Full Paper

SALAM CONTRACT WITH CREDIT RISK MODEL BY PARTIAL DIFFERENTIAL EQUATION APPROACH

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Graphical abstract

Introduction of Islamic derivative

Propose *salam* contract as Islamic derivative

Mathematical models for forwards and futures

Credit risk models

Set up the model for commodity *salam* contract with credit risk

Deriving the partial differential equation for the value of commodity *salam* contract with credit risk

Abstract

In the acute phase of global financial crisis, the risk management issue has become a major subject that attracts the interest of many financial institutions. Risk management in Islamic finance is proven to be more challenging than the conventional due to shariah principals and regulations. Therefore, there is a need for an alternative Islamic derivative product that can compete with the existing conventional derivatives. This study proposes a traditional Islamic contract, which is *salam,* that can be built as a new Islamic derivative product. Since there is lack of quantitative study regarding *salam* contract implementation, this study introduces a mathematical model of commodity *salam* contract by considering credit risk element. The structural approach is the best credit risk model to describe the structure and properties of *salam* contract. However, because of the unique structure and boundary condition of *salam* contract, some adjustments need to be considered. In deriving the partial differential equation that describes the dynamic behaviour of commodity *salam* contract with credit risk, the risk neutral valuation was employed.

Keywords: *Salam*, commodity *salam*, partial differential equation, credit risk, Islamic derivative

Abstrak

Dalam menghadapi krisis kewangan global yang kian meruncing, isu pengurusan risiko menjadi sebuah subjek penting bagi kebanyakan institusi kewangan. Pengurusan risiko dalam kewangan Islam terbukti lebih mencabar berbanding dengan konvensional disebabkan oleh prinsip dan peraturan syariah yang perlu dipatuhi. Lantaran itu, terdapat satu keperluan dalam pembangunan produk terbitan kewangan Islam yang boleh menandingi terbitan kewangan konvensional sedia ada. Kajian ini mencadangkan kontrak Islam tradisional, iaitu *salam,* yang boleh dibangunkan sebagai satu produk terbitan kewangan Islam yang baharu. Disebabkan terdapat kurang kajian kuantitatif berkenaan pelaksanaan kontrak *salam*, kajian ini memperkenalkan sebuah model matematik kontrak *salam* komoditi yang turut mempertimbangkan unsur risiko kredit. Pendekatan struktural merupakan model risiko kredit yang paling sesuai bagi memperincikan struktur dan ciri-ciri kontrak *salam*. Bagaimanapun, disebabkan oleh struktur dan syarat sempadan yang unik bagi kontrak *salam*, beberapa pelarasan perlulah dipertimbangkan. Dalam menerbitkan persamaan pembezaan separa yang menerangkan tingkah laku dinamik kontrak *salam* komoditi dengan risiko kredit, kaedah penilaian berkecuali risiko telah digunakan.

Kata kunci: Salam, salam komoditi, persamaan pembezaan separa, risiko kredit, terbitan kewangan Islam

1.0 INTRODUCTION

In facing the increasing globalisation pressure, the effective and efficient risk management tools are crucial to aid financial institutions in managing risks. The literature suggests that risk management is more challenging in Islamic finance as compared to conventional as the former is exposed to additional risks due to its features and nature of contracts [1-3]. Under shariah principles and regulations, all financial instruments must adhere to five shariah elements which do not involve *riba* (interest paid), *rishwah* (corruption), *gharar* (uncertainty or unnecessary risk), *maysir* (speculation or gambling) and *jahl* (trading upon the counterparty's ignorance) [1, 4-6]. Due to the needs for shariah compliance, Islamic finance has a limited risk management tools as compared to the conventional.

Derivatives have proven to be one of the most popular and effective risk hedging instruments in the financial institutions, however they remain controversial in the Islamic finance. This situation arises because of the difference between shariah scholars' perception on derivatives. According to Bacha [5], some of the traditional Islamic contracts are similar to the conventional derivatives. Therefore, the Islamic scholars' objection against derivatives need to be reviewed. In Malaysia, Shariah Advisory Council (SAC) [7] has endorsed three shariah-compliant derivatives namely crude palm oil futures, crude palm kernel oil futures and single stock futures. SAC has also approved the mechanism of stock index futures as long as the component comes from shariah-compliant securities. Islamic derivative transaction has been accepted in Malaysia since March 2010 [8]. This development shows that Islamic derivatives have gained positive feedbacks from the Islamic scholars when they are proven to be shariah-compliant.

For Islamic finance to remain competitive as conventional, more sophisticated products and risk management instruments such as derivatives are required. There are several Islamic contracts that can be developed as Islamic derivatives such as *salam*, *urbun*, *istijrar*, *wa'ad* and *murabaha* [5, 3, 9]. However, this study will focus on *salam* as there are discussions on acceptability of *salam* in the literature [1, 3, 10-12].

The preliminary concepts and qualitative studies of *salam* contract are reviewed. However, since there is lack of quantitative study on *salam* contract implementation, this study introduces a mathematical model of commodity *salam* contract by considering the credit risk element. To provide insight regarding the development of commodity *salam* contract as an Islamic derivative, the conventional forward and futures models are explored. Then, basic ideas on credit risk models are also discussed.

Salam contract has existed long time ago. Based on the hadith narrated by Bukhari and others, in Madinah Prophet Muhammad had observed the practice of people paying the price of dates that would be delivered within one, two or three years in advance [1].

Prophet Muhammad (PBUH) said: "*Whoever pays money in advance should pay it for a known quality, specified measure and weight along with the price and the date of delivery.*"

This hadith clarifies that *salam* contract is permissible in Islam on condition that it specifies the quality, measurement, weight, price and delivery date in the beginning of the contract.

Salam is the traditional Islamic contract that allows deferred sale. It is an agreement between the buyer and seller who agree to carry out a transaction in future (maturity) but at a price that is agreed earlier, in which a full payment is made at the beginning of the contract [3, 6]. In fact, the mechanism of *salam* contract resembles forwards and futures despite the full payment is settled up front in *salam* transaction. The condition of the full advance payment at the beginning of the contract will eliminate the *maysir* (speculation) element in the *salam* contract. The full advance payment also helps to finance the seller with working and extension capital, reduce leverage by minimising speculation and no default risk from the buyer.

In this *salam* transaction, the predetermined price is called *salam* price $\overline{S(0)}$ [1, 5, 6, 10, 11, 13]. Since the full *salam* payment is made at the beginning of the contract, the *salam* price is expected to be lower than market price if the sale is conducted via cash (spot) at maturity to compensate the buyer [5, 13]. This price behaviour of *salam* contract is distinguishable from the conventional forwards and futures because the price of both forwards and futures are higher than the spot at maturity [5]. Yaksick [13] in his study explained that the payoff or value of *salam* contract $F(S(t),t)$ at maturity or delivery date $t = T$ is given by:

$$F(S(T),T) = S(T) - \overline{S(0)} \tag{1}$$

where $S(T)$ is the underlying asset spot price at maturity. From equation (1), this *salam* agreement has gain if the underlying price at maturity is priced greater than the *salam* price $S(T) > \overline{S(0)}$ and incurs loss if the spot price at maturity is less than the *salam* price $S(T) < \overline{S(0)}$.

Since there are similarities of the mechanism between *salam* contract and conventional forwards and futures, some comparative studies are done. Bacha [5] evaluated the conventional derivatives' contract such as forwards and futures as well as Islamic finance instruments such as *salam* and *istijrar* contracts. Finally, it is concluded that the customised nature of *salam* contract is close to forwards rather than futures since the final condition or value of forwards at maturity $V(S(T),T)$ is quite similar to *salam* contract, indicated by:

$$V(S(T),T) = S(T) - K \qquad (2)$$

in which K is the fixed delivery price of forwards or the forward price [14]. However, the difference between forwards and *salam* contract is that the buyer will pay the fixed delivery price at contract maturity for forward transaction while in the *salam* contract, the buyer has to pay the full amount of the predetermined price during the contract initiation. This unique pricing behaviour of *salam* contract eliminates *gharar* (uncertainty) and *maysir* (speculation) elements that exist in forward contract. For a clearer vision on *salam* contract, there is an analogy of a simple transaction between a farmer who needs some working capital to harvest the commodity in future and a manufacturer who needs the commodity for his future production. Clearly, both parties face uncertain movement of the commodity price in future, which is known as price risk. The farmer worries the drop of the commodity price in future while the manufacturer is concerned with the increase of commodity spot price in future. Therefore, to eliminate the *gharar* (uncertainty) element in the transaction between them, both parties can proceed with a *salam* contract that entitles the farmer to deliver the commodity in future with a predetermined price, which is mutually agreed and fully paid at the beginning of the contract. At the initiation of the contract, the farmer will receive a predetermined price based on *salam* contract regardless of what happens to the commodity asset price in future. The manufacturer too has eliminated the price risk by fully paying a predetermined price at the beginning of the contract regardless of spot price in future. Since the manufacturer needs to hand over the full amount of payment at the initiation of the contract, the predetermined price is expected to be lower than the spot price if it involves a cash sale at the time of delivery [1, 5, 6, 10, 11, 13, 15]. The lower predetermined price as compared to spot price at maturity is the compensation given by the farmer to the manufacturer, which is a privilege for the latter [5, 15]. The price behaviour of *salam* contract is different from forwards, where the forward price is commonly higher than the spot price due to carrying cost [5, 15]. Furthermore, the *maysir* (speculation) element is also eliminated in *salam* contract transaction since the predetermined amount is paid at the beginning of the contract. Therefore, it will overcome the price speculation since the price of *salam* contract is mutually agreed and paid up front. Otherwise, the speculator can acquire substantial gains or losses from the speculative strategy in the forward and futures contracts, since the delivery price for both contracts are only paid at maturity [16].

Not only that, Ebrahin and Rahman [17] had investigated the pareto optimality of a synthetic futures over *salam* contract. The synthetic futures is created by combining the futures of permissible Islamic commodities and cost-plus sale contract (*Bai-Murabaha*). The proposed synthetic contract has outweighed *salam* in terms of efficiency and welfare issues. However, the result is contrary to the intuition that under competitive markets, arbitrage-free first order condition leads to pareto neutral (arbitrage-free) of both contracts. Bacha [6] once again reexamined the issues of conventional derivative instruments and shariah-compliant contract. He finds out that the payoff profile of *salam* contract is equivalent to the derivative instruments.

Aside from that, there are studies that propose *salam* contract as a new financing mode. Dali and Ahmad [18] proposes the application of *salam* in dinar economy to reduce price uncertainty and use capital from the advance prepayment rather than hedging the price. Then, Susanti [19] proposes the *salam* contract implementation in *murabahah* financing agreement to improve and introduce a new Islamic banking product. Putri and Dewi [20] evaluated a case study of *salam*-based financing product development in Indonesian Islamic rural bank. They found out that *salam*-based financing is feasible if the current *salam* financing mechanism is adjusted according to shariah. In addition, Muneeza, Nurul Atiqah Nik Yusuf and Hassan [21] highlights the potential of *salam* application in Malaysian banking industry to help farmers in terms of financing aspect with working capital. They also suggest a feasible way to implement *salam* contract in Malaysia.

In general, similar to forwards and futures, *salam* contract is an agreement between two parties who agree to carry out a transaction at a particular time in the future for a certain price that is agreed earlier. In terms of delivery, all the contracts shared similar delivery time, which is at maturity. However, the contracts are traded diffferently, especially in regards to the payment time. The forward contract is an over-the-counter (OTC) instrument in which its predetermined delivery price (forward price) is only paid at the maturity [14, 16]. Therefore, no money is transferred until the delivery date or at maturity. In contrast, the futures contract is usually traded on an exchange. The profit and loss for future position is calculated everyday and the change of value is paid from one party to another [14]. Thus, there is a gradual fund payment from the beginning to maturity in futures contract. Since the changes in the value of futures contract are settled everyday, its value remains zero throughout its life [14, 16]. The final condition or the value of futures contract at maturity $f(S(T),T)$ is given by [14, 22]:

$$f(S(T),T) = S(T) \qquad (3)$$

Equation (3) describes that although the futures price (predetermined delivery price) varies everyday, its value is settled daily. Hence, the value of futures contract at maturity must be similar to underlying asset price at maturity. The main criteria that makes *salam* contract different from forwards and futures is that the buyer needs to hand over the entire amount based on predetermined price at the contract initiation. Therefore, to compensate the buyer, the

predetermined price is expected to be lower than spot price at maturity [1, 5, 6, 10, 11, 13, 15].

In modelling a new mathematical model of *salam* contract, this study carried out an extensive review on mathematical models of conventional derivatives pertaining to forwards and futures. Forward and futures price are equal if the interest rate is nonstochastic [16, 23, 24]. There are several models that are used to predict price for forwards and futures. The cost of carry model is first formalised by Kaldor [25] and Working [26, 27]. It is developed based on arbitrage argument that the forward and futures price is equal to the spot price plus the carrying cost [23]. Under a multi-period economy condition, Chow *et al.* [23] have formalised the cost of carry model as:

$$f(t, t+k) = S(t)(1 + R(t, t+k) + W(t, t+k) - C(t, t+k) \qquad (4)$$

where $f(t, t+k)$ is the futures price at time t for delivery at time $t+k$, $R(t, t+k)$ is the risk-free rate over the period $(t, t+k)$, $W(t, t+k)$ is the marginal storage cost from time t to $t+k$ and $C(t, t+k)$ is the marginal convenience yield over k periods.

Instead of assuming a constant interest rate, Ramaswamy and Sundaresan [28] proposed a stochastic interest rate model for futures. It is assumed that the spot price follows a diffusion process:

$$dS(\mu - d)Sdt + \sigma_1 SdW_1 \qquad (5)$$

where μ is the drift rate, d is the dividend yield rate, σ_1 is the spot price volatility and W_1 is the Wiener process of spot price. The instantaneous risk-free interest rate is assumed to follow a mean reverting square root process that is given by:

$$dr = \kappa(u - r)dt + \sigma_2\sqrt{r}dW_2 \qquad (6)$$

where κ is the adjustment speed of interest rate, u is the long run mean of interest rate, σ_2 is the interest rate volatility and W_2 is the Wiener process of interest rate. Both Wiener processes are assumed to be correlated. Since there is no closed-form solution for this model, it needs to be solved numerically, subjected to the appropriate boundary condition as in (3) [29]. However, Ramaswamy and Sundaresan [28] had obtained a closed-form solution for a case when the correlation is equal to zero. The solution is given by:

$$f(t) = Sa(\tau)e^{[b(\tau)r]} \qquad (7)$$

where

$$a(\tau) = \left\{\frac{2\gamma \exp[(\gamma + \kappa)\tau / 2]}{2\gamma + (\gamma + \kappa)(\exp(\gamma\tau) - 1)}\right\}^{\frac{2\kappa u}{\sigma_2^2}} [\exp(-d\tau)]$$

$$b(\tau) = \frac{2\exp(\gamma\tau) - 1}{2\gamma + (\gamma + \kappa)(\exp(\gamma\tau) - 1)}$$

$$\gamma = \sqrt{(\kappa^2 - 2\sigma_2^2)} > 0 \qquad \text{by assumption}$$

$$\tau = T - t$$

All of the previously explained models have an assumption of a perfect market condition in pricing the futures contract. To relax this assumption, Hsu and Wang [30] proposed a pricing model for futures contract under imperfect market assumption. In this model, the market imperfection is measured by $\dfrac{\sigma_p}{\sigma}$ where σ_p is the instantaneous standard deviation of underlying asset return under imperfect market condition and σ is the instantaneous standard deviation of underlying asset. To describe the model, underlying asset is assumed to pay a continuous dividend rate d during its life and the stochastic process of the underlying asset is given by:

$$dS = (\mu - d)Sdt + \sigma SdW \qquad (8)$$

where μ is the drift of the underlying asset and W is the standard Wiener process. The solution for this model is given by:

$$f(S, t) = S(t)e^{(\mu_p - d)(T - t)} \qquad (9)$$

where μ_p is the instantaneous expected return of underlying asset under imperfect market that is derived from the induction process by using the concepts of price expectation and imperfect arbitrage.

Li [29] extended Hsu and Wang [30] model by considering two stochastic processes in the model. Under an imperfect market condition, Li [29] assumes that the underlying asset and volatility of the underlying asset follows a joint stochastic process as follows:

$$dS = (\mu - d)Sdt + \sigma SdW_1 \qquad (10)$$

$$d\sigma = a\sigma dt + \beta\sigma dW_2 \qquad (11)$$

where μ is the drift rate, d is the dividend rate, σ is the underlying asset volatility, a and β depend on σ and t and both Wiener processes are correlated with correlation ρ. To construct the imperfect hedge portfolio, Li [29] used the same induction process as Hsu and Wang [30]. Then, the market price concept of convenience yield risk proposed by Brennan and Schwartz [31] as well as Gibson and Schwartz [32] is used to obtain the partial differential equation for the futures contract. In solving the model, Li [29] uses an

explicit finite difference method as there are three variables available which are S, σ and T. However, the solution method is found to be unstable and highly vulnerable to the parameter change and grid division.

In a *salam* contract, the buyer needs to pay the full amount of *salam* price at the beginning of the contract and receive the agreed goods at maturity. From this situation, the buyer is exposed to the risk of not receiving the goods at maturity from the seller (credit default risk). Therefore, the credit risk model is an appropriate model since it can describe the credit default risk element in the commodity *salam* contract. Furthermore, shariah also allows the buyer to request for guarantee such as mortgage and collateral to compensate his condition from credit risk [10]. In general, there are two types of credit risk namely structural model and reduced form model. For the structural model, the default condition is determined based on the firm's structural variables which are assets, liabilities and equities. Meanwhile for the reduced form model, the default event is random, controlled by a Poisson process [33]. In this study, the structural model is fit to describe the commodity *salam* contract since the default event in a *salam* contract is only one-sided, in which it happens based on the *salam* writer's (seller) financial condition.

There are two main approaches to describe the structural model. The first approach is proposed by Merton [34] in modelling a risky discount bond, where it is assumed that the default event only occurs at the maturity of the contract. The second approach is introduced by Black and Cox [35], in which the default event can occur any time prior to maturity. In the studies, they assumed that the firm will only default when the firm asset value crosses a default barrier. In modelling the commodity *salam* contract with credit risk, the first approach which is structural model should be considered since the default event only happens at the maturity of the contract.

There are studies that were conducted on structural model. Johnson and Stulz [36] apply this approach to price an option with default risk. In their research, it is assumed that the option buyer claim is the sole liability. This research was extended by Klein [37], assuming that there are other liabilities instead of buyer claim only. The option value is described by two stochastic processes, which are the option writer's asset and the underlying asset. In this model, default will only happen if the option writer's asset falls below a fixed default boundary. Instead of assuming a fixed interest rate, Klein and Inglish [38] suggests a stochastic interest rate model for a defaultable option. Then, Klein and Inglis [39] extended the research by allowing the option writer's total liabilities to depend on the option holder claim value. This model is used to price a vulnerable European option.

2.0 METHODOLOGY

Although there are many researches on the application and implementation of *salam* contract in the existing financial instruments, the issues discussed only touch the financial and law aspects of the contract (qualitative). Since there is lack of quantitative study being done, this study introduces a mathematical model that will value a *salam* contract by considering credit risk. This model is anticipated to introduce a new alternative to Islamic derivative product that can compete with conventional forwards and futures. To construct the model, this study has derived a partial differential equation that describes the value of *salam* contract with credit risk.

The commodity asset is chosen as the scope in this study since the mechanism and structure of *salam* contract is suitable for commodity trading. In addition, the proposed commodity *salam* model is expected to help in hedging the commodity price and providing the commodity seller with working capital. To assure that the proposed commodity *salam* is shariah-compliant, it must follows all the five shariah prohibitions as stated earlier. The elements of *rishwah* (corruption) and *maysir* (speculation) are eliminated since the predetermined *salam* price, buyer and seller are clearly identified upon the contract signing. Furthermore, the quality, quantity and maturity are justified in the beginning of the contract, thus dropping the *jahl* (ignorance) element since both buyer and seller are well aware of the financial instrument. According to Conroy [40], commodity is a physical asset that needs to be stored, can be consumed and deteriorates over time. Therefore, in modelling the commodity *salam*, this study has taken into account another important variable, which is the storage cost. This study defines storage cost as the continuous compounding cost of commodity storage prior to its delivery per unit of spot price. This storage cost includes cost of handling the commodity (warehouse cost, shipping cost etc.) and spoilage [15]. By considering spoilage in the storage cost, this reduces the uncertainty (*gharar*) of goods not delivered with a required quality and quantity at maturity.

In modelling the commodity *salam* with credit risk, some basic assumptions on structural model are employed [34, 36-39]. However, some adjustments are needed based on the unique structure of *salam* contract. A commodity *salam* contract is written based on the underlying asset $S(t)$ with maturity time T. The underlying asset follows a lognormal stochastic differential equation:

$$dS(t) = \big(\mu(S) + \alpha(S)\big)S(t)dt + \sigma(S)S(t)dW(t) \qquad (12)$$

where $\mu(S)$ is the drift of the underlying asset, $\alpha(S)$ is the annualised storage cost as the proportion of spot price in percentage, $\sigma(S)$ is the underlying asset volatility and $W(t)$ is the standard Wiener process of the

underlying asset. The annualised storage cost $\alpha(S)$ is depicted by:

$$\frac{\text{total cost of storing the underlying asset per year (RM)}}{\text{underlying asset spot price (RM)}} \times 100\% \quad (13)$$

The commodity *salam* contract is issued by the *salam* writer, which is the seller with a predetermined *salam* price $\overline{S(0)}$. The buyer needs to hand over the entire amount based on *salam* price at the initiation of the contract $t = 0$ and is expected to receive the agreed goods from the seller at maturity. The dynamic in the value of *salam* writer's asset $V(t)$ at time t is given by:

$$dV(t) = \mu(V)V(t)dt + \sigma(V)V(t)dZ(t) \quad (14)$$

where $\mu(V)$ is the drift of the *salam* writer's, asset $\sigma(V)$ is the *salam* writer's asset volatility and $Z(t)$ is the Wiener process of the *salam* writer's asset. Both Wiener processes of $dW(t)$ and $dZ(t)$ are correlated with:

$$\text{cov}(dW(t), dZ(t)) = \rho(S,V)dt \quad (15)$$

where $\rho(S,V)$ is the correlation coefficient between the two Brownian motions in (12) and (14). Both stochastic processes in (12) and (14) are subjected to the assumptions as below:

Assumption 1
Both underlying and *salam* writer's assets are assumed to be traded over time. Although *salam* writer's asset is not directly traded, the market value of the *salam*'s writer asset behaves like a traded asset [37].

Assumption 2
Trading takes place in a prolonged time and perfect market assumption is employed (no transaction cost and taxes) [34, 41].

Assumption 3
Unrestricted borrowing and lending of fund with a similar instantaneous risk-free rate [34, 41]. To eliminate the *riba* (interest) element in this model, the interest rate will be replaced with Islamic interbank rate r.

Assumption 4
Claim by the *salam* holder (buyer) is the sole liability of the *salam* writer (seller) [36].

Assumption 5
Underlying asset and *salam* writer's asset have positive value since the value of any asset can only take nonnegative value [34].

$$S(t) \geq 0 \qquad\qquad V(t) \geq 0$$

Assumption 6
Since *salam* holder (buyer) is exposed to credit default risk from *salam* writer (seller), the *salam* writer's asset will be the collateral in a commodity *salam* contract [10].

Assumption 7
Commodity *salam* contract at maturity has a value of $S(T) - \overline{S(0)}$ if the *salam* writer (seller) dissolve his asset:

$$V(T) > S(T) - \overline{S(0)} \quad (16)$$

in case his total asset is greater than the *salam* holder (buyer) claim, the latter will receive all the *salam* writer's (seller) asset, which is $V(T)$ at maturity if there is seller's default by *salam* writer,

$$V(T) < S(T) - \overline{S(0)} \quad (17)$$

who fails to deliver the agreed goods at maturity when his total asset is less than the *salam* holder (buyer) claim. This condition is consistent with assumption 5 since the *salam* writer's (seller) asset will act as the collateral in the commodity *salam* contract.

Assumptions 1-5 are based on the basic assumptions of the structural model [34, 36-39], while assumptions 6 and 7 are due to the unique structure and boundary condition of the *salam* contract. The partial differential equation that describes the dynamic and behaviour of commodity *salam* contract with credit risk is constructed by using the risk neutrality approach, which is introduced by Cox and Ross [42] for option valuation. The significance of using this approach as compared to others is it does not involve delta hedging. It is based on the concept that an investment with zero risk of asset price movement due to arbitrage consideration will earn the same rate as the risk-free return. In fact, there are three steps in risk neutrality approach. First, the real diffusion process of all the state variables will be transformed to the risk-neutral process. Then, the process of state variables is identified. Finally, the replication portfolio is constructed to eliminate the uncertainty by the removal of risk and arbitrage argument.

3.0 RESULT AND DISCUSSIONS

By proposing the commodity *salam* contract model with credit risk, this results in a partial differential equation as in equation (41), which describes the dynamic behaviour of commodity *salam* contract. Then, under equivalent martingale probability measure, the current value of commodity *salam* contract with credit risk is illustrated by equation (42). To derive the partial differential equation, this study adopted the risk neutrality approach. Brief discussions on the derivation and the results from the model are extensively explained in the next sections.

3.1 Change of Measure

In the risk neutrality approach, the real stochastic process in equations (12) and (14) must first be transformed into the risk-neutral process. According to Cuthbertson and Nitzsche [16], the excess return of underlying asset is equal to risk market price multiply by risk quantity. Therefore it is written as:

$$\mu(S) - r = \lambda(S)\sigma(S) \tag{18}$$

where $\mu(S) - r$ is the excess return and $\lambda(S)$ is risk market price associated with the underlying asset. The quantity of risk in equation (18) is described by the underlying asset volatility $\sigma(S)$. Since the underlying asset is continuously traded and provides a return r ,, it is written as [43]:

$$r = \mu(S) - \lambda(S)\sigma(S) \tag{19}$$

thus,

$$\mu(S) = r + \lambda(S)\sigma(S) \tag{20}$$
$$\lambda(S) = \frac{\mu(S) - r}{\sigma(S)} \tag{21}$$

Substituting (20) and (21) into (12),

$$
\begin{aligned}
\frac{dS(t)}{S(t)} &= \big(\mu(S) + \alpha(S)\big)dt + \sigma(S)dW(t) \\
&= \big(r + \lambda(S)\sigma(S)\big)dt + \alpha(S)dt + \sigma(S)dW(t) \\
&= rdt + \lambda(S)\sigma(S)dt + \alpha(S)dt + \sigma(S)dW(t) \\
&= \big(r + \alpha(S)\big)dt + \sigma(S)\big(dW(t) + \lambda(S)dt\big)
\end{aligned}
$$
$$dS(t) = \big(r + \alpha(S)\big)S(t)dt + \sigma(S)S(t)\left(dW(t) + \frac{\mu(S) - r}{\sigma(S)}dt\right) \tag{22}$$

Comparing the terms in (12) and (22), the relationship of the Brownian motion between the underlying asset in true probability measure $dW(t)$ and martingale probability measure $dW^*(t)$ is given by:

$$dW^*(t) = dW(t) + \frac{\mu(S) - r}{\sigma(S)}dt$$
$$dW(t) = dW^*(t) - \frac{\mu(S) - r}{\sigma(S)}dt \tag{23}$$

where $\frac{\mu(S) - r}{\sigma(S)}$ deducts market price per unit of underlying asset risk [16][44][45]. Hence, from (22) and (23), the appropriate risk-neutral process of the underlying asset is given by:

$$dS(t) = \big(r + \alpha(S)\big)S(t)dt + \sigma(S)S(t)dW^*(t) \tag{24}$$

Based on assumption 1, the *salam* writer's asset also behaves like a traded asset. Therefore, adopting the same method as in underlying asset, the risk-neutral process of *salam* writer's asset is given by:

$$dV(t) = \mu(V)V(t)dt + \sigma(V)V(t)dZ^*(t) \tag{25}$$

where $dZ^*(t)$ is the Brownian motion of the *salam* writer's asset in martingale probability measure. Both Brownian processes in (24) and (25) are correlated by:

$$dW^*(t) \cdot dZ^*(t) = \rho(S,V)dt \tag{26}$$

3.2 The Process of State Variables

Under the Brownian motion properties, underlying asset has a lognormal stationary distribution [14]. Therefore, by defining $X = \ln S(t)$ where $X(S(t))$, then

$$\frac{dX}{dS(t)} = \frac{1}{S(t)} \tag{27}$$
$$\frac{d^2X}{dS^2(t)} = -\frac{1}{S^2(t)} \tag{28}$$

Applying one-dimensional Itŏ lemma [46] on $X(S(t))$

$$dX = \frac{\partial X}{\partial S(t)}dS(t) + \frac{1}{2!}\left[\frac{\partial^2 X}{\partial S^2(t)}\big(dS(t)\big)^2\right] + \ldots \tag{29}$$

where $(dt)^2 = 0$, $dtdW^*(t) = 0$ and $dW^*(t)dW^*(t) = dt$. By substituting equation (24), (27) and (28) into the Itŏ lemma in (29), the result is:

$$
\begin{aligned}
dX &= \frac{\partial X}{\partial S(t)}dS(t) + \frac{1}{2!}\frac{\partial^2 X}{\partial S^2(t)} \\
&\quad \big[\big(r + \alpha(S)\big)S(t)dt + \sigma(S)S(t)dW^*(t)\big]^2 \\
&= \frac{1}{S(t)}\big[\big(r + \alpha(S)\big)S(t)dt + \sigma(S)S(t)dW^*(t)\big] \\
&\quad + \frac{1}{2}\frac{\partial^2 X}{\partial S^2(t)}\big[\big(r + \alpha(S)\big)S(t)dt + \sigma(S)S(t)dW^*(t)\big]^2 \\
&= \big(r + \alpha(S)\big)dt + \sigma(S)dW^*(t) + \frac{1}{2}\left(-\frac{1}{S^2(t)}\right) \\
&\quad \big[\big(r + \alpha(S)\big)^2 S^2(t)(dt)^2 + 2rS^2(t)\sigma(S)dtdW^*(t) \\
&\quad + \sigma^2(S)S^2(t)\big(dW^*(t)\big)^2\big] \\
&= \big(r + \alpha(S)\big)dt + \sigma(S)dW^*(t) - \frac{1}{2}\sigma^2(S)dt
\end{aligned}
$$
$$d[\ln S(t)] = \left(r + \alpha(S) - \frac{1}{2}\sigma^2(S)\right)dt + \sigma(S)dW^*(t) \tag{30}$$

Therefore, the process of $\ln S(t)$ is described by (30). Since the *salam* writer's asset is also normally distributed, the result is $Y = \ln V$ where $Y(V(t))$. Therefore,

$$\frac{dY}{dV(t)} = \frac{1}{V(t)} \qquad (31)$$

$$\frac{d^2Y}{dV^2(t)} = -\frac{1}{V^2(t)} \qquad (32)$$

By applying one-dimensional Itŏ lemma on $Y(V(t))$

$$dY = \frac{\partial Y}{\partial V(t)} dV(t) + \frac{1}{2!}\left[\frac{\partial^2 Y}{\partial V^2(t)}\big(dV(t)\big)^2 \right] + \dots \qquad (33)$$

where $(dt)^2 = 0$, $dtdZ^*(t) = 0$ and $dZ^*(t)dZ^*(t) = dt$. Substituting (25), (31) and (32) into (33), the process of $\ln V(t)$ is described by:

$$dY = \frac{\partial Y}{\partial V(t)} dV(t) + \frac{1}{2!}\frac{\partial^2 Y}{\partial V^2(t)}$$
$$\left[rV(t)dt + \sigma(V)V(t)dZ^*(t) \right]^2$$
$$= rdt + \sigma(V)dZ^*(t) + \frac{1}{2}\left(-\frac{1}{V^2(t)} \right)$$
$$[r^2V^2(t)(dt)^2 + 2rV^2(t)\sigma(V)dtdZ^*(t)$$
$$+ \sigma^2(V)V^2(t)\big(dZ^*(t)\big)^2]$$
$$= rdt + \sigma(V)dZ^*(t) - \frac{1}{2}\sigma^2(V)dt$$

$$d\big[\ln V(t)\big] = \left(r - \frac{1}{2}\sigma^2(V) \right)dt + \sigma(V)dZ^*(t) \qquad (34)$$

Based on equations (30) and (34), it is shown that $\ln S(t)$ and $\ln V(t)$ follow log normal diffusion process.

3.3 Constructing the Replication Portfolio

Since both underlying asset and *salam* writer's asset follow log normal diffusion process, it is possible to construct a perfect hedge portfolio that can eliminate the uncertain element, represented by $dW^*(t)$ and $dZ^*(t)$. This results in a dynamic partial differential equation for a deterministic value of commodity *salam* contract with credit risk. In forming partial differential equation for the value of commodity *salam* contract with credit risk, this study adopted a contingent claim analysis method, as shown by Gibson and Schwartz [32], Bjerksund [44], Hosseini [45] and Tassis and Skiadopoulos [47]. Since the price of contingent claim $P(S(t), V(t), t)$ is twice of the continuous differential function of $S(t)$ and $V(t)$ [32, 44], the instantaneous change for the contingent claim value is searchable by applying the multi-dimensional Itŏ lemma [46]:

$$dP = \frac{\partial P}{\partial S(t)} dS(t) + \frac{\partial P}{\partial V(t)} dV(t) + \frac{\partial P}{\partial t} dt + \frac{1}{2!}[\frac{\partial^2 P}{\partial S^2(t)}$$
$$(dS(t))^2 + \frac{\partial^2 P}{\partial S(t)\partial V(t)} dS(t)dV(t) + \frac{\partial^2 P}{\partial S(t)\partial t} dS(t)dt$$
$$+ \frac{\partial^2 P}{\partial V^2(t)} (dV(t))^2 + \frac{\partial^2 P}{\partial V(t)\partial S(t)} dV(t)dS(t) +$$
$$\frac{\partial^2 P}{\partial V(t)\partial t} dV(t)dt + \frac{\partial^2 P}{\partial^2 t} (dt)^2 + \frac{\partial^2 P}{\partial t\partial S(t)} dtdS(t) +$$
$$\frac{\partial^2 P}{\partial t\partial V(t)} dtdV(t)] + \dots \qquad (35)$$

where $(dt)^2 = 0$, $dtdW^*(t) = 0$, $dtdZ^*(t) = 0$
$dW^*(t)dZ^*(t) = \rho(S,V)dt$, $(dW^*(t))^2 = dt$ and $(dZ^*(t))^2 = dt$.
By substituting the correlated risk-neutral process of (24) and (25) into (35),

$$dP = [(r + \alpha(S))S(t)\frac{\partial P}{\partial S(t)} + rV(t)\frac{\partial P}{\partial V(t)} + \frac{\partial P}{\partial t} +$$
$$\frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 P}{\partial S^2(t)} + \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)$$
$$\frac{\partial^2 P}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 P}{\partial V^2(t)}]dt + \sigma(S)S(t)$$
$$\frac{\partial P}{\partial S(t)} dW^*(t) + \sigma(V)V(t)\frac{\partial P}{\partial V(t)} dZ^*(t) \qquad (36)$$

Equation (36) describes the instantaneous change of the value of contingent claim $dP(S(t), V(t), t)$. Based on the perfect market assumption which implies no arbitrage and nonstochastic Islamic interbank rate, the instantaneous change of the value of contingent claim will has a deterministic term,

$$dP = [(r + \alpha(S))S(t)\frac{\partial P}{\partial S(t)} + rV(t)\frac{\partial P}{\partial V(t)} + \frac{\partial P}{\partial t} +$$
$$\frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 P}{\partial S^2(t)} + \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)$$
$$\frac{\partial^2 P}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 P}{\partial V^2(t)}]dt \qquad (37)$$

Under no arbitrage argument, the risk-free portfolio is being set up as:

$$dP = rPdt \qquad (38)$$

Equation (38) describes that a completely risk-free change dP in the portfolio value P must be similar to the growth if an equivalent amount of cash is put in the risk-free interest bearing account. Therefore, by substituting (37) into (38), the market value of contingent claim is described as:

$$(r + \alpha(S))S(t)\frac{\partial P}{\partial S(t)} + rV(t)\frac{\partial P}{\partial V(t)} + \frac{\partial P}{\partial t} +$$

$$\frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 P}{\partial S^2(t)} + \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)$$

$$\frac{\partial^2 P}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 P}{\partial V^2(t)} - rP = 0 \qquad (39)$$

According to Cuthbertson and Nitzsche [16] and Hosseini [45], the change in the replication portfolio value must be matching with the change in the derivative contract value $dF(S(t),V(t),t)$

$$dP(S(t),V(t),t) = dF(S(t),V(t),t) \qquad (40)$$

Therefore, under similar market condition with neither risk nor arbitrage condition, the value of commodity *salam* contract with credit risk $F(S(t),V(t),t)$ satisfies the following two dimensional partial differential equations:

$$(r + \alpha(S))S(t)\frac{\partial F}{\partial S(t)} + rV(t)\frac{\partial F}{\partial V(t)} + \frac{\partial F}{\partial t} +$$

$$\frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 F}{\partial S^2(t)} + \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)$$

$$\frac{\partial^2 F}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 F}{\partial V^2(t)} - rF = 0 \qquad (41)$$

Based on equivalent martingale theory [16, 44], the current value of commodity *salam* contract with credit risk and claim of future delivery at time T is given by:

$$F(S(t),V(t),t) = e^{-r(T-t)}E^*(S,V,t)[\text{payoff at } T] \qquad (42)$$

where $E^*(S,V,t)[.]$ is the expectation under equivalent martingale probability measure. Therefore, equation (42) is solved by considering the payoff of commodity *salam* contract with credit risk as in equation (1), the claim as in assumption 4, the nonnegativity condition as in assumption 5, the collateral as in assumption 6 and the default boundary condition as in assumption 7.

4.0 CONCLUSION

In managing risk, all Islamic financial institutions need to ensure that the risk management technique employed is complying with the shariah principals, that has no *riba*, *rishwah*, *gharar*, *maysir* and *jahl*. Due to these prohibitions, Islamic finance has a very limited risk management tools as compared to conventional. For Islamic finance to remain competitive as the conventional, there is a need to develop a shariah-compliant product such as Islamic derivative that is useful to manage the risk. This study has proposed a traditional Islamic contract namely *salam* that can be built as an Islamic derivative product. The condition of

full advance payment at the beginning of the *salam* contract will eliminate the *maysir* (speculation) element. Moreover, the full advance payment also helps to finance the seller with working and extension capital, reduce leverage by minimising speculation and no default risk from the buyer. By considering the commodity as the underlying asset, this study has taken into account another important variable which is the storage cost. This consideration helps to reduce the uncertainty (*gharar*) of not delivering the commodity in certain quantity and quality at the maturity of the contract.

Nevertheless, the condition of the prepayment at the beginning of the contract poses another problem which is the credit default risk from the seller. Therefore, in modelling the commodity *salam* contract, an appropriate credit risk model should be considered. Since there is lack of quantitative study on *salam* contract, this study has introduced a mathematical model that can valuate the commodity *salam* contract with credit risk. The structural model is chosen to describe the commodity *salam* contract since the default event of a *salam* agreement only occurs at the maturity of the contract. However, because of the unique structure of *salam* contract, some adjustments regarding the collateral and terminal boundary condition are made. In constructing a partial differential equation that describes the dynamic behaviour of the commodity *salam* contract with credit risk, the risk-neutral valuation is employed. In general, there are three steps in risk neutrality approach. First, the real diffusion process of state variables are transformed into risk-neutral process. Then, the process of state variables is identified. Finally, the replication portfolio is constructed to eliminate the uncertain element by considering no risk and no arbitrage argument.

Acknowledgement

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