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NONLINEAR OBSERVER VIA EXTENDED KALMAN FILTER ALGORITHM FOR EULER ANGLES ESTIMATION WITHOUT ATTITUDE SENSOR MEASUREMENTS

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Abstract

This paper designs and investigates an observer system via Extended Kalman Filter algorithm to estimate the satellite's Euler angles attitude during the absence of the attitude sensor measurement. This work contributes as a backup or an alternative system during unavailable attitude sensor measurement due to malfunction sensor or for cost reduction by reducing the number of sensors. In this work, the observer model for satellite attitude is presented in their non-simplified nonlinear form by combining the Euler's Moment Equation and kinematics Euler angles parameter. The performance of the designed observer via Extended Kalman Filter algorithm is analyzed and verified using real flight data of Malaysian satellite.

Keywords: Nonlinear observer, Extended Kalman Filter, Satellite attitude estimation

Abstrak

Kertas kerja ini merekabentuk dan menyiasat sistem pemerhati menggunakan algoritma Extended Kalman Filter untuk menganggar sudut Euler bagi sesebuah satelit semasa ketiadaan pengukuran sensor. Usaha ini mampu menjadi sistem alternatif semasa ketiadaan sensor pengukuran sikap yang disebabkan oleh kerosakan sensor atau sebagai langkah pengurangan kos dengan cara mengurangkan bilangan sensor. Dalam kertas kerja ini, model pemerhati bagi sikap satelit dipersembahkan dalam bentuk bukan linear dengan menggabungkan Persamaan Moment Euler dan kinematik berparameter sudut Euler. Prestasi sistem pemerhati yang direkabentuk menggunakan algoritma Extended Kalman Filter itu disahkan dan dianalisis menggunakan data penerbangan sebenar satelit Malaysia.

Kata kunci: Sistem pemerhati bukan linear, Extended Kalman Filter, Anggaran sikap satelit

1.0 INTRODUCTION

Satellite attitude determination is one of the important aspects in Attitude Determination and Control System (ADCS) of a satellite. Satellite attitude is important to be determined in a satellite to be fed back to controller in accomplishing a specific satellite mission such as Earth observation, communication, scientific research and many other missions. However not all states are directly available may be due to faulty sensor or as a way to obtain a substantial reduction of sensors which represents a cost and hardware complexity reduction.

In most practical implementations of ADCS, the angular velocity and attitude information of a spacecraft are obtained respectively from measurement of rate sensor such as gyroscopes and also attitude sensor such as sun sensor, star sensor, or magnetometer. However, attitude sensors are generally expensive and are often prone to degradation or failure. Therefore, as an alternative or backup system to circumvent the problem of attitude measurement absence, an observer can be designed to provide the information of Euler angles attitude by using only the measurement of angular velocity from gyroscope sensor.

Since decades, a great number of research works have been devoted to the problem of estimating the attitude of a spacecraft based on a sequence of noisy vector observations such as [1]–[4]. Different algorithms have been designed and implemented in satellite attitude estimation problem. Early applications relied mostly on the Kalman filter for attitude estimation. Kalman filter was the first applied algorithm for attitude estimation for the Apollo space program in 1960s. Due to limitation of Kalman filter which work optimal for linear system only, several famous new approaches have been implemented to deal with the nonlinearity in satellite attitude system including Extended Kalman Filter (EKF) [3], [5], [6], Unscented Kalman Filter (UKF) [7]–[9], Particle Filter [10]–[12], and predictive filtering [13]-[14]. EKF is an extended version of Kalman filter for nonlinear system whereby the nonlinear equation is approximated by linearized equation through Taylor series expansion. UKF, an alternative to the EKF uses a deterministic sampling technique known as the unscented transform to pick a minimal set of sample points called sigma points to propagate the non-linear functions. EKF and UKF approaches are restricted assume the noise in the system is Gaussian white noise process. While, Particle Filter is a nonlinear estimation algorithm that approximates the nonlinear function using a set of random samples without restricted to a specific noise distribution as EKF and UKF.

In the open literature, spacecraft attitude estimation use different attitude representation either Euler angles, Rodrigues parameter, or quarternion parameter as their kinematic model [15]. Each kinematic model of different parameter is governed by different differential equation [16]. The researchers also studied the performance of estimated states by varying different type of sensor measurement such as gyroscope, magnetometer, sun sensor or star sensor.

To the best of authors' knowledge, there is no study that designs an observer to estimate the satellite's Euler angles attitude by using non-simplified nonlinear Euler angles attitude kinematics model. Hence, an observer to estimate the Euler angles attitude of a satellite using angular velocity measurements only is designed and investigated in this paper. The performance of the nonlinear observer to estimate the states is also verified using real flight data of Malaysian satellite, RazakSAT.

The organization of this paper proceeds as follows. Section 2 presents mathematical model of nonlinear satellite attitude dynamics. Section 3 describes briefly the concept of an observer and also estimation algorithm used in the observer system which is EKF algorithm. Section 4 presents and discusses the results of the observer system which was tested and verified using actual flight data and Section 5 presents the paper's conclusions.

2.0 MATHEMATICAL MODEL OF NONLINEAR SATELLITE ATTITUDE DYNAMICS

Mathematical model of satellite attitude dynamics is described by both the dynamics equation of motion and kinematics equation of motion [17].

Dynamic equation of motion relates the angular velocity to the exerted torque as defined by Euler's Moment Equation [16], [17]

$$I\dot{\omega} + \omega \times I\,\omega = T \tag{1}$$

or similarly is written in component-wise

$$\dot{\omega}_x = -\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + \frac{T_x}{I_x} \tag{2}$$

$$\dot{\omega}_y = -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + \frac{T_y}{I_y} \tag{3}$$

$$\dot{\omega}_z = -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + \frac{T_z}{I_z} \tag{4}$$

with

$I = diag[I_x, I_y, I_z]$ is satellite moment of inertia matrix.

$\dot{\omega} = [\dot{\omega}_x,\ \dot{\omega}_y,\ \dot{\omega}_z]$ is angular acceleration vector. $\omega = [\omega_x,\ \omega_y,\ \omega_z]$ is angular velocity vector.

$T = [T_x, T_y, T_z]$ is space environmental disturbances torque vector.

For low Earth orbit satellite, gravity gradient torque must be taken into consideration as part of external torque since it is continuously acting on the spacecraft body and influence the satellite's attitude motion. The external torque dominated by gravity gradient torque is written as [16], [18]

$$T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} 3\omega_0{}^2(I_z - I_y)s\emptyset c\emptyset c^2\theta \\ 3\omega_0{}^2(I_z - I_x)s\theta c\theta c\emptyset \\ 3\omega_0{}^2(I_x - I_y)s\emptyset c\theta s\theta \end{bmatrix} \tag{5}$$

While, kinematic equation of motion relates the attitude parameter to the angular velocity. In this work, Euler angles parameter is preferred to represent the satellite's attitude as its straightforward physical interpretation for analysis. Euler angles are defined as the rotational angles about the body axis as follows: $\emptyset$ is rotational angle about X-axis (roll); θ is rotational angle about Y-axis (pitch); and φ is rotational angle about Z-axis (yaw). The kinematic equation of Euler angles parameter using $\varphi - \theta - \emptyset$ (or some literature use notation 3-2-1) sequence rotation is

$$\dot{p} = \begin{bmatrix} \dot{\emptyset} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} \tag{6}$$

with

$$\begin{aligned} \dot{\emptyset} = {} & [\omega_x + \omega_0 c\theta s\varphi\,] \\ & + s\emptyset t\theta[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] \\ & + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \end{aligned} \tag{7}$$

$$\dot{\theta} = c\emptyset[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)]$$

$$-s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \quad (8)$$

$$\dot{\varphi} = \frac{s\emptyset}{c\theta}[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \quad (9)$$

where *c*, *s* and *t* denote cosine, sine, and tangent functions, respectively. While, ω_0 is the orbital rate of the spacecraft.

A complete formulation of the satellite attitude dynamics for low Earth orbit satellite is obtained by combining both the dynamics equation of motion under influence of gravity gradient torque with the kinematics equation of motion such that

$$\begin{bmatrix}\dot{\omega}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\\ \dot{\emptyset}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix} \quad (10)$$

with

$$\dot{\omega}_x = -\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + 3{\omega_0}^2\frac{(I_z - I_y)}{I_x}s\emptyset c\emptyset c^2\theta \quad (11)$$

$$\dot{\omega}_y = -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + 3{\omega_0}^2\frac{(I_z - I_x)}{I_y}s\theta c\theta c\emptyset \quad (12)$$

$$\dot{\omega}_z = -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + 3{\omega_0}^2\frac{(I_x - I_y)}{I_z}s\emptyset c\theta s\theta \quad (13)$$

$$\dot{\emptyset} = [\omega_x + \omega_0 c\theta s\varphi] + s\emptyset t\theta[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \quad (14)$$

$$\dot{\theta} = c\emptyset[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] - s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \quad (15)$$

$$\dot{\varphi} = \frac{s\emptyset}{c\theta}[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \quad (16)$$

3.0 NONLINEAR OBSERVERS FOR SPACECRAFT ATTITUDE ESTIMATION

3.1 Nonlinear Observers

A nonlinear observer is a nonlinear dynamic system that is used to estimate the unknown states from one or more noisy measurements. Mathematically, the nonlinear observer design is described as follows. Given the actual nonlinear system dynamics and measurement described by continuous-time model [18], [19]

$$\dot{x} = f(x) + w \quad (17)$$
$$y = h(x) + v \quad (18)$$

Then, the observer is modeled as

$$\dot{\hat{x}} = f(\hat{x}) \quad (19)$$
$$\hat{y} = h(\hat{x}) \quad (20)$$

In Equation (17)-(20), $x \in R^n$ is the state vector and $y \in R^p$ is the output vector, w and v denote the noise or uncertainty vector in the state and measurement respectively. While $\hat{x}$ and $\hat{y}$ denotes the corresponding estimate.

In this work, the system is designed to estimate the satellite's Euler angles by using angular velocity measurement only. Hence the state vector of the system is $x = [\omega_x, \omega_y, \omega_z, \emptyset, \theta, \varphi]^T$, while the state equation is described by

$$\dot{x} = f(x) = \begin{bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\\ \dot{\emptyset}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix} \quad (21)$$

with its component is described in Equation (11)-(16). While the measurement equation for the system is

$$y = h(x) = \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} \quad (22)$$

3.2 Extended Kalman Filter Algorithm

In this work, EKF is used as the estimation algorithm in the nonlinear observer system due to its well-known and established algorithm and theoretically attractive in the sense that it minimizes the variance of the estimation error. EKF is an on-line, recursive algorithm trying to estimate the true state of an observable nonlinear system where only some noisy measurements are available. EKF algorithm is described as below. [17]

Let the continuous-time model in Equation (17) and (18) is transformed into the discrete-time model such that

$$x_k = f(x_{k-1}) + w_{k-1} \quad (23)$$
$$y_k = h(x_k) + v_k \quad (24)$$

Here the subscript of the variables denotes the time step, while w_{k-1} and v_k are restricted assumed as Gaussian distributed noises with mean zero and covariance R_w and R_v respectively such that $w_{k-1} \sim N(0, R_w)$ and $v_k \sim N(0, R_v)$. Then, the estimated state is obtained through the following step:

Step 1:Set the initial state estimate $\hat{x}_0 = \hat{x}_{0|0}$ and variance $P_0 = P_{0|0}$.

Step 2: Repeat

(i) Prediction step (priori estimate)

- Jacobian of $f(x_{k-1})$: $F_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}}$ (25)
- Predicted state estimate: $\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1})$ (26)
- Predicted covariance estimate: $P_{k|k-1} = F_{k-1}P_{k-1|k-1}F_{k-1}^T + R_w$ (27)

(ii) Update step (posteriori estimate)

- Jacobian of $h(x_k)$: $H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k|k-1}}$ (28)

- Kalman gain:

$$K_k = P_{k|k-1}H_k^T\left[H_k P_{k|k-1}H_k^T + R_v\right]^{-1} \quad (29)$$

- Updated state estimate:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left[y_k - h(\hat{x}_{k|k-1})\right] \quad (30)$$

- Updated covariance estimate:

$$P_{k|k} = [I - K_k H_k]P_{k|k-1} \quad (31)$$

4.0 RESULTS AND DISCUSSION

In this section, the performance of the EKF as an observer to estimate the Euler angles attitude without attitude sensor measurement situation is investigated.

The performance of the nonlinear observer via EKF designed in this work is analyzed and validated using real sensors data from the RazakSAT satellite. RazakSAT is a Malaysian satellite which was launched into Low Earth Orbit near Equatorial in 2009. In the mission, the attitude was provided directly using sun sensor, one of the attitude sensors, while the angular velocity was provided by gyroscope sensor. The satellite's characteristics of RazakSAT are given in Table 1, which was provided by Astronautic Technology Sdn Bhd (ATSB), the Malaysian company that responsible for RazakSAT's mission.

Table 1 RazakSAT's characteristics

Parameter	Values
Moment of inertia, I_x	25.4 kg.m2
Moment of inertia, I_y	26.2 kg.m2
Moment of inertia, I_z	21.0 kg.m2
Orbital rate, ω_0	0.001063 rad/s or 0.0609 deg/s

Figures 1, 2 and 3 show the real measurements of angular velocity respectively around X-axis, Y-axis, and Z-axis as obtained by using gyroscope sensors in RazakSAT mission. The available measurements are for about 5 orbits sequentially, available to the ground system at a sampling rate of about 1 minute.

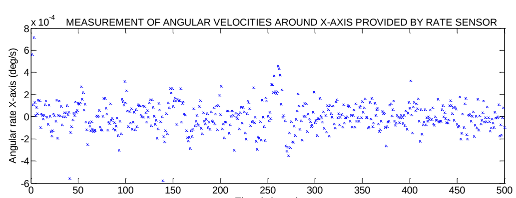

Figure 1 Real measurement of angular velocity around X-axis provided by gyroscope sensor

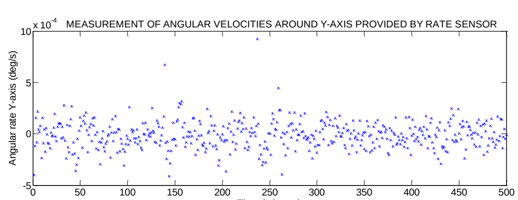

Figure 2 Real measurement of angular velocity around Y-axis provided by gyroscope sensor

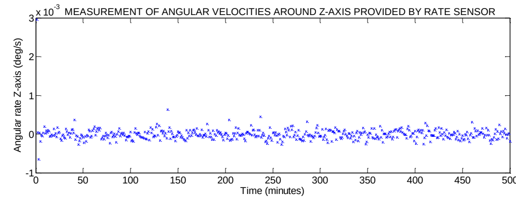

Figure 3 Real measurement of angular velocity around Z-axis provided by gyroscope sensor

In order to validate and analyze the performance of the designed system, the estimated result is compared with the real sensor data of RazakSAT. Figures 4, 5, and 6 show comparison between the estimated Euler angles through the designed observer and the real Euler angles measurements provided by sun sensor in RazakSAT mission.

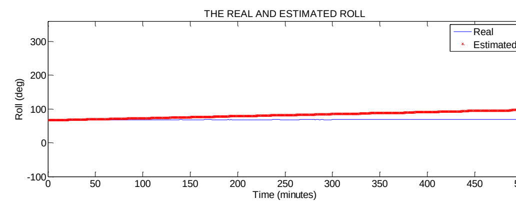

Figure 4 Comparison between the estimated and the real roll angle

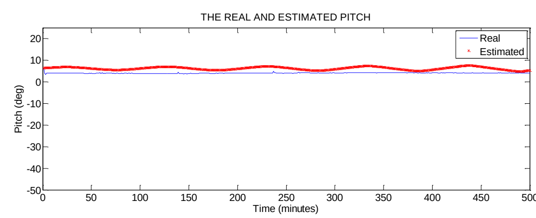

Figure 5 Comparison between the estimated and the real pitch angle

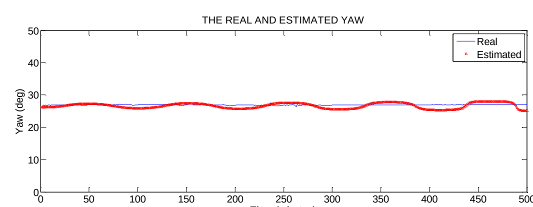

Figure 6 Comparison between the estimated and the real yaw angle

The accuracy of the estimated states is validated using Root Mean Squared Error (RMSE) with

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(x_{i,ESTIMATE} - x_{i,REAL}\right)^2}{n}} \quad (32)$$

The RMSE is a frequently used measure of the differences between values estimated by an estimator and the values actually observed. It is a representative of the size of an average error. The RMSE value of the estimated Euler angles attitude is tabulated in Table 2.

Hence from the table, the average error between the estimated and the real states are 16.5205 degree, 2.3565 degree, 0.8558 degree respectively for roll, pitch, and yaw. As overall it can be said that the EKF are able to provide the information of Euler angles attitude in less than 20 degree average error which can be considered as coarse accuracy attitude. Hence, the designed system is suitable for coarse accuracy attitude determination mode such as during detumbling task, but unsuitable for mode that requires more precise accuracy such as during imaging and housekeeping task.

Table 2 RMSE of the estimated Euler angles attitude

States	RMSE value
Roll, ϕ	16.5205 degree
Pitch, θ	2.3565 degree
Yaw, φ	0.8558 degree

5.0 CONCLUSION

In this paper, a nonlinear observer to provide the Euler angles attitude information during the absence of attitude sensor measurement was designed and developed using EKF algorithm. The performance of the designed observer by employing the EKF algorithm is then validated and investigated using real flight data of RazakSAT, the Malaysian satellite. The result shows that the system is able to provide the information of Euler angles within 20 degrees accuracy, which is suitable only for coarse accuracy attitude determination such as during detumbling mode, but unsuitable for mode that requires more precise accuracy such as during imaging and housekeeping task. The designed observer can be used as an alternative or backup attitude determination system of a Low Earth Orbit satellite during unavailable attitude measurement due to faulty sensor or reduction of sensor hardware for cost reduction.

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