

IMPACT OF FLUCTUATIONS OF GRADED-INDEX OPTICAL FIBER PARAMETERS ON PROPAGATION OF GAUSSIAN BEAMS

Hassan Pakarzadeh*, Jamshid Dousti

Department of Physics, Shiraz University of Technology, Shiraz, Iran

Article history

Received

15 August 2015

Received in revised form

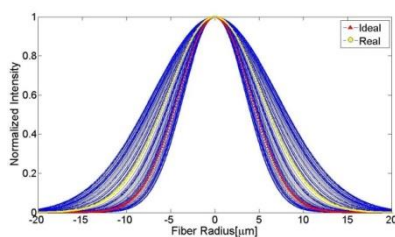
15 November 2015

Accepted

30 December 2015

*Corresponding author
pakarzadeh@sutech.ac.ir

Graphical abstract



Abstract

In this paper, the impact of fluctuations of graded-index (GRIN) optical fiber parameters on propagation of Gaussian beams is investigated by using ABCD matrix approach and a stochastic process. Since GRIN optical fibers have many applications in the optical communications, variations of fiber parameters such as core refractive index, core-cladding index difference and core radius affect the propagation of optical signals. Results show that the variations of the mentioned parameters change the beam spot size as well as the periodicity of the beam. Also, the fluctuations of such fiber parameters increase the spot size of the output Gaussian beam in comparison with the case when the fluctuations are ignored.

Keywords: Graded-index (GRIN), optical fiber, fluctuations, ABCD matrix, Gaussian beam

Abstrak

Dalam kertas ini, kesan turun naik digred-index (GRIN) parameter gentian optik pada pembiakan rasuk Gaussian disiasat dengan menggunakan ABCD matriks pendekatan dan proses stokastik. Sejak GRIN gentian optik mempunyai banyak aplikasi dalam komunikasi optik, variasi parameter serat seperti indeks teras, teras pelapisan perbezaan indeks dan jejari teras menjejaskan penyebaran isyarat optik. Keputusan menunjukkan bahawa variasi parameter disebut menukar saiz tempat rasuk dan juga jangka masa untuk rasuk. Selain itu, turun naik parameter serat seperti meningkatkan saiz spot output rasuk Gaussian berbanding dengan kes itu apabila turun naik diabaikan.

Kata kunci: Digred-index (GRIN), gentian optic, turun naik, ABCD matriks, rasuk Gaussian

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1.0 INTRODUCTION

A lens-like medium, in which the refractive index is gradually varied as a function of radius, can behave as a positive lens and hence focus an optical beam. A good example of a lens-like medium is a graded-index (GRIN) fiber [1, 2]. In practice, during the fabrication of optical fibers, owing to unwanted variations of environmental parameters and existing strain and stress, the refractive index and core radius

of the fiber are randomly changed with length [3]. These fluctuations of fiber parameters affect the propagation of optical beams. Therefore, for a more realistic and precise modeling of beam propagation, it is important to include such fiber fluctuations.

The ABCD matrix is a well-known approach to describe the characteristics of the propagating beam [4-7]. Although the propagation of Gaussian beams in lens-like medium by means of the ABCD matrix has been vastly studied; in this paper, the

impact of fluctuations of graded-index (GRIN) optical fiber parameters on propagation of Gaussian beams is investigated for the first time using the ABCD matrix approach and considering fluctuations as a stochastic process.

The paper is organized as follows. In Section 2.0, the ABCD matrix approach for propagation of Gaussian beams in GRIN fibers is described and the stochastic model for modeling fiber fluctuations is presented. Then, the main results of the paper are given in Section 3.0 where the characteristics of a propagating Gaussian beam in presence of fiber fluctuations are investigated. Finally, the paper is concluded in Section 4.0 where the remarks are summarized.

2.0 THEORY

A GRIN optical fiber exhibits a refractive index profile in the cylindrical coordinate given by [8]:

$$n^2(r) = n_0^2 \left[1 - \left(\frac{r}{l_G} \right)^2 \right] \quad (1)$$

where n_0 is the center refractive index calculated at the fiber axis ($r=0$) and l_G is a length scale which depends on the core radius a , center refractive index and core-cladding index difference Δn as below [8]:

$$l_G = a \sqrt{\frac{n_0}{2\Delta n}} \quad (2)$$

A 2×2 ABCD matrix for the beam propagation in a GRIN fiber is given as [8]:

$$T = \begin{pmatrix} \cos\left(\frac{z}{l_G}\right) & l_G \sin\left(\frac{z}{l_G}\right) \\ -\frac{1}{l_G} \sin\left(\frac{z}{l_G}\right) & \cos\left(\frac{z}{l_G}\right) \end{pmatrix} \quad (3)$$

Also, a Gaussian field can be considered as:

$$E = \exp\left(-\frac{r^2}{w^2}\right) \quad (4)$$

where w is the spot size of the beam and r is the radial distance from the fiber axis. The q parameter of the Gaussian beam is related to the w and the beam curvature R via [8]:

$$\frac{1}{q} = \frac{1}{R} - j \frac{\lambda}{\pi n_0 w^2} \quad (5)$$

where λ is the wavelength of the lightwave. By defining the beam parameter $q(0)$ at the fiber input and using the ABCD matrix, the beam parameter $q(z)$ at any arbitrary fiber length z can be determined through [8]:

$$q(z) = \frac{A(z)q(0) + B(z)}{C(z)q(0) + D(z)} \quad (6)$$

where $A(z)$, $B(z)$, $C(z)$ and $D(z)$ are the matrix elements of the GRIN fiber given by Eq. (3). As it can be readily seen from Eqs. (2) and (3), the ABCD matrix elements explicitly depend on the main fiber

parameters such as the core refractive index and the core radius. Therefore, any change or fluctuations of such fiber parameters will change and affect the characteristics of the beam propagating along the fiber. In presence of any change or fluctuation, the core refractive index and radius respectively change or fluctuate about their mean values as below:

$$n_0 = \bar{n}_0 + \delta n_0 \quad (7)$$

$$a = \bar{a} + \delta a \quad (8)$$

Where \bar{n}_0 and δn_0 are the mean value and the variation or fluctuation of the core refractive index, respectively. Similarly, \bar{a} and δa are the mean value and the variation or fluctuation of the core radius, respectively. δn_0 can be modeled as a Gaussian distribution [9, 10]:

$$f_G(\delta n_0) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\delta n_0}{\sigma}\right)^2\right] \quad (9)$$

where σ is the standard deviation of δn_0 . A similar distribution can be proposed for δa .

3.0 RESULTS AND DISCUSSION

In this section, we investigate the propagation of a Gaussian beam in a GRIN fiber for two cases. In subsection 3.1 we consider the propagation of a Gaussian beam in presence of intentional variations of fiber parameters which do not vary along the fiber; and in subsection 3.2, the stochastic and longitudinal fluctuations of fiber parameters are considered and the propagation of a Gaussian beam through such random fluctuations is studied.

3.1 Intentional Variations of Fiber Parameters

Here, we have performed our simulations for a fiber with parameters as: $n_0 = 1.5$, $\Delta n = 0.05$, and $a = 25 \mu\text{m}$ [8]. We also assume a Gaussian beam with the wavelength of $\lambda = 1.064 \mu\text{m}$ and the spot size of $w_0 = 10 \mu\text{m}$ at the fiber input. The input beam is considered to have a flat wave front with the curvature of ∞ . Figure 1 shows the variation of the beam spot size as it propagates down the fiber for various $n_0 = 1.45, 1.50$ and 1.55 . Other parameters of the fiber are fixed at $a = 25 \mu\text{m}$ and $\Delta n = 0.05$. As it is seen, with variation of core index, the beam spot size is changed. The periodicity of the beams (which is about 3mm at the initial steps of propagation) is also changed and beams drift apart as the propagation length increases. The periodic behavior of the beam is the consequence of the lens-like medium introduced by the GRIN fiber.

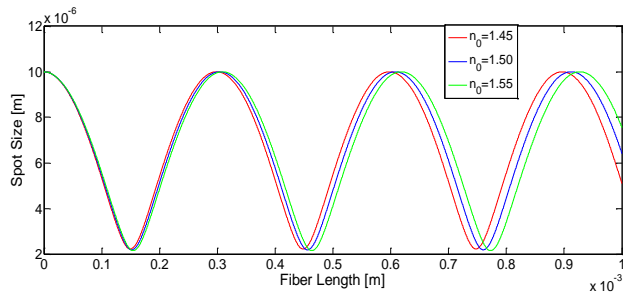


Figure 1 Variation of the beam spot size as a function of fiber length for different core indices n_0 ; and fixed values of $a = 25 \mu\text{m}$ and $\Delta n = 0.05$

Figure 2 has been plotted for a various core-cladding index differences $\Delta n = 0.045, 0.050$ and 0.055 and fixed $n_0 = 1.5$ and $a = 25 \mu\text{m}$. By the increase of Δn , the variation of the spot size increases and the periodicity decreases. This is because when the core-cladding index difference increases, the lens-like property of the fiber increases as well and hence the beam is focused and de-focused more quickly. Here, by small variations of Δn as small as 0.005 , the significant changes in beam characteristics is observed.

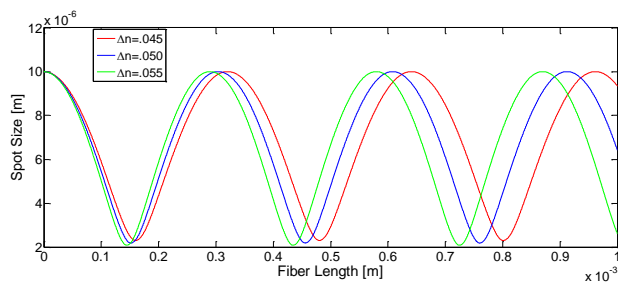


Figure 2 Variation of the beam spot size as a function of fiber length for different core-cladding index differences $\Delta n = 0.045, 0.050$ and 0.055 ; and fixed $n_0 = 1.5$ and $a = 25 \mu\text{m}$

Figure 3 shows the variation of the spot size for various core radii $a = 24, 24$ and $26 \mu\text{m}$ as beam propagates down the fiber. Other parameters of the fiber are fixed at $n_0 = 1.5$ and $\Delta n = 0.05$. As the core radius increase, the spot size at the periodicity of the beam increases as well. This is because according Eq. (2), by the increase of a the l_c increases, too; hence the radial changes of refractive index $n(r)$ will be more slowly and consequently the lens-like property of medium decreases. Therefore, we have shown that any intentional variations of the fiber parameters such as core refractive index, core radius and core-cladding index difference will affect the characteristic of the propagating Gaussian beam, i.e., the beam spot size and the periodicity. It is also shown that the variation of core-cladding index

difference has more impact on the beam propagation compared to other fiber parameters.

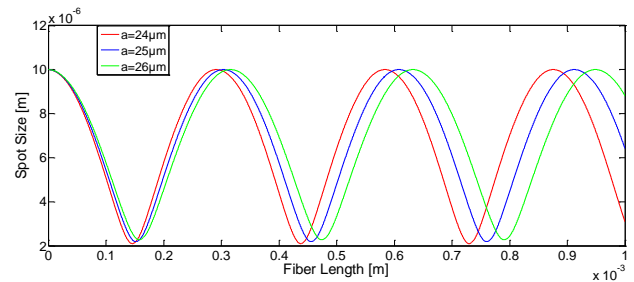


Figure 3 Variation of the beam spot size as a function of fiber length for various core radii $a = 24, 24$ and $26 \mu\text{m}$; and fixed $n_0 = 1.5$ and $\Delta n = 0.05$

3.2 Stochastic Fluctuations of Fiber Parameters

In this part, we consider the impact of stochastic longitudinal fluctuations of the fiber parameters on the propagation of a Gaussian beam. The difference between this part and the previous one is that this analysis is more realistic since the variations of fiber parameters do not actually remain unchanged along the fiber; but they instead fluctuate randomly. We consider the core index fluctuations as $\delta n_0 = \sigma \times n_1$ where n_1 is an output of the normal distribution of Eq. (9) in the interval of $[-1, 1]$ with the standard deviation of σ . A similar relation can be also assumed for other fiber parameters such as a and Δn .

The longitudinal fluctuations of n_0 can be modeled using the relation $z = -L_c \times \ln(n_1)$, where L_c is the correlation length over which the fluctuations remain unchanged. Figure 4 shows the stochastic distribution of the core index fluctuations along the 1000 m of a GRIN fiber with a fixed $L_c = 10\text{m}$ and $\sigma = 0.01\%$. Based on Figure 4, the fiber length is randomly divided to $N = \frac{1000}{10} = 100$ segments. For each segment, the fiber parameters are fixed and the ABCD matrix is given by Eq. (3). When a Gaussian beam is input to the real fiber (a fiber which contains fluctuations), the q parameter of the beam at the output of the first segment is given by Eq. (6). Then, the output of the first segment is the input of second segment. Finally, the beam parameter at the output of the whole fiber is calculated by successive use of Eq. (6) via N iterations. When the output parameter of the beam is calculated, then the output spot size can be determined as well. However, it should be noted that because of the random nature of fluctuations, for any given distribution with a fixed L_c and σ , there are huge number of events or realizations. Therefore, the real output spot size of the beam should be calculated by averaging over all possible events.

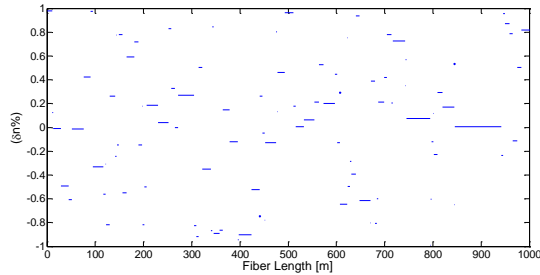


Figure 4 Stochastic distribution of the core index fluctuations along the 1000 m of a GRIN fiber with a fixed $L_c = 10m$

Figure 5 shows the variations of the output spot size of the Gaussian beam owing to the 100 different events of core index fluctuations associated with Figure 4. As it is seen, for each event or realization of the fluctuation distribution, the output spot size is randomly varied between the minimum of 5 and maximum of $10\mu m$. For better visualization of the output beam shape, the normalized intensity of the Gaussian beam is plotted in Figure 6 for 100 different events.

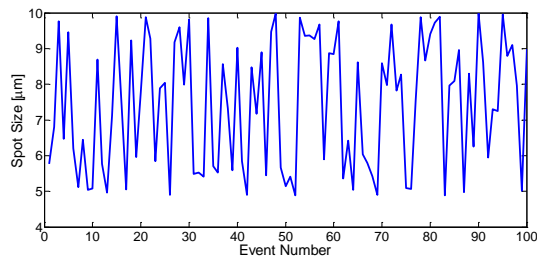


Figure 5 Variation of the output spot size of the Gaussian beam for 100 different events of core index fluctuations associated with Figure 4

As mentioned earlier, the real spot size is determined via averaging over large numbers of events normally more than 5000. Therefore, in Figure 6 we have indicated the average profile (in presence of fluctuations) by the symbol of circle. For better comparison, the ideal profile (in absence of fluctuations) is also shown by the symbol of triangle. As it is evident, for a real fiber where fluctuations exist, the spot size increases compared with that of ideal fiber where the fluctuations are ignored. The real spot size is $7.23\mu m$ and the ideal one is $5.74\mu m$. This explicitly shows that the impact of random fluctuations is to increase the spot size of the output beam or to spread the beam diameter.

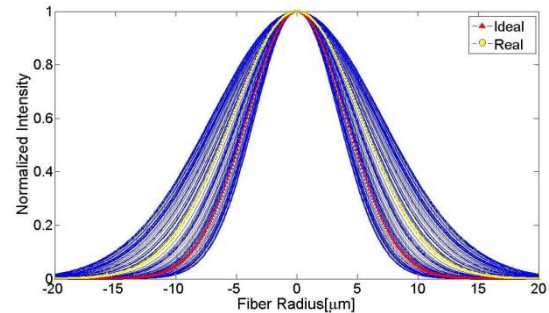


Figure 6 Normalised profile of the output Gaussian beam for 100 different events of core index fluctuations associated with Figure 4. The triangle and circle symbols indicate the ideal (without fluctuations) and the real ((with fluctuations)) profiles, respectively

For investigating the impact of core radius fluctuations, a figure similar to 4 can be considered for the stochastic fluctuations of the core radius along the fiber. Figure 7 shows the variations of the output spot size of the Gaussian beam owing to the 100 different events of core radius for a 1000-m long GRIN fiber. Similar to figure 5, fluctuations of core radius lead to the random variations of the output spot size between 5 and $10\mu m$. This is also shown in Figure 8 where the output profile of the Gaussian beam is plotted for 100 different events of core radius fluctuations, all for the same correlation length $L_c = 10m$ and standard deviation of $\sigma = 0.01\%$. Here, in presence of core radius fluctuations, the output spot size increases from its ideal value of $5.74\mu m$ (triangle symbol) to its real value of $7.70\mu m$ (circle symbol). It should be noted that although Figures 5 and 7 are plotted for 100 random events, in practice we have used more than 5000 events or realizations to calculate the mean (real) value of the spot size.

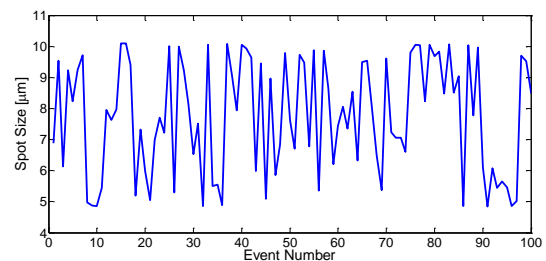


Figure 7 Variation of the output spot size of the Gaussian beam for 100 different events of core radius fluctuations similar to Figure 4

In general, according to the above discussion, the spot size of a Gaussian beam propagating along a real GRIN fiber, in which the fluctuations are present, is increased. This means that the random fluctuations can be considered as the scattering centres that scatter the beam and hence widen the spot size.

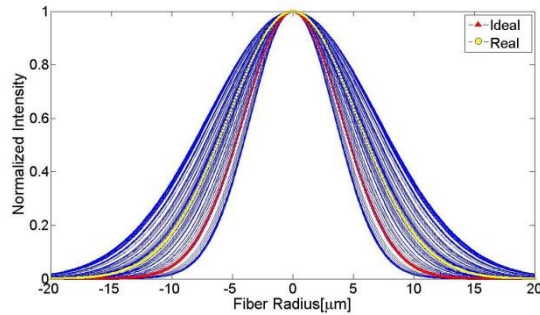


Figure 8 Normalised profile of the output Gaussian beam for 100 different events of core radius fluctuations after propagating a 1000-m GRIN fiber. The triangle and circle symbols indicate the ideal (without fluctuations) and the real ((with fluctuations)) profiles, respectively

4.0 CONCLUSION

We have investigated the impact of fluctuations of GRIN fiber parameters on the propagation of Gaussian beams. We have used the ABCD matrix approach for the beam propagation and a stochastic process for modeling the random fluctuations of the fiber parameters such as the core refractive index and the core radius. We have observed that the variations or fluctuations of the fiber parameters change the propagation characteristic of the Gaussian beam like the spot size and the periodicity. More specifically, the random fluctuations lead to ~ 30% increase in beam spot size compared with the case when the fluctuations are ignored. The results of this paper are important for

exact modeling of the beam propagation in GRIN fiber, since in practice the longitudinal fluctuations exist for realistic fibers.

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