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THE SERIES OF FUZZIFIED FUZZY BEZIER CURVE

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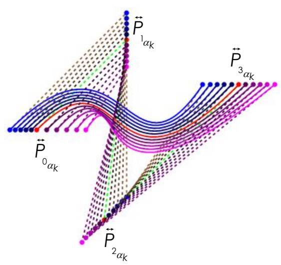
Graphical abstract



Abstract

This paper discusses about the series of fuzzified Bezier curves. This series of fuzzified Bezier curves constructed based on the fuzzy set theory especially fuzzy number concepts. The series of fuzzified represented by the alpha-cut of fuzzy number with various values of alpha. Then, the results were blended with Bezier curve function to produce the series of fuzzified fuzzy Bezier curves.

Keywords: Fuzzy number, fuzzification, alpha-cut operation, Bezier curve

Abstrak

Kertas kerja ini membincangkan tentang siri pengkaburan lengkung Bezier. Siri pengkaburan lengkung Bezier dibina berdasarkan kepada teori set kabur terutama konsep nombor kabur. Siri pengkaburan ini diwakili oleh potongan-alfa bagi nombor kabur dengan pelbagai nilai bagi alfa. Kemudian, hasilnya akan disuaikan denagn fungsi lengkung Bezier untuk menghasilkan siri pengkaburan lengkung Bezier kabur.

Kata kunci: Nombor kabur, pengkaburan, operasi potongan-alfa, lengkung Bezier

1.0 INTRODUCTION

Curve modeling is useful in representing a set of data in a graphical form for better understanding of the nature of these data. The model can be analyzed for decision making. Therefore, a set of real data are needed in certain forms. But, if the data become uncertain, then it is difficult to model or represent the data in the form of a curve function. Usually, uncertain data are removed and we only consider the certain data points. However, the resulting curve model lacks accuracy due the removal the uncertainty data points.

In order to model the uncertainty data points, the fuzzy set theory is used to define the uncertainty data points especially by using the definition of fuzzy number since we are dealing the uncertainty in real numbers. Thus, the uncertainty data points become fuzzy data points as discussed in [1-5].

After the fuzzy data points are obtained, the next step is to model these data by using Bezier curve function which can referred in [2, 6] and known as fuzzy Bezier curve model. It is followed by fuzzification process; the alpha-cut operation of triangular fuzzy number is used in order to find the fuzzy interval of fuzzy Bezier curve model. Indeed, the outcome of fuzzification process which is applied to fuzzy Bezier curve is now known as fuzzified fuzzy Bezier curves.

This paper discusses on modeling the series of fuzzified fuzzy Bezier curves through the application of alpha-cut operation of triangular fuzzy number and it is organized as follows: Section II review the basic properties including fuzzy set theory, fuzzy number concept, fuzzy relation and alpha-cut operation. Section III discusses on the process of defining uncertainty data point through fuzzy number concept and the alpha-cut operation against fuzzy data/control points. Section IV discusses on the construction of the series of fuzzified fuzzy Bezier curves which the definition of crisp Bezier curve is introduced first.

2.0 BASIC PROPERTIES

In this part, we will discuss about the definition of fuzzy set theory, fuzzy number concept, fuzzy relation and the alpha-cut operation as the basics fundamental. Therefore, the definitions can be given as follows.

Definition 1. Let *X* be a universal set and $A \subset X$. Set *A* called fuzzy set denoted by $\ddot{A}$ if for every $x \in X$ there exists $\mu_A : X \to [0,1]$ a form of membership function that characterizes the membership grade for every element of *A* in *X* defined by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \text{ (full membership)} \\ c \in (0,1) & \text{if } x \ddot{\in} A \text{ (partial membership)} \\ 0 & \text{if } x \notin A \text{ (non-membership)} \end{cases} \quad (1)$$

So, fuzzy set $\ddot{A}$ can be written as $\ddot{A} = \{(x, \mu_A(x))\}$ which is $\ddot{A}$ in X is a set of order pair denoted generically by x in X with grade of membership in ; see reference [3,4,6].

Definition 2. Let *R* be a universal set which *R* is a real number and *A* is subset to *R*. Fuzzy set, $\ddot{A}$ in *R* (number around *A* in *R*) called fuzzy number which explained through the α-level set(strong and normal α-cut) that is if for every $\alpha \in (0,1]$, there exist set $\ddot{A}_\alpha$ in R where $\ddot{A}_\alpha = \{x \in R : \mu_{\ddot{A}_\alpha}(x) > \alpha\}$ and $\ddot{A}_\alpha = \{x \in R : \mu_{\ddot{A}_\alpha}(x) \geq \alpha\}$ [5].

Definition 3. If triangular fuzzy number as shown in Figure 1 represent as $\ddot{A} = (a,d,c)$ and $\ddot{A}_\alpha$ be a α-cut operation of triangular fuzzy number, then crisp interval by α-cut operation is obtained as $\ddot{A}_\alpha = [a^\alpha, b^\alpha] = [(d-a)\alpha + a, -(c-d)\alpha + c]$ with $\alpha \in (0,1]$ where the membership function, $\mu_A(x)$ given by [1,4,7]

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{d-a} & \text{for } a \leq x \leq d \\ \dfrac{c-x}{c-d} & \text{for } d \leq x \leq c \\ 0 & \text{for } x > c \end{cases} \quad (2)$$

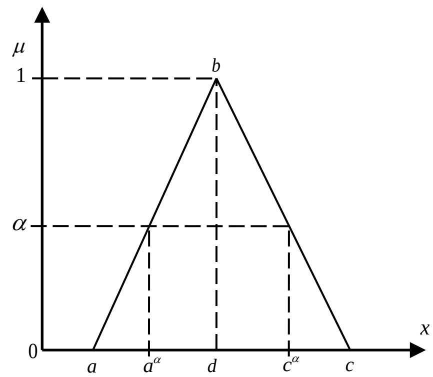


Figure 1 Triangular fuzzy number, $\ddot{A} = (a,d,c)$

Definition 4. Let $X, Y \subseteq R$ be universal sets then

$$\ddot{R} = \{((x,y), \mu_R(x,y)) \,|\, (x,y) \subseteq X \times Y\}$$

is called a fuzzy relation on $X \times Y$ [8].

Definition 5. Let $X, Y \subseteq R$ and $\ddot{A} = \{(x, \mu_A(x) \,|\, x \in X\}$ and $\ddot{B} = \{(y, \mu_B(y) \,|\, y \in Y\}$ are two fuzzy sets. Then $\ddot{R} = \{[(x,y), \mu_R(x,y)], (x,y) \in X \times Y\}$ is a fuzzy relation on $\ddot{A}$ and $\ddot{B}$ if $\mu_R(x,y) \leq \mu_A(x), \forall (x,y) \in X \times Y$ and $\mu_R(x,y) \leq \mu_B(y), \forall (x,y) \in X \times Y$.

Definition 6. Let $X, Y \subseteq R$ with $\ddot{M} = \{(x, \mu_M(x) \,|\, x \in X\}$ and $\ddot{N} = \{(y, \mu_N(y) \,|\, y \in Y\}$ are two fuzzy data. Then, the fuzzy relation between both fuzzy data is given by $\ddot{P} = \{[(x,y), \mu_P(x,y)], (x,y) \in X \times Y\}$.

3.0 FUZZY DATA/CONTROL POINTS

This section defines uncertainty data point. These uncertainty data will become fuzzy data points or more specifically called as fuzzy control points. The fuzzy data points (FDPs) and fuzzy control points (FCPs) are the same due to nature of the usage of these data in approximation curve model using Bezier curve function.

Definition 7. Let $D = \{(x,y), x \in X, y \in Y \,|\, x, y : \text{fuzzy data}\}$ and $\ddot{D} = \{P_i \,|\, P \text{ is data point}\}$ are the set of FDPs which is $D_i \in D \subset X \times Y \subseteq R$ with *R* is universal set and $\mu_P(D_i) : D \to [0,1]$ is membership function defined as $\mu_P(D_i) = 1$ in which $\ddot{D} = \{(D_i, \mu_D(D_i)) \,|\, D_i \in R\}$. Therefore,

$$\mu_P(D_i) = \begin{cases} 0 & \text{if } D_i \notin R \\ c \in (0,1) & \text{if } D_i \ddot{\in} R \\ 1 & \text{if } D_i \in R \end{cases} \quad (3)$$

with $\mu_D(D_i)=\left\langle \mu_P(D_i^{\leftarrow}), \mu_P(D_i), \mu_P(D_i^{\rightarrow})\right\rangle$ where $\mu_D(D_i^{\leftarrow})$ and $\mu_D(D_i^{\rightarrow})$ are left-grade and right-grade membership values respectively. This can be written as

$$\ddot{D}=\left\{\ddot{D}_i=(x_i,y_i)\,|\,i=0,1,..,n\right\}$$

for all i, $\ddot{D}_i=\left\langle \ddot{D}_i^{\leftarrow}, D_i, \ddot{D}_i^{\rightarrow}\right\rangle$ with $\ddot{D}_i^{\leftarrow}$, D_i and $\ddot{D}_i^{\rightarrow}$ are left FDP, crisp data point and right FDP respectively [5]. The procedure in defining FDP is illustrated in Figure 2 as follows.

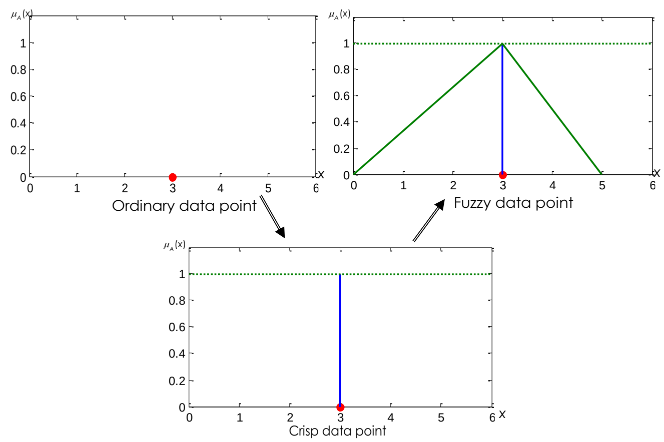


Figure 2 The process of defining FDPs

Definition 8. Let $\ddot{P}_i$ be the set of FCPs where $i=0,1,...,m$ and $j=0,1,...,n$. Then, $\ddot{P}_{i_{\alpha_k}}$ is the alpha-cut operation of fuzzy control point which is given as (4) where $\alpha_k \in (0,1]$ with $k=1,2,...,l$ and k is the series of fuzzification of FCPs.

$$\begin{aligned}\ddot{P}_{i_{\alpha_k}} &= \left\langle \ddot{P}^{\leftarrow}_{i_{\alpha_k}}, P_i, \ddot{P}^{\rightarrow}_{i_{\alpha_k}}\right\rangle \\ &= \left\langle \left[\left(P_i-\ddot{P}_i^{\leftarrow}\right)\alpha_k+\ddot{P}_i^{\leftarrow}\right], P_i, \left[-\left(\ddot{P}_i^{\rightarrow}-P_i\right)\alpha_k+\ddot{P}_i^{\rightarrow}\right]\right\rangle\end{aligned} \quad (4)$$

The Def. 8 of fuzzification is also the operation of alpha-cut in triangular forms. The series of fuzzified FCPs are based on the various alpha value, $\alpha_k \in (0,1]$ for each $k=1,2,...,l$.

4.0 THE SERIES OF FUZZIFIED FUZZY BEZIER CURVE

In this part, the series of fuzzified fuzzy Bezier curve is discussed based on the series of fuzzified FCPs. Before we construct the series of fuzzified fuzzy Bezier curve, the definitions of crisp Bezier curve as shown in Figure 3 and fuzzy Bezier curve as shown in Figure 4 are given as follows.

Definition 9. Generally Bezier curve [9-11] defined by

$$B(t)=\sum_{i=0}^{n} P_i B_i^n(t) \quad (5)$$

where $B_i^n(t)=\binom{n}{i} t^i(1-t)^{n-i}$ and $0\le t\le 1$. $B_i^n(t)$ is the ith Bernstein's polynomial of degree n.

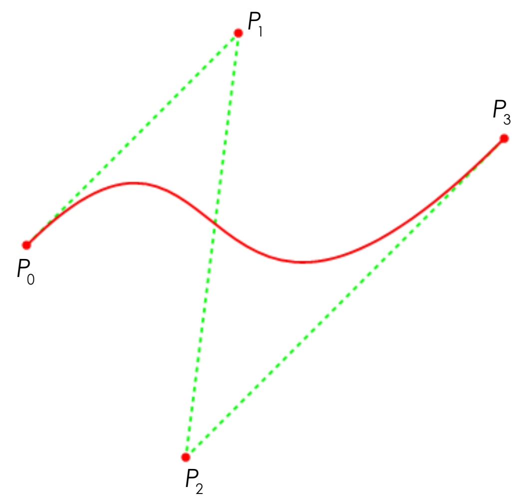


Figure 3 Crisp cubic Bezier curve

Definition 10. Let $\ddot{P}_i$ be the FCPs, then the fuzzy Bezier curve can be defined as

$$\ddot{B}(t)=\sum_{i=0}^{n} \ddot{P}_i B_i^n(t) \quad (6)$$

with $B_i^n(t)$ is the ith Bernstein's polynomial of degree n and $i=0,1,2,...,m$.

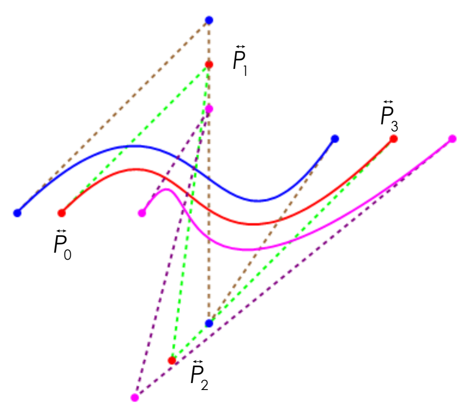


Figure 4 Fuzzy cubic Bezier curve

The next definition is the fuzzification process of fuzzy Bezier curve which used the alpha-cut operation of triangular fuzzy number.

Definition 11. Let α_k be the operation of alpha-cut towards FCPs which represented as $\ddot{P}_{i_{\alpha_k}}$, then the fuzzified fuzzy Bezier curve can be given as follow

$$\ddot{B}_{\alpha_k}(t)=\sum_{i=0}^{n}\ddot{P}_{i_{\alpha_k}}B_i^n(t). \qquad (7)$$

The series of fuzzified fuzzy Bezier curve in cubic form is illustrated in Figure 5 as follows.

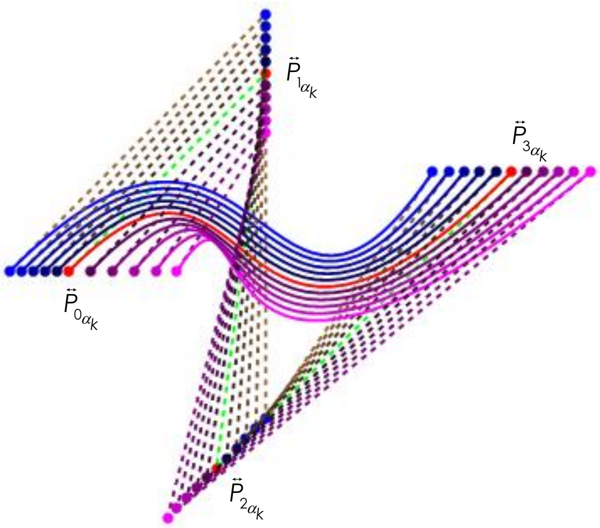


Figure 5 Series of fuzzified fuzzy cubic Bezier curve

Based on Figure 5, the series of fuzzified fuzzy cubic Bezier curve was constructed by the series of fuzzification process (alpha-cut operation) towards left/upper and right/lower FCPs which the alpha values are $\alpha_1=0.2$, $\alpha_2=0.4$, $\alpha_3=0.6$ and $\alpha_4=0.8$ where $\alpha_k\in[0,1]$ with $k=1,2,...,m$.

The series of fuzzified left/upper and right/lower of FCPs which had been modeled through Bezier curve function gives the series left/upper and right/lower fuzzified fuzzy Bezier curves. The illustration for both series of fuzzified fuzzy curves can be given via Figure 6 and Figure 7 respectively.

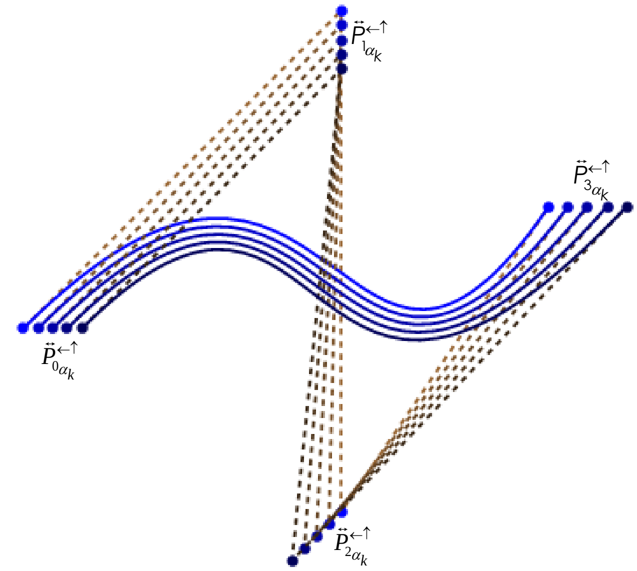


Figure 6 Series of left/upper fuzzified fuzzy cubic Bezier curve

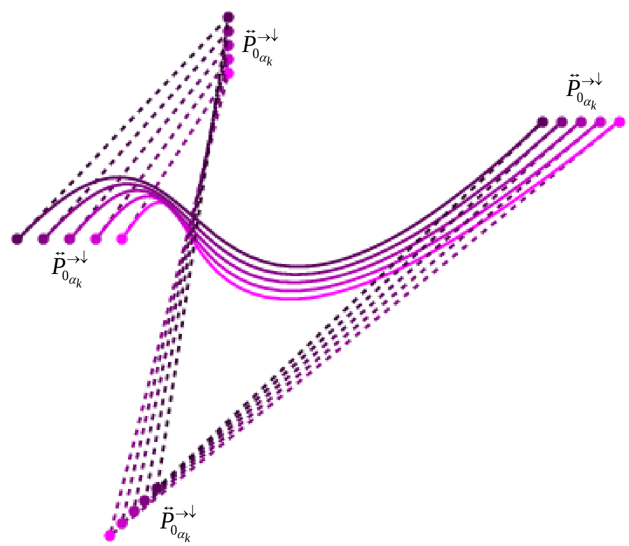


Figure 7 Series of right/lower fuzzified fuzzy cubic Bezier curve

5.0 DISCUSSION AND CONCLUSION

In this paper, we constructed the series of fuzzified fuzzy Bezier curve based on the modeling uncertainty data points through Bezier curve function and also the alpha-cut operation as the fuzzification process. The implementation of alpha-cut operation in order to construct the series of fuzzified fuzzy Bezier curve result with the fuzzy interval of crisp fuzzy Bezier curve of FDPs.

Based on modeling a series fuzzified fuzzy Bezier curve, the operation of alpha-cut can be used to represent the degree of membership FCPs. For example, if the alpha values increase and tend to 1, then the fuzzified fuzzy Bezier curve move to a crisp Bezier curve. The used of alpha-cut operation in defining uncertainty data is to obtain the fuzzy interval of FDPs which can decided by the users based on their observation and analysis.

The result of this series of fuzzified fuzzy Bezier curve which was constructed by the application of the alpha-cut operation in triangular form give the advantage to the users in defining the fuzzified fuzzy Bezier curve on the specific of the alpha value. The flexibilities of using this method also can give the user in defining the alpha value on the left/lower or right/upper fuzzy Bezier curve either the value are same or not based on the nature of fuzzy data.

Future research include modeling fuzzy curves using other complicated curves functions such as B-spline, rational Bezier and rational B-spline. In addition, this research also can be extended to represent surfaces.

The series of fuzzified fuzzy Bezier curve can be applied in 3d curve modeling to determine the road surface boundaries, closed boundaries of map perimeter modeling mainly calculate the area region based on the series of alpha level values. Also, this method can be applied in reverse engineering modeling with has the uncertainty collective data for reshape an object based on its original which has the uncertainty matter.

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