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# OPTIMAL SHAPES OF WEIRS FOR TRAPPING MIGRATORY FISH

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**Graphical abstract**

**Abstract**

Use of fishing weirs can be seen throughout the world to trap migratory fish in rivers. In rivers flowing into Lake Biwa, Shiga prefecture, Japan, a peculiar type of traditional fishing weir has been operated, targeting anadromous fish species such as Plecoglossus altivelis and Oncorhynchus masou rhodurus. That type of fishing weir has the advantage of catching fish alive, adding an extra value to the fish. Hence the structure of such a weir is considered to be historically optimized so that physiological damages to the fish are minimum. In this study, assuming that the horizontal shape is designed to minimize the traveling time of fish from any point along the downstream side of the weir to the gate of no return situated at the bank, as well as to minimize the size of structure, a mathematical problem is formulated in the framework of dynamic programming to determine the optimal shape. Geometric consideration results in the traveling time as a functional of the shape, whose slope of the tangent is dealt with as the control variable. The value function and the optimal control solve the Hamilton-Jacobi-Bellman equation, which represents the principle of optimality. The system of the Hamilton-Jacobi-Bellman equation is finally reduced to an ordinary differential equation with an initial condition. Some computational results are in good agreement with the actual shapes of the fishing weirs installed across the rivers flowing into Lake Biwa. This mathematical approach is also applicable to other problems such as optimal design of fish ladders.

*Keywords*: Fishing weirs, Shapes, Migratory fish, Optimal control, Hamilton-Jacobi-Bellman equation



## 1.0 INTRODUCTION

Fishing weirs are traditionally used for trapping migratory fish, and many types of the fishing weirs are operated around the world. Fishing by means of the weirs has been playing an important role for a long time to harvest animal protein without damaging local environment. The shapes and materials of fishing weirs differ, depending on geomorphological, social and ecological conditions. Erickson (2000) studied a complex artificial network of hydraulic earthworks covering 525 km$^2$ in the Baures region of Bolivia to categorize particular zigzag earthwork structures used for fishing, based on form, orientation, location, association with other hydraulic works, and ethnographic analogy [1]. Tveskov *et al*. (2003) focused on estuarine wood stake fishing weirs in the southern Cascadia coast and summarized native oral traditions and archaeological researches indicating that fishing by means of the weirs had been primarily household, day-to-day activity with a wide range of anadromous and resident target fish species [2]. Operation of fishing weirs in Japan is important not only for catching fish as food but also for developing tourism industry and fisheries. Landscapes with fishing weirs in rural areas themselves come to special attractions for tourists. Some types of fishing weirs are physiologically least damaging the fish, which have an extra gastronomic value and also can be utilized in pisciculture or stocked in other rivers for maintenance of fishery resources.

This study focuses on the fishing weirs installed across the rivers flowing into Lake Biwa, Japan. Since that type of fishing weirs has been traditionally operated for a long time, it is assumed that the horizontal shapes are designed to optimize performance of the structures in terms of harvesting rate, stress to the fish, structural strength, and so on. A mathematical problem is formulated in the framework of dynamic programming to determine the optimal shape, and the governing Hamilton-Jacobi-Bellman (HJB) equation is presented and solved.

## 2.0 FISHING WEIRS AROUND LAKE BIWA

In Japanese rivers, two types of fishing weirs are common: Kudari-yana and Nobori-yana. Kudari-yana is set in rapid flows and targeting catadromous fish species. In major rivers flowing into Lake Biwa, a peculiar type of traditional fishing weir called Nobori-yana has been operated [3]. This type of fishing weir is targeting anadromous fish species such as *Plecoglossus altivelis* and *Oncorhynchus masou rhodurus*. *P. altivelis* caught alive is mainly transported to other basins for being stocked. *O. masou rhodurus* is a species endemic to Lake Biwa. In order to conserve the species, eggs are collected from adult *O. masou rhodurus* caught alive by the fishing weirs, before hatched artificially and then stocked in the river for growth.

We focus on Kattori-yana (Figure 1) which is a kind of Nobori-yana in the rivers flowing into Lake Biwa. This traditional type of fishing weir has unique features for trapping anadromous fish. Fish ascending to downstream side of the weir jump toward upstream, hit against the weir and then get back to downstream. Repeating this jumping and bouncing, fish move toward the bank. At the bank, a special structure is set in order to catch fish alive without failing. The flow from an edge of the weir is divided into two directions by a plate; one is to the downstream of the river and the other is to the gate of no return, called Kattori-guchi (Figure 2). Fish jumping toward the vicinity of the edge of the weir are reflected toward the gate, and then kept alive in the fish preserve.

Different horizontal shapes of Kattori-yana are found in the different rivers flowing into Lake Biwa. We hypothesize that this difference is due to different intention of design, such as durability of weirs themselves or efficiency for the catchment of fish. Here, the traveling time of fish from any point along the downstream side of the weir to the gate of no return situated at the bank, as well as the size of structure, is considered as the performance index to be minimized in dynamic programming.

**Figure 1** A Fishing weir in a river flowing into Lake Biwa (http://shigabun4.shiga-saku.net/e568476.html)


**Figure 2** Gate of no return called Kattori-guchi

## 3.0 OPTIMAL CONTROL MODEL

A mathematical problem is formulated in the framework of dynamic programming to determine the optimal shape. Geometric consideration results in the traveling time as a functional of the shape, whose slope of the tangent is dealt with as the control variable. Figure 3 shows the schematic diagram for trajectory of an indivisual fish in a fishing weir. A Cartesian coordinate system is taken so that the $x$-axis directs the right bank of the river across the flow and the $y$-axis directs downstream. The right bank B, where the gate of no return is installed, is located at $x = B$. The shape is represented as a function $y = f(x)$. An ascending fish at $f(x) + L$, where $L$ is the representative distance of a jump of the fish, jumps toward the structure. Then, the fish is reflected to another position closer to the right bank. That position is assumed to be the distance $L$ apart from the structure, and the trajectory has the angle $2\alpha$, where $\alpha$ is the angle between the $y$-axis and the normal vector of $f(x)$ at the hitting point of the fish. The resulting moving distance $\Delta x$ of the fish is deduced as

$$\Delta x = -\frac{2f'}{1+f'^2}L . \tag{1}$$

For the fish jumping $C/L$ times per unit time, the moving dynamics is governed by

$$\mathrm{d}x = -\frac{2Cu}{1+u^2}\mathrm{d}t \tag{2}$$

where $t$ is the time, and the slope $f'$ is dealt with as the control variable $u$. The fish positioned at $x$ at the current time $s$ arrives at the gate of no return at the first exit time $\tau$ such that

$$\int_s^\tau -\frac{2Cf'}{1+f'^2}dt = B . \tag{3}$$

The traveling time of fish from $x$ to the gate of no return is

$$\tau - s = \int_s^\tau 1dt \tag{4}$$

where $s$ is the current time. While, the shape $f(x)$ of structure is also given by

$$f(x) = f(B) + \int_B^x u\mathrm{d}x = \int_s^\tau \frac{2Cu^2}{1+u^2}\mathrm{d}t . \tag{5}$$

An optimal control problem is formulated to maximize the performance index with the weight $\varepsilon$

$$J^u(s,x) = -(\tau - s) - \varepsilon f(x) = \int_s^\tau \left(-1 - \frac{2\varepsilon Cu^2}{1+u^2}\right)\mathrm{d}t . \tag{6}$$

A control variable $u = u(s,x)$ that achieves the maximum of the performance index $J^u(s,x)$ is referred to as the optimal control $u^*$, while the value function $\Phi = \Phi(s,x)$ is defined as

$$\Phi(s,x) = J^{u^*}(s,x) = \sup_u J^u(s,x) . \tag{7}$$

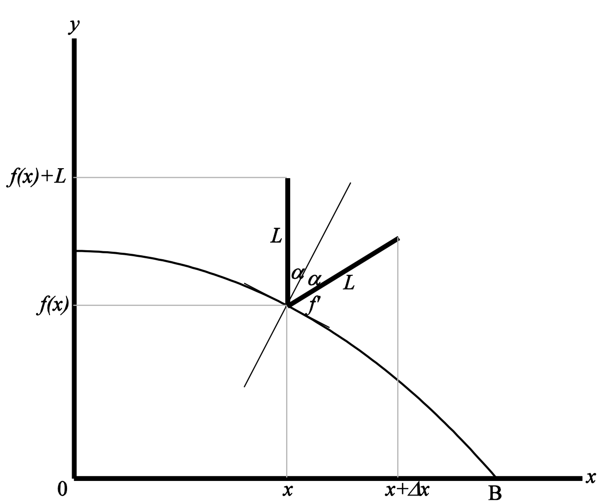


**Figure 3** Schematic diagram of the model

According to [4], the value function $\Phi$ and the optimal control $u^*$ are governed by the HJB equation

$$\begin{aligned} &\frac{\partial \Phi}{\partial s} - \frac{2Cu^*}{1+u^{*2}}\frac{\partial \Phi}{\partial x} - 1 - \frac{2\varepsilon Cu^{*2}}{1+u^{*2}} \\ &= \sup_u \left\{ \frac{\partial \Phi}{\partial s} - \frac{2Cu}{1+u^2}\frac{\partial \Phi}{\partial x} - 1 - \frac{2\varepsilon Cu^2}{1+u^2} \right\} = 0 \end{aligned} . \tag{8}$$

A steady solution $u^*$ to (8), if any, represents the optimally designed shape of the fishing weir. Therefore, the ordinary differential equation

$$\frac{\mathrm{d}\Phi}{\mathrm{d}x} = \Phi' = -\frac{1+(2\varepsilon C+1)u^{*2}}{2Cu^*} \tag{9}$$

with the initial condition

$$\Phi(B) = 0 \tag{10}$$

is examined. The optimal control is analytically computed as

$$u^* = -\frac{\Phi'}{\varepsilon + \sqrt{\varepsilon^2 + \Phi'^2}} . \tag{11}$$

For any point $x$ (9) and (11) constitute a non-linear algebraic equations system, which is solved at each step of numerical integration of (9) with (10).

## 4.0 COMPUTATIONAL RESULTS AND APPLICATION

Specifying five different values of $\varepsilon$ as 0, 0.1, 1, 10, and 100, the system consisting of (9), (10), and (11) is computed for six different functions of jumping rates $C_0 = 1$, $C_1 = 1 + B/2$, $C_2 = 1 + x$, $C_3 = 1 + (B - x)$, $C_4 = 1 + (B - x)^2$, and $C_5 = \min(1 + (B - x)^2, 1 + (B/5)^2)$ with $B$ = 100 m. The varying jumping rates $C_2$, $C_3$, $C_4$, and $C_5$ assume cross-sectional heterogeneity of the flow fields in the river. The computational grid size is set as $B/1000$.

The computational results of the value function $\Phi$, the optimal control $u^*$, and $y$ for the different $\varepsilon$ and $C$ are shown in Figures 4–9. For every case of $C$, $\Phi$ increases monotonically as $x$ increases. As the weight $\varepsilon$ increases, $\Phi$ decreases and $u^*$ increases monotonically, and $u^* = -1$ with the weight $\varepsilon = 0$. When $C$ is constant as $C_0$ and $C_1$, $u^*$ is constant, and the optimal shape is a straight line for every $\varepsilon$. $\Phi$ and $u^*$ for $C_0$ are smaller than those for $C_1$ for any $x$ for $\varepsilon > 0$. For varying $C_2$, $u^*$ is monotone increasing convex upward for $\varepsilon > 0$. For varying $C_3$ and $C_4$, $u^*$ is monotone decreasing convex upward, and $\Phi$ and $u^*$ for $C_3$ are smaller than those for $C_4$ for any $x$ for $\varepsilon > 0$. The resulting optimal shapes are convex downward for $C_2$, and convex upward for $C_3$ and $C_4$. For $C_5$, $u^*$ levels off when $0 \le x \le 0.8B$, and monotone decreasing convex upward when $0.8B \le x \le B$. The resulting optimal shape is a straight line when $0 \le x \le 0.8B$ and monotone decreasing convex upward when $0.8B \le x \le B$.

The shapes of the fishing weirs actually found i n the rivers flowing into Lake Biwa are gene rally classified into two types. One has a straight line shape except in the vicinity of the gate of no return as shown in Figure 10 (Ane River), and the other has a dominantly curved shape as shown in Figure 11 (Ado River). The optimal shapes for $C_5$ is in good accordance with the fishing weirs shown in Figure 10 if $\varepsilon$ is taken as 0.005. However, the curved shape as shown in Figure 11 is not computationally reproduced in any case of $C$. The operators of the weir shown in Figure 11 claim that the curved shape has been established so as not to be destroyed by flood events. The balance between the tension applied to the structure and the forces from the water flows might better ex plain that catenary-like shape, rather than the assumption in this study.

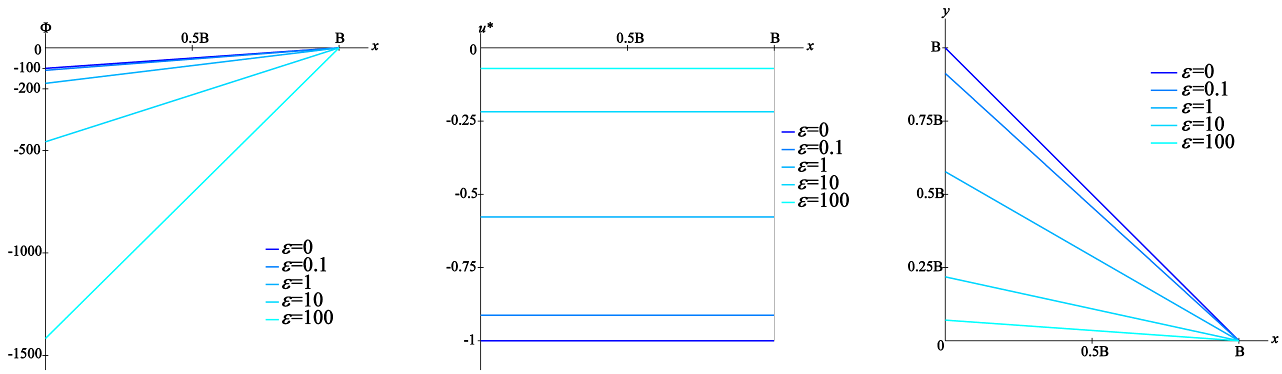


**Figure 4** Computational results of $\Phi$, $u^*$, and $y$ for $C_0$

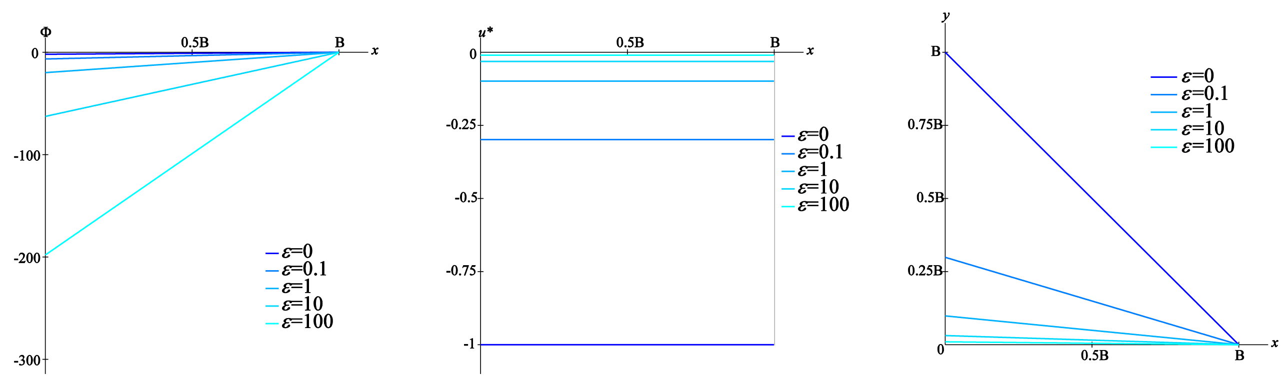


**Figure 5** Computational results of $\Phi$, $u^*$, and $y$ for $C_1$

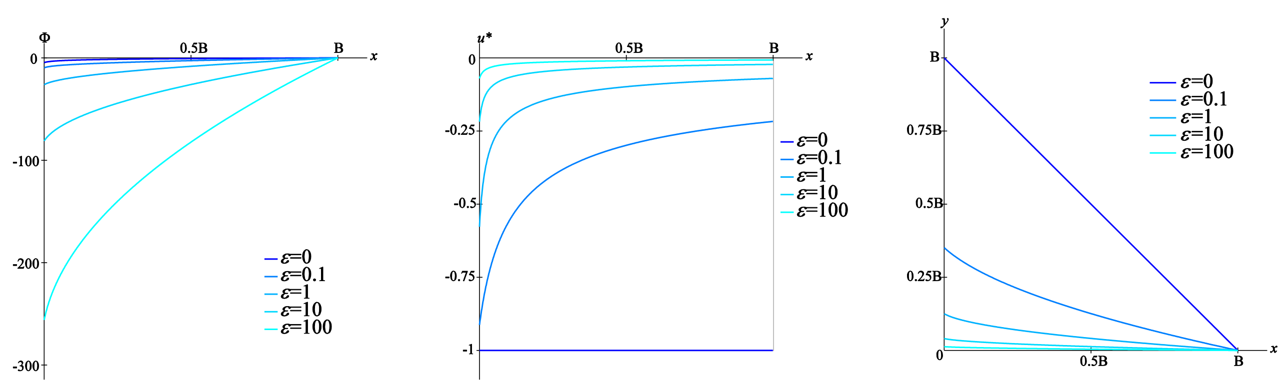


**Figure 6** Computational results of $\Phi$, $u^*$, and $y$ for $C_2$

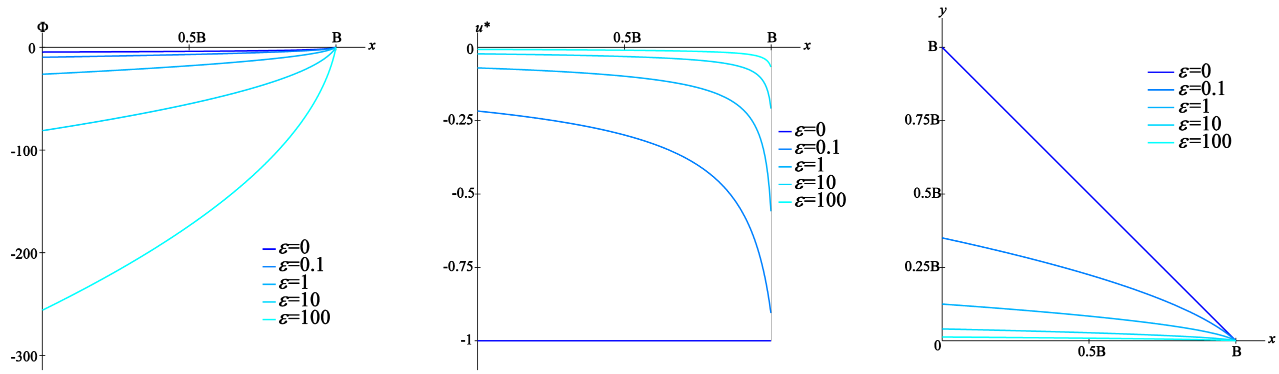


**Figure 7** Computational results of $\Phi$, $u^*$, and $y$ for $C_3$

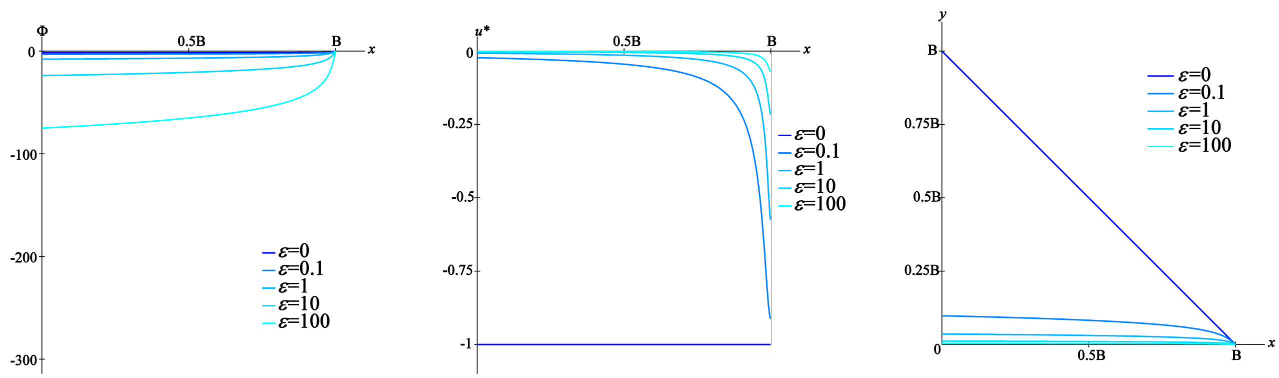


**Figure 8** Computational results of $\Phi$, $u^*$, and $y$ for $C_4$

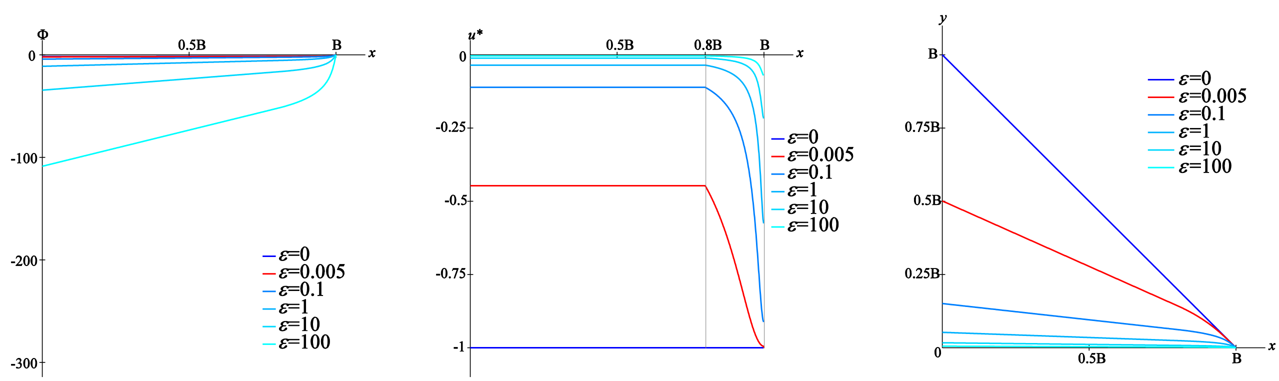


**Figure 9** Computational results of $\Phi$, $u^*$, and $y$ for $C_5$


**Figure 10** Fishing weir in Ane River (Map Data: Google, DigitalGlobe)


**Figure 11** Fishing weir in Ado River (Map Data: Google, DigitalGlobe)

## 5.0 CONCLUSIONS

The optimal shapes of the peculiar type of fishing weir are discussed, assuming that the horizontal shape is designed to minimize the traveling time of fish as well as to minimize the size of weir itself. The slope $f'$ of the shape function is dealt with as the control value $u$ , and the optimal shapes are deduced as the solutions of HJB equations, which represent the principle of optimality. The computational results indicate that a varying jumping rate $C$ controls curvature of the optimal shape. The optimal shape for $C_5$ with $\varepsilon$ = 0.005 well explains the actual shape of the fishing weir installed in Ane River.

For further studies, the performance index shall be improved to include mechanical interaction between the structure and water flows considering hydro-environmental characteristics of the river. Then, this mathematical approach shall be applicable to other problems such as optimal design of fish ladders to facilitate fish migration without hindering their hydraulic and structural functions.

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