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FAST POSITIONING PERFORMANCE IN BALL SCREW MECHANISM WITH DISTURBANCE OBSERVER

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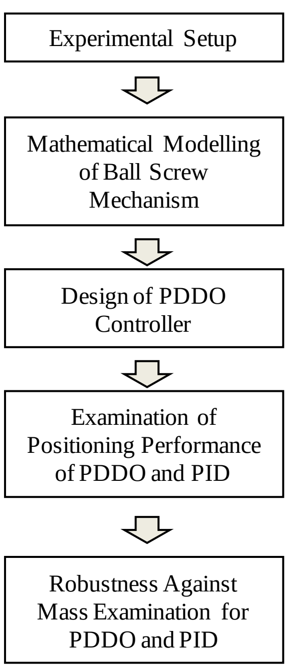
Graphical abstract



Abstract

Ball screw mechanisms are widely applied in different industries due to their capability in achieving precise positioning performance as well as its long travel range for positioning, travelling and contouring actions. However, this mechanism exhibits nonlinearities in micro movement. In this paper, a disturbance observer and PD controller (PDDO) is proposed in ball screw mechanism to achieve fast and precise positioning performance. A macrodynamic mathematical model of the mechanism is derived. PDDO controller is designed to achieve fast positioning in micro travel range. The robustness of the controller against mass is examined. The experimental results demonstrated that the PDDO controller achieves better performance in fast tracking (3 Hz) with working range at 100 μm, 1 mm and 3 mm as compared to the PID controller. Besides that, the PDDO controller also demonstrated its robustness in the presence of mass changes.

Keywords: Ball screw mechanism, disturbance observer, PD controller

Abstrak

Mekanisme skru bola sering digunakan dalam pelbagai industri kerana kebolehannya dalam mencapai prestasi ketepatan kedudukan yang tinggi dan pergerakan kontur serta jarak pergerakan yang panjang. Namun begitu, mekanisme ini menunjukkan sifat tidak linear dalam pergerakan mikro. Dalam kertas kerja ini, sebuah pengawal pemerhati gangguan dan pengawal PD (PDDO) telah dicadangkan untuk mekanisme skru bola bagi mencapai prestasi kedudukan yang tepat dan laju. Sebuah model makrodinamik telah diperoleh. Pengawal PDDO telah direka untuk mencapai prestasi kedudukan yang laju dalam pergerakan mikro. Keteguhan pengawal ini terhadap perbezaan berat beban juga dikaji. Hasil eksperimen menunjukkan bahawa pengawal PDDO mempunyai prestasi yang lebih baik dalam pergerakan laju (3 Hz) untuk jarak 100 μm, 1 mm, 3 mm berbanding dengan pengawal PID. Selain itu, pengawal PDDO juga teguh terhadap perbezaan berat beban.

Kata kunci: Mekanisme skru bola, Pengawal pemerhati gangguan, pengawal PD

1.0 INTRODUCTION

A ball screw mechanism is a mechanical actuator that translates rotary motion of the driver motor into linear displacement. It is widely applied in various automated industries such as semiconductor industries and CNC machineries due to its high stiffness, efficiency and capability in achieving high precision performance. However, past studies indicated that the mechanism exhibits nonlinear behaviours in micro movement that affects the positioning performances [1, 2]. Such behaviours are caused by hysteresis, Stribeck effect and nonlinear frictions along the screw shaft [3]. To model these non linear behaviours, Kim and Chung presented a friction model that describes the behaviour of ball screw mechanism [4]. In [5], Xiang, Qiu and Li modeled and described the non-linear frictions in the ball screw mechanism with LuGre friction model. In [2], Dong and Tang proposed a hybrid modelling with to demonstrate the dynamics of a ball screw mechanism.

From these researches, it can be seen that it requires huge effort to model the ball screw mechanism while considering all the microdynamic parameters. Due to this, many had suggested to lump the system parameters together [1, 6, 7]. This approach is also known as reduced order modelling or macrodynamic modelling. Though this modelling method is easier, a controller that can adapt to model mismatch and parameter variations is highly necessary. Such mismatch or variations are often considered as unwanted disturbances, and must be suppressed or removed in order to achieve high precision positioning performance.

Over the years, conventional controllers remain as the popular choice of industries due to their practical and simple applications. These conventional controllers, however, have to be tuned frequently due to the parameter variations or model mismatches. To overcome this limitation, different advance controllers such as H_∞ controller[8], sliding mode controller (SMC) [9], fuzzy logic controllers [7, 10] were proposed for positioning control of ball screw mechanisms. Despite their effectiveness of friction compensation and positioning control, Mendez Cubillos and de Souza pointed out that the H_∞ controller requires one to have deep understanding on this controller in order to design the form of weighting function [11]. A sliding mode control is often proposed for nonlinear control, but the discontinuous structure of the controller itself causes chattering problem and may lead to instability [12].

Unlike these controllers, a disturbance observer with PD controller (PDDO controller) appears to have a simple feedback structure. Despite its simplicity, the PDDO controller performs better than conventional controllers. A PDDO controller is capable of estimating the disturbances that arise from the nonlinear characteristics of the system and subsequently rejects these disturbances in order to achieve positioning accuracy. Differ from sliding mode controller, PDDO controller does not create chattering problem instability issue. The PDDO controller was first introduced by Ohnishi in 1983 to perform torque-speed regulation in DC motor [13]. In later years, PDDO controller was applied in various mechanisms such as robotic manipulators [14], hard disk servo system [15] and ball screw mechanisms. In [16], a PDDO controller was proposed to perform disturbance compensation on ball screw mechanism. This research has shown that PDDO is robust towards different form of disturbances, which are represented in sinusoidal and step disturbance signal. In another research, Ro *et al.* presented the performance of PDDO controller in achieving submicrometer positioning control in ball screw mechanism [17]. Ro *et al*. have pointed out that the proposed PDDO controller is capable of performing well in tracking motion though the friction estimation was not tuned accurately.

The main contribution of this research is to demonstrate the effectiveness of PDDO controller in achieving fast positioning control in ball screw mechanism. In order to do so, a macrodynamic modelling is first derived to demonstrate the dynamics involved in the ball screw mechanism. The design procedures of the PDDO controller are discussed. In this paper, the tracking performance of the PDDO controller is validated in fast sinusoidal input for different amplitudes, and compared with the conventional PID controller. Besides that, the robustness of PDDO controller against mass is also examined.

This paper is organized as follows: section 2 discusses the experimental setup of the ball screw mechanism. Section 3 derives the mathematical model of the ball screw mechanism and Section 4 explains the design procedures of the PDDO controller. Section 5 shows and compares the positioning performances of PID controller and PDDO controller while Section 6 summarizes this paper.

2.0 EXPERIMENTAL SETUP

In this paper, the ball screw mechanism is constructed as shown in Figure 1. The ball screw is driven by a DC servomotor (RS Component: RS 263-2011), with a back electromotive force constant of 0.09 Vs/rad. The ball screw lead is measured as 8mm/rev. A voltage amplifier with voltage gain of 2 is used to drive the motor. The input voltage to the amplifier is limited to ±10 V. The ball screw lead is given as 8 mm/rev with a maximum range of 160 mm. A linear encoder (Renishaw: RGH22A30L00A) with resolution of 5 μm is used in this system to measure the displacement of the table. The sampling frequency of the experiments is set as 1 kHz.

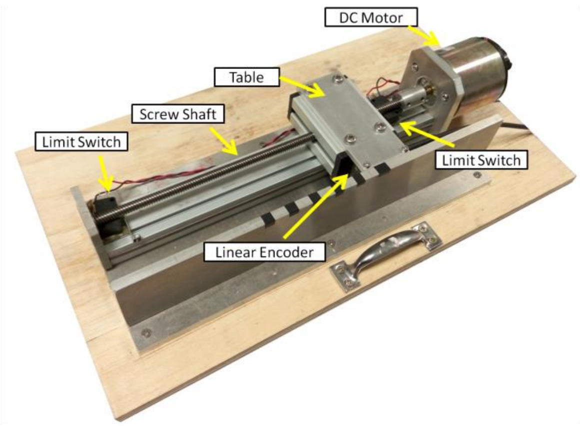


Figure 1 Ball screw mechanism driven by DC motor

3.0 MATHEMATICAL MODELLING

To construct mathematical model of the ball screw mechanism, a free body diagram describing the macrodynamics of the setup is shown in Figure 2. τ_m denotes the driving torque of the motor while J_m is the motor moment of inertia; D_m is the viscous friction of the motor; M is the mass of the table; and C_m denotes the viscous friction applied by the mass on the screw lead.

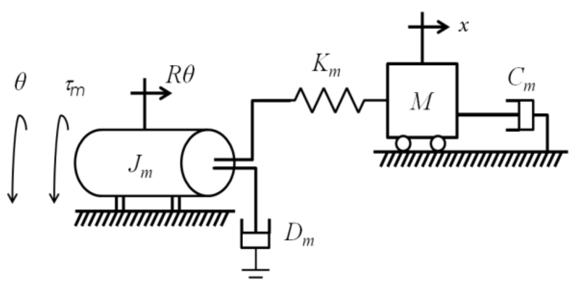


Figure 2 Free body diagram of ball screw mechanism

In this mechanism, the rotating screw shaft drives the table such that it moves in a linear manner. The contact of the screw shaft and table raises a contact force given as f_c. This force acts as a bridge in energy transmission from motor to the table movement. The contact force is dependent on the rotational stiffness of the screw shaft, K_m. The contact force then produces a driving force, f_d to move the table in a linear motion. The transmission ratio of motor rotation to linear displacement is given as $R = r/2\pi$, where r represents the ball screw lead. Thus, three equations of motion for each component are given in equation (1)-(3):

Motor:

$$J_m\ddot{\theta} + D_m\dot{\theta} + RK_m(R\theta - x) = \tau_m \qquad (1)$$

Contact force between screw shaft and table:

$$f_c = K_m(R\theta - x) \qquad (2)$$

Table:

$$M\ddot{x} + C_m\dot{x} - f_c = f_d \qquad (3)$$

Based on (1), (2), (3), a structure describing the dynamics of the mechanism is modelled as shown in Figure 3 while the parameters are summarized in Table 1. The dynamic mathematical form of the mechanism is represented as in equation (4):

$$\begin{bmatrix} J_m & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} D_m & 0 \\ 0 & C_m \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} R^2K_m & -RK_m \\ -RK_m & K_m \end{bmatrix}\begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} \tau_m \\ f_d \end{bmatrix} \qquad (4)$$

$$\begin{bmatrix} J_m s^2 + D_m s + R^2K_m & -RK_m \\ -RK_m & Ms^2 + C_m s + K_m \end{bmatrix}\begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} \tau_m \\ f_d \end{bmatrix} \qquad (5)$$

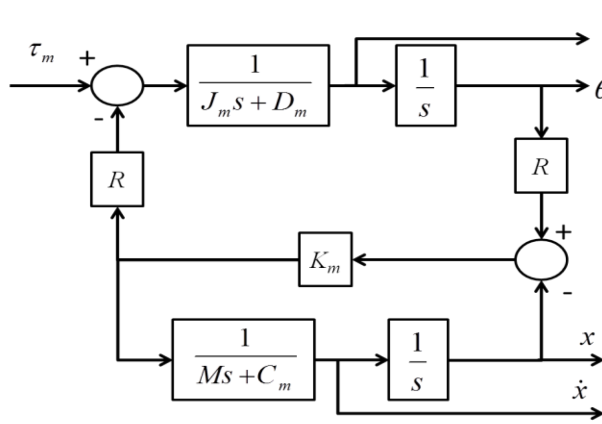


Figure 3 Block diagram of ball screw mechanism

Table 1 Parameters of the ball screw mechanism

Parameters	Description	Value	Unit
J_m	Motor inertia	$6\text{x}10^{-5}$	kgm^2
D_m	Motor Viscous Friction	$3.85\text{x}10^{-4}$	Nm/rad/sec
K_m	Rotational Stiffness of Screw Shaft	$1.82\text{x}10^{3}$	N/m
M	Mass of Table	$5\text{x}10^{-1}$	kg
R	Transmission Ratio of Rotary to Linear Motion	$1.273\text{x}10^{-3}$	m/rad
C_m	Viscous Friction applied by mass	$5\text{x}10^{1}$	N/m/sec

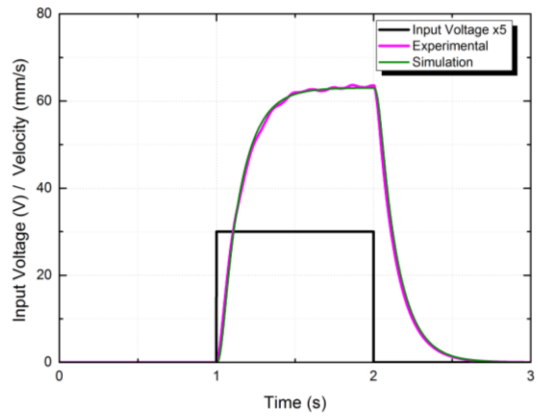


Figure 4 Comparisons of velocity curve

Figure 4 shows the comparisons for open loop response in simulation and experiment. It can be seen that the simulation results agree well with the experimental results.

In Figure 5, the comparisons of open loop frequency response of the experiment and simulation results are presented. It can be seen that the model is unable to capture the microdynamic behaviours, thus exhibiting the slight difference between the experimental and simulation results. Besides that, the unmodelled time and position varying frictions may also induce this phenomenon.

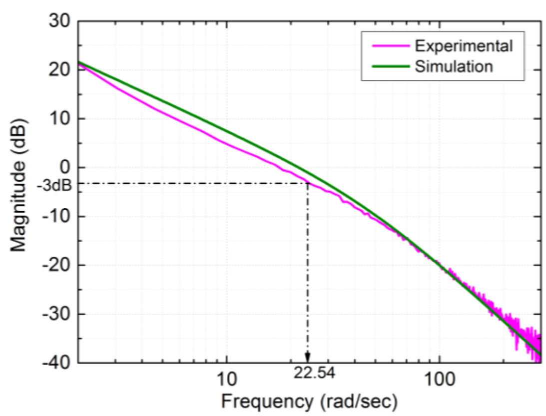


Figure 5 Comparisons of open loop frequency response for experimental and simulation results

4.0 DISTURBANCE OBSERVER WITH PROPORTIONAL DERIVATIVE CONTROLLER (PDDO) DESIGN

The PDDO controller demonstrated in Figure 6 consists of two major parts: the disturbance observer (DOB) and a PD controller. Within the DOB, an inverse nominal plant, $P_n^{-1}(s)$ and a low pass filter, $Q(s)$ is included. When the disturbance, d arises from the surrounding or due to parameter variations occurs in the system, the output of the mechanism is fed into $P_n^{-1}(s)$. By comparing u_o with the input to the mechanism, u, an estimate of the disturbance, $\hat{d}$ can be obtained and then fed back to the inner loop to perform disturbance rejection. Though theoretically agreeable, this direct estimation is infeasible as the inverse nominal plant is a non proper transfer function. To overcome this issue, a low pass filter, $Q(s)$ is added into the control structure. Since the inner loop DOB is only capable of disturbance rejection [18], a PD controller is included in the outer loop of the control scheme to satisfy the positioning performance of the ball screw mechanism.

To design the PDDO controller, the procedures began with the nominal plant, $P_n(s)$ determination. To model the nominal plant, a general second-order model is first considered:

$$P_n(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n + \omega_n^2} \tag{6}$$

where the poles, S_d are set according to desired specifications such that the overshoot percentage of the system is 2% and the settling time is 0.5 seconds. The equation of the poles, S_d is given as:

$$S_d = -\varsigma\omega_n \pm j\omega_n \tag{7}$$

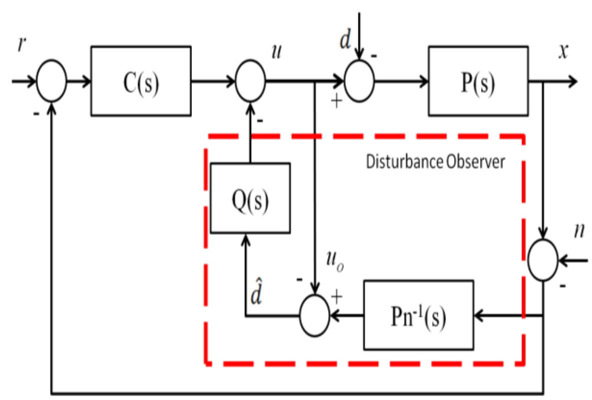


Figure 6 Block diagram of PDDO controller

In Figure 7, the structure of PDDO is presented. To design the observer, the observer gain, L is selected 5 times larger than the desired poles such that the estimation error will reduce to zero quickly, i.e. performing in fast response. The observer gain, L and state feedback gain, K is determined with Ackermann's formula (Appendix). With the Ackerman's formulation, the observer gain, L and state feedback gain, K are given as:

$$\begin{aligned} L &= [1600 \quad 64]^T \\ K &= [0 \quad 1] \end{aligned} \tag{8}$$

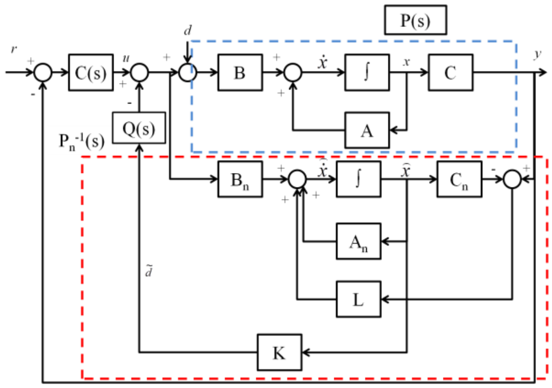


Figure 7 State space representation of PDDO

To design the low pass filter, *Q(s)* the cut off frequency, ω_d is determined from the frequency of P(s). The transfer function of *Q(s)* is given as:

$$Q(s) = \frac{\omega_d}{s + \omega_d} \quad (9)$$

The cut-off frequency of the low pass filter is selected such that the high frequency measurement noise, *n* can be attenuated. The filter is important in the system as it ensures that the inverse nominal plant is valid. The cut-off frequency of the low pass filter is selected as about half of the bandwidth of *P(s)*, where ω_d is 14 rad/sec. In the design of PD controller, the equation of PD controller is defined as follows:

$$C(s) = K_p + \frac{K_D s}{T_d s + 1} \quad (10)$$

From (10), a low pass filter with time constant, T_d is added to filter the measurement noise picked up from derivative part of the controller. In this filter, T_d is selected as $1/\omega_d$. The controller gain, K_p and K_d were obtained via root locus method with the desired settling time set as 0.5 seconds and overshoot percentage of 2%. The gains were then fine-tuned experimentally. The gains of PD controller are presented in Table 2.

5.0 EXPERIMENTAL RESULTS

According to Figure 5, the bandwidth of the ball screw positioning system is 22.54 rad/sec (3.6 Hz), thus the experiments were conducted at the limit of 3 Hz, which is considered relatively high frequency for the system. In this paper, the positioning performances of the PDDO controller were examined under high speed tracking motion within varying small amplitudes. The positioning performance of the controller was then compared with a conventional PID. The PID controller was designed based on equation (11), where the gains, K_p, K_I and K_D were obtained through root locus approach with desired specifications. The gains were then fine-tuned experimentally. The gains of the ID controller are shown in Table 2. The control structure of the PID controller is shown in Figure 8.

$$C(s) = K_p + \frac{K_I}{s} + K_D s \quad (11)$$

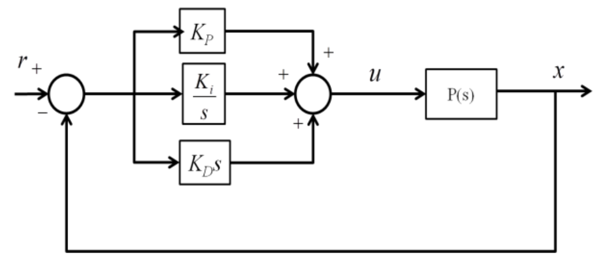


Figure 8 Control structure of PID controller

Table 2 The controller gains for PDDO controller and PID controller

Controller	K_P	K_I	K_D
PID	5	12	0.05
PDDO	10	-	0.5

Based on the tracking responses in Figure 9 to Figure 11, it can be observed that the PDDO controller performs better than the PID controller at high frequency. At high speed motion, the viscous friction of the system becomes significant. Since PID controller is very sensitive towards parameter variations, thus it has trouble following the reference signal and exhibits large tracking error.

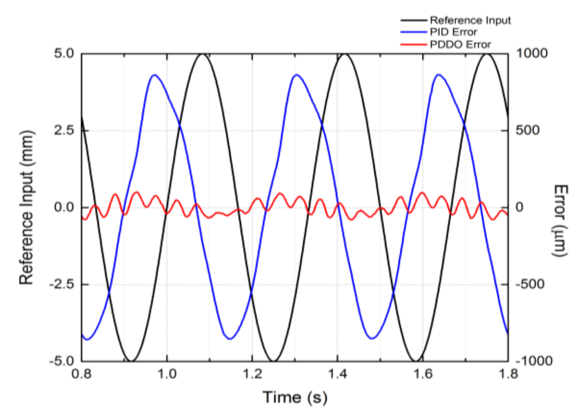


Figure 9 Experimental tracking results of the PDDO controller and PID controller at sinusoidal input of 3 Hz and 5 mm amplitude

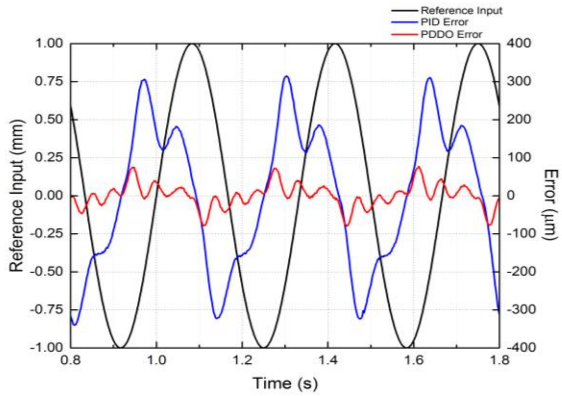


Figure 10 Experimental tracking results of the PDDO controller and PID controller at sinusoidal input of 3 Hz and 1 mm amplitude

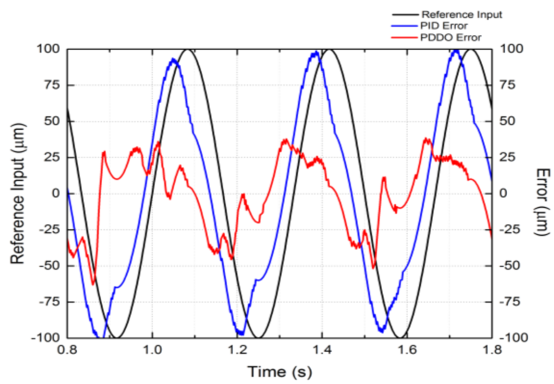


Figure 11 Experimental tracking results of the PDDO controller and PID controller at sinusoidal input of 3 Hz and 100 µm amplitude

In order to examine the robustness of the controllers under mass variations, loads of different mass are added to the table of ball screw mechanism. The first load is 1 kg, which is 2 times the table mass, whereas the second load is 5kg, which is 10 times the table mass. The experiments are conducted with a 3 Hz reference signal with amplitude of 100 µm. The outputs of the two controllers are presented in Figure 12 and 13.

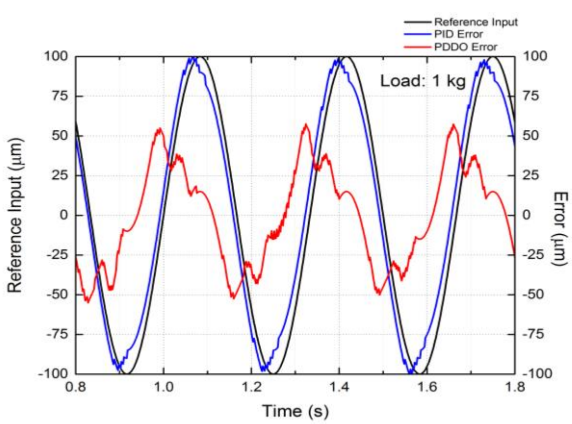


Figure 12 Experimental tracking results of PDDO and PID controller when subjected to 1 kg load in 3 Hz with 100 µm sinusoidal input

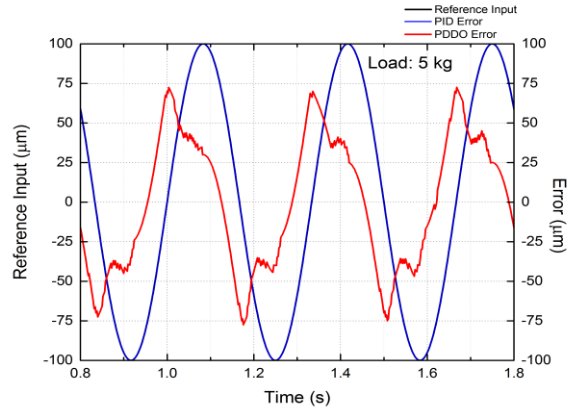


Figure 13 Experimental tracking results of PDDO and PID controller when subjected to 5 kg load in 3 Hz with 100 µm sinusoidal input

From the results, it can be observed when the 1 kg load is added, the performance of PID controller starts to deteriorate. When the load is further increased to 5kg, the PID controller is unable to cope with such large changes of parameter, i.e. mass, thus it remained stationary and produces such large error. Based on the tracking results of PDDO controller, despite the rise of error when the mass of load is increased, the PDDO controller is still capable of performing better than the PID controller. Unlike PID controller, the PDDO controller is less sensitive towards the changes of parameters, thus it produces better positioning performance than the PID controller. From these responses, it can be concluded that the PDDO controller is more robust towards mass changes than the PID controller.

6.0 CONCLUSION

In this paper, a ball screw mechanism was constructed and the macrodynamic modelling of the ball screw mechanism was derived. A PDDO controller was designed for the ball screw mechanism to achieve fast and precise positioning performance. The performance of the proposed controller was then compared with a PID controller in tracking motion. It is concluded that a PDDO controller has better tracking performance than PID controller at high speed and small working range. When the load mass is added to the mechanism, the PDDO controller is still capable of following the reference compared to PID controller. In other word, the PDDO controller is robust towards mass changes. However, it can be seen that the tracking error of the PDDO controller is still considerably large. Therefore, in future work, a feedforward controller is proposed to be added into the system in order to reduce the large tracking error that cannot be compensated by the PDDO controller.

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Appendix

To derive the state feedback gain, K and observer gain, L, consider the system where

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \quad \text{(A1)}$$

And given the state feedback equation as

$$u = -Kx \quad \text{(A2)}$$

Such that (A1) becomes

$$\dot{x} = (A - BK)x$$

$$\text{Let } \tilde{A} = A - BK \quad \text{(A3)}$$

Thus, the desired characteristic equations become

$$|sI - A + BK| = |sI - \tilde{A}| = (s - \mu_1)(s - \mu_2)\cdots(s - \mu_n) = 0 \quad \text{(A4)}$$

According to Cayley-Hamilton Theorem, $\tilde{A}$ meets its own characteristic equations, thus

$$\phi(\tilde{A}) = \tilde{A}^n + \alpha_1 \tilde{A}^{n-1} + \cdots \alpha_{n-1}\tilde{A} + \alpha_n I = 0 \quad \text{(A5)}$$

Consider the case of second order model, where n=2,

$$\begin{aligned}I &= I \\ \tilde{A} &= A - BK \\ \tilde{A}^2 &= (A - BK)^2 = A^2 - ABK - BK\tilde{A}\end{aligned} \quad \text{(A6)}$$

Multiplying (A6) with (A5), the equation is obtained

$$\begin{aligned}&\alpha_2 I + \alpha_1 \tilde{A} + \tilde{A}^2 \\ &= \alpha_2 I + \alpha_1 (A - BK) + A^2 - ABK - BK\tilde{A}\end{aligned} \quad \text{(A7)}$$

Referring to Equation (A5)

$$\begin{aligned}&\phi(\tilde{A}) = \tilde{A}^2 + \alpha_1 \tilde{A} + \alpha_2 I = 0 \\ &\text{and} \\ &\phi(A) = A^2 + \alpha_1 A + \alpha_2 I \neq 0\end{aligned} \quad \text{(A8)}$$

Substituting (A7) into (A8)

$$\phi(\tilde{A}) = \phi(A) + \alpha_1 (A - BK) + A^2 - ABK - BK\tilde{A} = 0 \quad \text{(A9)}$$

Given $\phi(\tilde{A}) = 0$, therefore

$$\phi(\tilde{A}) = B(\alpha_1 K) + ABK = [B \vdots AB]\begin{bmatrix}\alpha_1 K \\ K\end{bmatrix} \quad \text{(A10)}$$

Since system is controllable, multiplying the (A10) at both side with inverse matrix gives

$$[B \vdots AB]^{-1}\phi(A) = \begin{bmatrix}\alpha_1 K \\ K\end{bmatrix} \quad \text{(A11)}$$

Through multiplication on both side with [0 0 1], K is obtained as

$$[0 \quad 1][B \vdots AB]^{-1}\phi(A) = \begin{bmatrix}\alpha_1 K \\ K\end{bmatrix}[0 \quad 1] = K \quad \text{(A12)}$$

To obtain observer gain, L, for the system derived in equation (A1), the previous equation in (A12) is modified as

$$[0 \quad 1][B^* \vdots AC^*]^{-1}\phi(A^*) = K_1 \quad \text{(A13)}$$

Where K_1* gives the observer gain matrix, L.