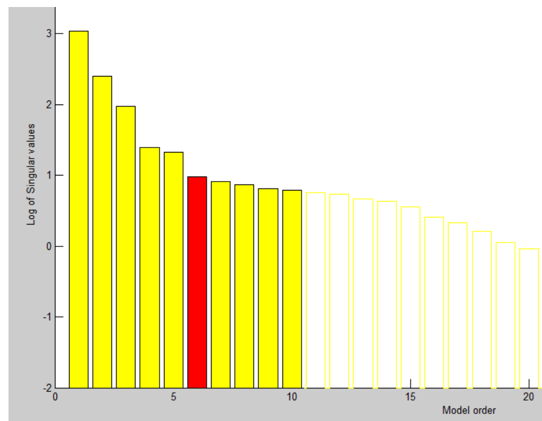



**Figure 14** Singular Value Decomposition (SVD) based on MOESP

From Figure 13 and Figure 14 its shows that the best order is six for both N4SID and MOESP. Figure 15 show the result of the validation based on N4SID however Figure 16 show the results of the validation based on MOESP.

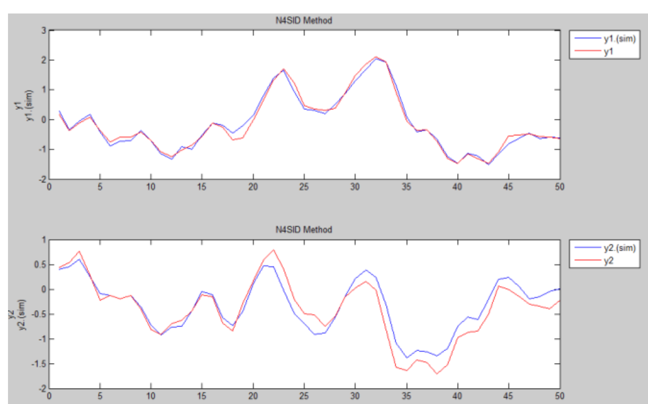



**Figure 15** Validation based on N4SID

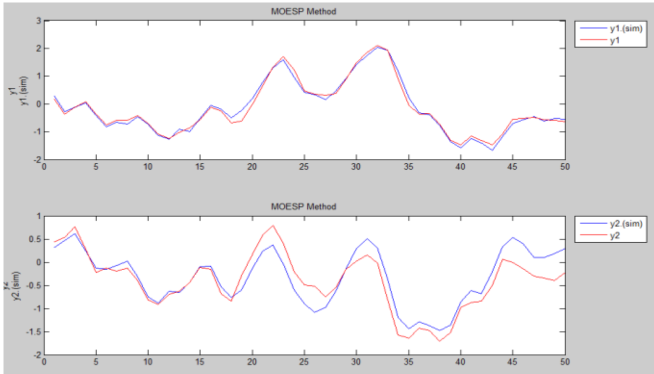



**Figure 16** Validation based on MOESP

For Figure 15, the best fitness for output 1 is 86.32% and output 2 is 80.09%. However, for Figure 16 the best fitness for output 1 is 84.66% and for output 2 is 72.63%. Figure 17 and Figure 18 below show poles of the obtained system based on N4SID and MOESP respectively.

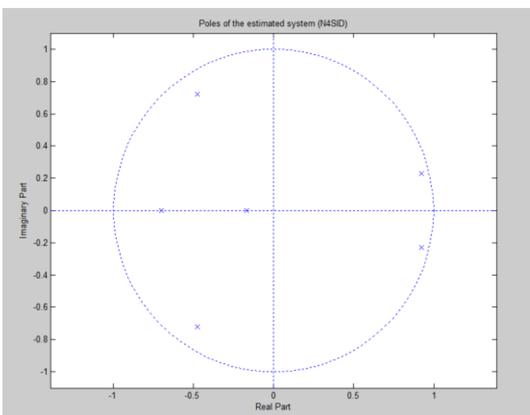



**Figure 17** Poles of the estimated system based on N4SID

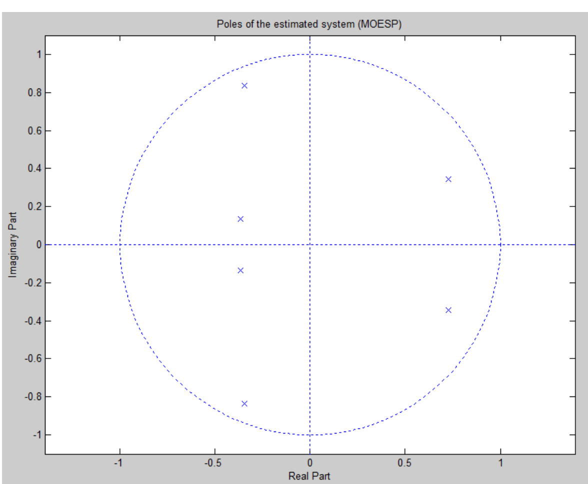



**Figure 18** Poles of the estimated system based on MOESP

Below is the eigenvalues of the A matrices for N4SID and MOESP respectively.

$$\begin{pmatrix} 0.9245 + 0.2288i \\ 0.9245 - 0.2288i \\ -0.4766 + 0.7198i \\ -0.4766 - 0.7198i \\ -0.7002 \\ -0.1715 \end{pmatrix}$$

$$\begin{pmatrix} 0.7293 + 0.3426i \\ 0.7293 - 0.3426i \\ -0.3470 + 0.8357i \\ -0.3470 - 0.8357i \\ -0.3669 + 0.1341i \\ -0.3669 - 0.1341i \end{pmatrix}$$

## 5.0 CONCLUSION

In this paper, On- and Off- line identification of a state space models of a glass tube manufacturing process system are obtained based on two subspace algorithm. N4SID and MOESP are chosen for this approach. The system matrices are identified by only applying numerically stable SVD techniques to a block Hankel matrix which is it's constructed with noisy input-output data. As it turns out that it is only a left singular basis is required and both the computational load and the noise sensitivity are considerable reduced. For Off-line identification the order of the system is fixed and need to gather the data, means that its only consists fixed order and it's only cover a certain point. However, for the On-line identification it can track every change of order and system matrices in the system, means that, when changes the value of weighting factor (a) it will give different results and need to estimate every change in sequence data, k. The algorithm is easily converted into an adaptive version for slowly time-varying systems. The suggestions for future works are recommended such as use Particle Swarm Optimization (PSO) to get the best value for weighting factor (a) and use another subspace algorithm such as CVA.

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## References


[1] Moonen, M., B. De Moor, L. Vandenberghe and J. Vandewalle. 1989. On and Off-line identification of Linear State Space Models. *Internal Journal Control.* 49(1): 219-232.

[2] Irma Wani Jamaludin, Norhaliza Abdul Wahab and M.S. Gaya. 2013. Data- Driven Subspace Predictive Control for a MIMO System. *The International Conference on Advanced Material Engineering & Technology (ICAMET)*. 28 November 2013, Bandung, Indonesia.

[3] Baram Y. 1981. Realization and Reduction of Markovian Models from Nonstationary Data. *IEEE Trans. on Automatic Control*. AC-26(6): 1225-1231.

[4] Akaike H. 1974. Stochastic Theory of Minimal Realization. *IEEE Trans. on Automatic Control.* AC-19(6): 667-674.

[5] Van Overschee, P. and B. De Moor. 1994. Subspace Algorithms for the Identification of Combined Deterministic-Stochastic Systems. *Automatica*. 30(1): 75-93.

[6] Van Overschee, P. and B. De Moor. 1991. Subspace Algorithms for the Stochastic Identification Problem. *30th IEEE Conference on Decision and Control*. Brighton, England. 1321-1326.

[7] De Moor B., Moonen M., Vandenberghe L., Vandewalle J. 1987. Identification of Linear State Space Models with Singular Value Decomposition Using Canonical Correlation Analysis. *Proc. of the International Workshop on SVD and Signal Processing*. Grenoble. France. North Holland, Elseviers Science Publishers B.V.

[8] Klema V. C. and Laub A. J. 1980. The Singular Value Decomposition: Its Computation and Some Applications. *IEEE Transactions on Automatic Control*. 25(2): 164-176.

[9] B. Wayne Bequette. 2003. *Process Control: Modeling, Design and Simulation.* Prentice Hall. 631-634.