Jurnal Teknologi

Investigation of Frequency Noise and Spectrum Linewidth in Semiconductor Optical Amplifier

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Graphical abstract
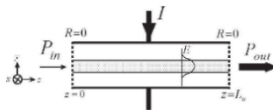
Abstract

The characteristics of FM noise and linewidth of semiconductor optical amplifier without facet mirrors were theoretically analyzed and experimentally confirmed. The concept of discrete longitudinal mode for the spontaneous emission was introduced as the basis of quantum mechanical characteristics, allowing the quantitative examination of noise sources. The continuously broaden output spectrum profile of the amplified spontaneous emission (ASE) was well explained as a spectrum broadening of each longitudinal mode. We found that the linewidth of the inputted signal light hardly changes by the optical amplification in the SOA. The FM noise increases proportional to square value of the noise frequency and less affected by the electron density fluctuation, the linewidth enhancement factor and the ASE. The higher FM noise in the higher noise frequency is caused by the intrinsic phase fluctuation on the optical emission. The characteristics of the linewidth and the noise frequency dependency were experimentally confirmed.

Keywords: Optical fiber communication, semiconductor optical amplifier, noise, frequency noise, linewidth

Abstrak

Ciri-ciri hingar FM dan *linewidth* yang terdapat pada penguat optik semikonduktor yang tidak mempunyai permukaan cermin, telah dianalisis dan disahkan melalui teori dan eksperimen. Konsep diskret mod membujur pada pancaran spontan telah diperkenalkan sebagai asas kepada ciri quantum mekanikal dimana ia membenarkan kuantitatif penilaian terhadap sumber hingar. Profil spektrum yang meluas secara berterusan, yang terdapat pada pancaran spontan yang dikuatkan, telah dijelaskan sebagai spektrum yang meluas bagi setiap mod membujur. Kami mendapati bawa *linewidth* isyarat cahaya tidak berubah dengan kehadiran penguatan optik yang terdapat didalam SOA. Hingar FM meningkat berkadaran dengan nilai persegi frekuensi hingar dan kurang dipengaruhi oleh turun-naik ketumpatan elektron, faktor peningkatan *linewidth* dan ASE tersebut. Hingar FM lebih tinggi dalam frekuensi hingar tinggi adalah disebabkan oleh turun-naik fasa intrinsik pada pemancar optik. Ciri-ciri *linewidth* dan kebergantungan frekuensi hingar telah disahkan secara eksperimen.

Kata kunci: Komunikasi gentian optik, penguat optik semikonduktor, hingar, hingar frekuensi, linewidth

1.0 INTRODUCTION

The semiconductor optical amplifier (SOA) is a useful optical device achieving high gain and be able to integrate with a semiconductor laser and a photodetector. The more wide spread applications of SOAs in optical communication system and other optoelectronics are expected. Many studies about SOAs have been reported in order to improve the characteristics and performance of the amplifier. The noise problem of SOAs is one of the most important key issues which needs to be resolved [1-16].

The origin of noise in almost all optical devices is considered to be quantum noise generated by temporal fluctuations associated with electron transition between two energy levels. The noise source is included in the rate equations of photon and electron as Langevin noise source [17]. Property of the Langevin noise source can be determined when optical mode is well defined with a discrete mode number under orthogonal relation among modes. In the case of lasers, the optical mode and the photon are well defined with the help of standing waves because lasers have closed cavities.

One of the authors of this paper proposed a model of SOA in ref. [14], in which the longitudinal mode is defined with periodic boundary condition where the length of period is the propagating length of a spontaneously emitted field with fixed discrete optical frequency. Excellent correspondences of the intensity noised between the theoretical analyses and experimental data have been obtained [14–16].

Analysis of frequency noise (FM noise) and spectral linewidth of the light signal are important especially in wavelength-division-multiplexing (WDM) and coherent optical communication systems. Many analysis and evaluations of SOAs have been done but the intrinsic characteristics of frequency noise and linewidth have not been clear yet.

In this paper, we define the longitudinal modes of the amplified spontaneous emission (ASE) and analyze effect of ASE on the FM noise and spectrum linewidth in the SOA. Analytical manner follows the previous paper [14], but several improvements are added to analyze more accurately the high frequency region and the ASE properties. We demonstrate that continuous ASE spectrum can be represented with supposed longitudinal modes. We also evaluate the theoretical analysis of the noise frequency characteristics with the experimentally measured data.

This paper is organized as follows. In Section 2, the basic model for analysis, equations describing the operation of SOA and the boundary condition to define longitudinal modes are introduced. The basic theory starts from introducing the spontaneous emission in the classical Maxwell's wave equation. Then quantum mechanical discussion is given to introduce the Langevin noise sources. In Section 3, fluctuation of electron density and phase fluctuation are expanded with angular frequency component to express the frequency noise and spectrum linewidth.

In Section 4, the numerical calculations are performed. In Section 5, the experimentally measured data are presented and the comparisons with theoretical analysis are shown. In Section 6, the conclusions are given.

2.0 MODEL FOR ANALYSIS AND BASIC EQUATIONS
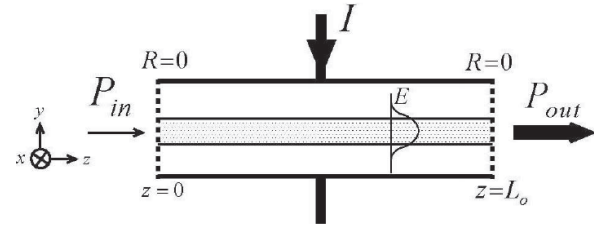
2.1 Introduction of ASE Modes

Figure 1 Structure of semiconductor optical amplifier (SOA)

Structure of semiconductor optical amplifier (SOA) used in this analysis is shown in Figure 1. The front and back facets of the SOA are anti-reflection coated to prevent reflections. The length of SOA is L_o, the width and thickness of the active region are w and d, respectively. The signal wave propagates in the z-direction. The driving current is I and the input power of the signal is P_{in}.

For this analysis, we introduce two types of electric polarization into the classical Maxwell's wave equation,

$$P = \varepsilon_0 \chi E + P_{sp} , \qquad (1)$$

where χ is the laser susceptibility giving the stimulated emission (amplification) and P_{sp} is another polarization giving the spontaneous emission. Then, the Maxwell's wave equation is given as

$$\nabla^2 E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \varepsilon_0 \chi \frac{\partial^2 E}{\partial t^2} + \mu_0 \sigma \frac{\partial E}{\partial t} + \mu_0 \frac{\partial^2 P_{sp}}{\partial t^2} . \qquad (2)$$

The electric field component E is given with $E^{(+)}$ and $E^{(-)}$ as the forward and the backward propagating optical waves to be

$$E = E^{(+)} + E^{(-)} , \qquad (3)$$

where

$$E^{(+)} = \sum_m A_m(t,z)\Phi(x,y)e^{-j\beta_m z + j\omega_m t} + c.c. , \qquad (4)$$

$$E^{(-)} = \sum_m B_m(t,z)\Phi(x,y)e^{j\beta_m z + j\omega_m t} + c.c. . \qquad (5)$$

Here, m is the longitudinal mode number which cover all modes including the signal mode and the ASE modes, $\Phi(x,y)$ is normalized field distribution function in the transverse cross- section, ω_m is angular frequency, β_m is propagation constant and c.c. indicates to take the complex conjugate. $A_m(t,z)$ and $B_m(t,z)$ are amplitudes of the forward and the

backward propagating waves of the mode *m*, respectively.

Although we are investigating properties of the forward propagating wave in the SOA, we need to take into account existence of the backward propagating wave, because the spontaneous emission generates in both the forward and the backward direction simultaneously. In this paper, we adopt three improvements from our previous published papers in refs. [14, 16]. This additional count of the backward propagating wave is the first improvement.

2.2 Photon and Longitudinal Modes

The spontaneous emission is caused by the zero point energy of the quantized optical field [17]. We define the photon and longitudinal modes for the ASE as followings: We suppose a finite length L_f with which the photon number is defined. The optical energy W in the supposed length L_f is given as

$$W = \int_{z-L_f/2}^{z+L_f/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varepsilon \left\{ E^{(+)2} + E^{(-)2} \right\} dx\,dy\,dz$$
$$= \sum_m \left\{ S_m^{(+)} + S_m^{(-)} + \frac{1}{2} \right\} \hbar \omega_m \qquad (6)$$

Here, $S_m^{(+)}$ and $S_m^{(-)}$ are photon numbers of the forward and backward propagating optical wave of mode *m*, respectively. The zero-point energy of the quantized optical wave $(1/2)\hbar\omega_m$ is originally defined with a standing wave, which is a superposed mode of the forward and the backward waves, and should be uniquely defined with the quantum number 1/2 without doubling the quantum number. Other zero-point energy at same spatial position must be belong to different longitudinal modes.

Then, we can define longitudinal mode *m* with condition of

$$2\beta_m L_f = 2m\pi \qquad (7)$$

Orthogonal relations among the signal mode and all ASE modes are guaranteed. The propagation constant is characterized with equivalent dielectric constant or equivalent refractive index $\varepsilon_{eq} = \varepsilon_o n_{eq}^2$, and propagation speed v and optical frequency f_{mo} as in the followings :

$$\beta_m = \sqrt{\mu_0 \varepsilon_{eq}} \, \omega_m = \frac{2\pi f_{mo}}{v} \qquad , \qquad (8)$$

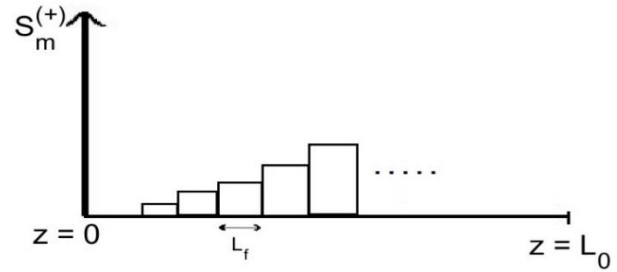$$v = \frac{c}{n_{eq}} \qquad . \qquad (9)$$

Figure 1 Introduction of finite length L_f to define longitudinal mode

Our idea for introduction of finite length L_f is illustrated in Figure 2. Criterions on the supposed length L_f will be given in Subsection 2.4.

Although discrete values of the mode number *m* are supposed, the output ASE profile reveals continuous spectrum as shown in Figure 3 because of the spectrum broadening by each ASE mode. Figure 3 is a calculated example based on our proposed model. The center peak is the signal mode, fine Lorentzian shape spectrum show contribution from each ASE mode and flat spectrum is summed up output of the ASE modes. Calculation manner is given in Sections 3 and 4.
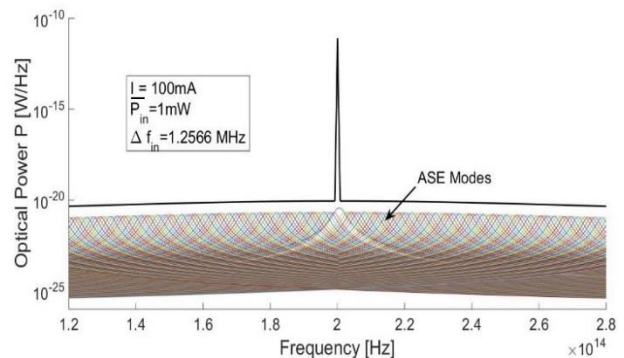

Figure 2 An example calculated of light output spectrum from an SOA

2.3 Gain and Spontaneous Emission

The amplitude $A_m(t,z)$ is factorized to an absolute value and phase term:

$$A_m(t.z) = |A_m(t,z)| e^{j\theta_m(t,z)} , \qquad (10)$$

where θ_m *(t,z)* is the optical phase

The carrying optical power $P_m(t,z)$ of the forward propagating mode along the waveguide is

$$P_m(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_m^{(+)} \times H_m^{(+)} dxdy = 2n_{eq}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|A_m(t,z)\right|^2 \quad [W].$$

$$= \frac{S_m^{(+)}\hbar\omega_m v}{L_f} \tag{11}$$

where $E_m^{(+)}$ and $H_m^{(+)}$ are electric and magnetic components of the mode m, respectively.

Dynamic equation of optical power $P_m(t,z)$ is obtained from Maxwell's wave equation given in equation (2) with suitable manipulation and quantum mechanical modification [14]:

$$\frac{\partial P_m}{\partial z} + \frac{1}{v}\frac{\partial P_m}{\partial t} = (g_m - \kappa)P_m + \frac{\hbar\omega_m v g_{em}}{L_f} + \frac{\hbar\omega_m}{L_f}F_m(t,z) , \tag{12}$$

where g_m is the gain coefficient, κ is the guiding loss coefficient and F_m is the Langevin noise source for photon number. Note that vg_{em} indicates the spontaneous emission.

In this paper, we take up an SOA which is made of InGaAsP system having a quantum well structure. Then, the gain coefficient g_m shows saturation phenomena with increasing of electron density n. We propose here an approximated expression of the gain coefficient as [16].

$$g_m = \frac{\xi a(n - n_g)}{1 + bn} , \tag{13}$$

where ξ is the field confinement factor, a and b are the coefficient to characterize the gain coefficient.

From quantum mechanical point of view, the gain coefficient g_m consists of two parts, g_{em} for optical emission and g_{am} for optical absorption, corresponding to the electron transition from the conduction band to the valence band and that from the valence band to the conduction band, respectively:

$$g_m = g_{em} - g_{am}. \tag{14}$$

Then, these parts are written as

$$g_{em} = \frac{\xi a n}{1 + bn} \tag{15}$$

and

$$g_{am} = \frac{\xi a n_g}{1 + bn} \tag{16}$$

2.4 Examination of Supposed Length L_f

Since the term vg_{em} gives emitting probability of the spontaneous emission, $\dfrac{1}{vg_{em}}$ is a time period in which one photon is emitted in the mode m as the spontaneous emission. During this time period, the photon propagates length L_f with velocity v. After this time period, the next spontaneous emission is generated in the mode m.

Based on discussion mentioned above, we can conclude that the supposed length L_f must be

$$L_f = \frac{v}{vg_{em}} = \frac{1}{g_{em}} . \tag{17}$$

The frequency separation of the longitudinal mode is,

$$\Delta_m \equiv f_{(m+1)o} - f_{mo} = \frac{v}{2L_f} = \frac{vg_{em}}{2} . \tag{18}$$

2.5 Electron Density Variation

The three dimensional volume V_f in the active region corresponding to the supposed length L_f is

$$V_f = wd L_f , \tag{19}$$

and total volume of the active region is

$$V_0 = wd L_0 \tag{20}$$

The variation of electron density n in the volume V_f is given as [14],

$$\frac{dn}{dt} = -\sum_m \frac{g_m P_m}{\hbar\omega_m wd} - \frac{n}{\tau} + \frac{I}{eV_0} + \frac{W(t,z)}{V_f} , \tag{21}$$

where τ is the electron lifetime, e is the electron charge and $W(t,z)$ is the Langevin noise source for the electron number fluctuation.

2.6 Optical Phase Variation

A dynamic equation for variation of the optical phase $\theta_m(t,z)$ is also derived from Maxwell's wave equation given in equation (2) with suitable quantum mechanical approximation [14]:

$$\frac{d\theta_m}{dt} = \frac{\partial\theta_m}{\partial t} + v\frac{\partial\theta_m}{\partial z}$$
$$= \frac{v\alpha\,g'_m}{2}\{n(z) - \bar{n}(0)\} + T_m(t,z) \tag{22}$$

Here, α is the linewidth enhancement factor which indicates change of the refractive index with the electron density [18]. g'_m is derivative of the gain coefficient on the electron density defined by

$$g'_m = \frac{\partial g_m}{\partial n} = \frac{\xi a}{1 + b\bar{n}} . \tag{23}$$

$\bar{n}$ is a temporally averaged (CW) value of the electron density. $T_{mz}(t,z)$ is a noise source giving an intrinsic phase fluctuation associating the electron transition.

The FM noise is driven from the phase variation.

3.0 NOISE ANALYSIS

3.1 Frequency Expansion of Fluctuated Terms

As found from equation (22), the phase fluctuation suffers the fluctuation on the electron density n as well as the intrinsic noise source $T_m(z)$. The fluctuation on the electron density are determined with fluctuation on the optical intensity as shown in equations (12) and (21). Therefore, we need to analyze fluctuations on all temporally changing variables to determine the FM noise.

We expand the noise sources with noise frequency components;

$$F_m(t,z) = \int F_{m\Omega}(z) e^{j\Omega(t-z/v)} d\Omega \quad, \qquad (24)$$

$$W(t,z) = \int W_\Omega(z) e^{j\Omega(t-z/v)} d\Omega \quad. \qquad (25)$$

$$T_m(t,z) = \int T_{m\Omega}(z) e^{j\Omega(t-z/v)} d\Omega \quad. \qquad (26)$$

Here,

$$\Omega = 2\pi f_N \qquad (27)$$

is an angular frequency corresponding to the noise frequency f_N.

In this paper, we expands the temporally changing variables with $\exp[j\Omega(t-z/v)]$ not with $\exp[j\Omega t]$, although we used $\exp[j\Omega t]$ in [14, 16], because the fluctuations should propagate with velocity v same as the optical field in the SOA. Expansion with $\exp[j\Omega(t-z/v)]$ must be suitable to represent the traveling wave, and $\exp[j\Omega t]$ is suitable for the standing wave. This replacing is the second improvement from our previous analyses in [14, 16].

Then the optical power, electron density and gain coefficients are represented with continuous wave (CW) terms and fluctuating terms as follows:

$$P_m(t,z) = \overline{P}_m(z) + \int P_{m\Omega}(z) e^{j\Omega(t-z/v)} d\Omega \quad. \qquad (28)$$

$$n(t,z) = \overline{n}(z) + \int n_\Omega(z) e^{j\Omega(t-z/v)} d\Omega \quad, \qquad (29)$$

$$g_m(t,z) = \overline{g}_m(z) + g'_m \int n_\Omega(z) e^{j\Omega(t-z/v)} d\Omega \quad, \qquad (30)$$

$$g_{em}(t,z) = \overline{g}_{em}(z) + g'_m \int n_\Omega(z) e^{j\Omega(t-z/v)} d\Omega \quad. \qquad (31)$$

3.1 Propagation of Intensity Noise

By substitution of equations (24), (28) – (31) into equation (12), we get equations for spatial variations of the CW term and the fluctuating term of the optical power along z-direction respectively as:

$$\frac{\partial \overline{P}_m}{\partial z} = \{\overline{g}_m - \kappa\} \overline{P}_m \frac{\hbar\omega_m v}{L_f} \overline{g}_{em}, \qquad (32)$$

$$\frac{\partial P_{m\Omega}}{\partial z} = \{\overline{g}_m - \kappa\} P_{m\Omega} + \left\{ \overline{P}_m + \frac{\hbar\omega_m v}{L_f} \right\} g'_m n_\Omega + \frac{\hbar\omega_m}{L_f} F_{m\Omega}$$

$$\tag{33}$$

We also substitute equations (25), (28) - (30) into equation (21) and obtain equations for the CW term and the fluctuating term of the electron density respectively as:

$$0 = \sum_m \frac{\overline{g}_m \overline{P}_m}{\hbar\omega_m wd} + \frac{\overline{n}}{\tau} - \frac{I}{eV_0} , \qquad (34)$$

$$n_\Omega = \frac{\dfrac{W_\Omega}{V_f} - \sum_m \dfrac{\overline{g}_m P_{m\Omega}}{\hbar\omega_m wd}}{j\Omega + \sum_m \dfrac{g'_m \overline{P}_m}{\hbar\omega_m wd} + \dfrac{1}{\tau}} . \qquad (35)$$

Substitution of equation (35) to equation (33) gives,

$$\frac{\partial P_{m\Omega}}{\partial z} = \{\overline{g}_m - \kappa\} P_{m\Omega} + \left\{ \overline{P}_m + \frac{\hbar\omega_m v}{L_f} \right\} g'_m$$

$$\times \left\{ \frac{\dfrac{W_\Omega}{V_f} - \sum_p \dfrac{\overline{g}_p P_{p\Omega}}{\hbar\omega_p wd}}{j\Omega + \sum_p \dfrac{g'_p \overline{P}_p}{\hbar\omega_p wd} + \dfrac{1}{\tau}} \right\} + \frac{\hbar\omega_m}{L_f} F_{m\Omega} , \qquad (36)$$

Now, we derive an equation for auto-correlated value of the intensity fluctuation, because amount of the fluctuation are evaluated with auto-correlated value. From equation (36), we get

$$\frac{\partial \langle P_{m\Omega}^2 \rangle}{\partial z} = \left\langle \frac{\partial P_{m\Omega}}{\partial z} P_{m\Omega}^* + P_{m\Omega} \frac{\partial P_{m\Omega}^*}{\partial z} \right\rangle$$

$$= 2\left[\overline{g}_m - \kappa - \frac{g'_m \overline{g}_m \left\{ \overline{P}_m + \dfrac{\hbar\omega_m v}{L_f} \right\} \left\{ \sum_p \dfrac{g'_m \overline{P}_p}{\hbar\omega_p wd} + \dfrac{1}{\tau} \right\}}{\left\{ \Omega^2 + \left(\sum_p \dfrac{g'_m \overline{P}_p}{\hbar\omega_p wd} + \dfrac{1}{\tau} \right)^2 \right\} \hbar\omega_m wd} \right] \times \langle P_{m\Omega}^2 \rangle$$

$$+ \frac{2g'_m \left\{ \overline{P}_m + \dfrac{\hbar\omega_m v}{L_f} \right\}}{V_f} \times \mathrm{Re}\left\{ \frac{\langle P_{m\Omega} W_\Omega \rangle}{-j\Omega + \sum_p \dfrac{g'_m \overline{P}_p}{\hbar\omega_p wd} + \dfrac{1}{\tau}} \right\}$$

$$+ \frac{2\hbar\omega_m}{L_f} \times \mathrm{Re}\langle P_{m\Omega} F_\Omega \rangle$$

$$\tag{37}$$

Although the auto-correlated value $\langle P_{m\Omega}^2 \rangle$ is given with a real number, the mutual correlated values $\langle P_{m\Omega} W_\Omega \rangle$ and $\langle P_{m\Omega} F_{m\Omega} \rangle$ are given with complex numbers. Varying equations of these mutual correlated values are also obtained from (36) as

$$\frac{\partial \langle P_{m\Omega} W_\Omega \rangle}{\partial z} = (\overline{g}_m - \kappa)\langle P_{m\Omega} W_\Omega \rangle$$

$$- \frac{g'_m \left(\overline{P}_m + \frac{\hbar \omega_m v}{L_f} \right) \overline{g}_m}{\left\{ j\Omega + \sum_p \frac{g'_m \overline{P}_p(z)}{\hbar \omega_p wd} + \frac{1}{\tau} \right\} \hbar \omega_m wd} \sum_p \langle P_{p\Omega} W_\Omega \rangle \quad ,$$

$$+ \frac{g'_m \left(\overline{P}_m + \frac{\hbar \omega_m v}{L_f} \right)}{\left\{ j\Omega + \sum_p \frac{g'_m \overline{P}_p(z)}{\hbar \omega_p wd} + \frac{1}{\tau} \right\} V_f} \langle W_\Omega^2 \rangle + \frac{\hbar \omega_m}{L_f} \langle F_{m\Omega} W_\Omega \rangle$$

(38)

$$\frac{\partial \langle P_{m\Omega} F_{m\Omega} \rangle}{\partial z} = \left[\overline{g}_m - \kappa - \frac{g'_m \overline{g}_m (\overline{P}_m + \hbar \omega_m v/L_f)}{\left\{ j\Omega + \sum_p \frac{g'_m \overline{P}_p}{\hbar \omega_p wd} + \frac{1}{\tau} \right\} \hbar \omega_m wd} \right]$$

$$+ \frac{g'_m (\overline{P}_m + \hbar \omega_m v/L_f)}{\left\{ j\Omega + \sum_p \frac{g'_m \overline{P}_p}{\hbar \omega_p wd} + \frac{1}{\tau} \right\} V_f} \langle W_\Omega F_{m\Omega} \rangle + \frac{\hbar \omega_m}{L_f} \langle F_{m\Omega}^2 \rangle$$

. (39)

Here, we have supposed that $\partial W_\Omega / \partial z$ and $\partial F_{m\Omega} / \partial z$ are very small as a first order approximation.

Auto-correlated value of the carrier density fluctuation is given from equation (35) as

$$\langle n_\Omega^2 \rangle = \left\{ \langle W_\Omega^2 \rangle - \sum_m \frac{2 L_f \overline{g}_m}{\hbar \omega_m wd} \text{Re} \langle P_{m\Omega} W_\Omega \rangle + \sum_m \left(\frac{L_f \overline{g}_m}{\hbar \omega_m} \right)^2 \langle P_{m\Omega}^2 \rangle \right\}$$

$$\div \left[V_f^2 \left\{ \Omega^2 + \left(\sum_m \frac{g'_m \overline{P}_m}{\hbar \omega_m wd} + \frac{1}{\tau} \right)^2 \right\} \right]$$

. (40)

The correlated values of Langevin noise sources are given with CW components of the electron density and the optical power as:

$$\langle F_{m\Omega}^2 \rangle = \frac{\overline{g}_{em} + g_{am} + \kappa}{\hbar \omega_m} L_f \overline{P}_m + v \overline{g}_{em} ,$$

(41)

$$\langle W_\Omega^2 \rangle = \sum_m \frac{\overline{g}_{em} + g_{am}}{\hbar \omega_m} L_f \overline{P}_m + \frac{V_f \overline{n}}{\tau} + \frac{V_f I}{V_0 e} ,$$

(42)

$$\langle F_{m\Omega} W_\Omega \rangle = \langle W_\Omega F_{m\Omega} \rangle = -\frac{\overline{g}_{em} + g_{am}}{\hbar \omega_m} L_f \overline{P}_m - v \overline{g}_{em} ,$$

(43)

3.2　Propagation of FM Noise

We define the shift of operating optical frequency of mode m from the original frequency f_{mo} to be f_m.

The shift of the frequency f_m is derived from the optical phase $\theta_m(t,z)$ in equation (10) as,

$$\frac{\partial \theta_m(t,z)}{\partial t} = 2\pi f_m = 2\pi \left\{ \overline{f}_m(z) + \int f_{m\Omega}(z) e^{j\Omega(t - z/v)} d\Omega \right\},$$

(44)

where $\overline{f}_m$ is temporally averaged component and $f_{m\Omega}$ is fluctuating term. The FM noise is given by this fluctuating term as $\langle f_{m\Omega}^2 \rangle$.

The optical phase is expressed from equation (44) to be

$$\theta_m(t,z) = 2\pi \left\{ \overline{f}_m(z) t + \int \frac{f_{m\Omega}(z) e^{j\Omega(t - z/v)}}{j\Omega} d\Omega \right\}.$$

(45)

By substituting equations (26), (29), (44) and (45) to equation (22), we get an equation for spatial variation of the fluctuating term of the optical frequency as:

$$\frac{\partial f_{m\Omega}(z)}{\partial z} = \frac{j\Omega}{2\pi} \left\{ \frac{\alpha g'_m}{2} n_\Omega(z) + \frac{T_{m\Omega}(z)}{v} \right\}.$$

(46)

Then, variation of the auto-correlated value of $\langle f_{m\Omega}^2 \rangle$ is given as

$$\frac{\partial \langle f_{m\Omega}^2 \rangle}{\partial z} = \left\langle \frac{\partial f_{m\Omega}}{\partial z} f_{m\Omega}^* + f_{m\Omega} \frac{\partial f_{m\Omega}^*}{\partial z} \right\rangle$$

$$= \frac{\Omega \alpha g'_m}{2\pi} \text{Im} \langle f_{m\Omega} n_\Omega \rangle + \frac{\Omega}{\pi v} \text{Im} \langle f_{m\Omega} T_{m\Omega} \rangle$$

(47)

Equations for variation of $\text{Im} \langle f_{m\Omega} n_\Omega \rangle$ and $\text{Im} \langle f_{m\Omega} T_{m\Omega} \rangle$ are obtained from equation (46) as

$$\frac{\partial \langle f_{m\Omega} n_\Omega \rangle}{\partial z} = \frac{j\Omega \alpha g'_m}{4\pi} \langle n_\Omega^2 \rangle,$$

(48)

$$\frac{\partial \langle f_{m\Omega} T_{m\Omega} \rangle}{\partial z} = \frac{j\Omega}{2\pi v} \langle T_{m\Omega}^2 \rangle.$$

(49)

Here, we have assumed that $\partial n_\Omega / \partial z$ is very small and are using the fact that mutual correlation between the fluctuation on the electron density n_Ω and the generating term $T_{m\Omega}$ is zero.

From equations (48) and (49), the terms $\text{Im} \langle f_{m\Omega} n_\Omega \rangle$ and $\text{Im} \langle f_{m\Omega} T_{m\Omega} \rangle$ are given as,

$$\text{Im} \langle f_{m\Omega}(z) n_\Omega(z) \rangle = \frac{\Omega \alpha g'_m}{4\pi} \int_0^z \langle n_\Omega^2(z') \rangle dz', \quad (50)$$

$$\text{Im} \langle f_{m\Omega}(z) T_{m\Omega}(z) \rangle = \frac{\Omega}{2\pi v} \int_0^z \langle T_{m\Omega}^2(z') \rangle dz'. \quad (51)$$

Therefore, the frequency noise $\langle f_{m\Omega}^2(z) \rangle$ at position z is expressed by next equation:

$$\langle f_{m\Omega}^2(z) \rangle = \frac{\Omega^2}{2\pi^2} \int_0^z \int_0^{z'} \left\{ \left(\frac{\alpha_m g'_m}{2} \right)^2 \langle n_\Omega^2(z'') \rangle + \frac{1}{v^2} \langle T_{m\Omega}^2(z'') \rangle \right\} dz'' dz'$$

$$+ \langle f_{m\Omega}^2(0) \rangle$$

[Hz²/Hz].　(52)

We find from this equation that the FM noise is proportional to square value of the noise frequency $f_N = \Omega/2\pi$. If the integrand variables are almost uniformly distributed along z-direction, the double integral gives $\int_0^z \int_0^{z'} dz'' dz' = z^2/2$. Then the FM noise of the output light is almost proportional to $(\Omega L_o)^2$.

Amount of the intrinsic phase noise source $\langle T_{m\Omega}^2 \rangle$ is obtained as a divided value of the intensity noise source $\langle F_{m\Omega}^2 \rangle$ by intensity of the electric field [14]. As discussed with equation (6) in Subsection 2.2 of this paper, the zero-point energy $(1/2)\hbar\omega_m$ is originally obtained for a standing wave which is superposed field of the forward and the backward propagating waves. Then we suppose in this paper, the zero-point energy can be shared with the forward and the backward waves by $(1/4)\hbar\omega_m$ each.

$$\langle T_{m\Omega}^2 \rangle = \frac{\langle F_{m\Omega}^2 \rangle}{4\left\{ S_m^{(+)} + \frac{1}{4} \right\}^2} = \frac{\dfrac{\overline{g}_{em} + g_{am} + \kappa}{\hbar\omega_m} L_f \overline{P}_m + v\overline{g}_{em}}{4\left(\dfrac{L_f}{\hbar\omega_m} \overline{P}_m + \dfrac{1}{4} \right)^2}$$

(53)

This equation is slightly differ from similar equation in ref. [14] on the treatment of the zero-point energy. This change is the third improvement from our previous analysis in ref. [14].

3.3 Spectrum Linewidth and Measurable Spectrum Profile

The spectrum linewidth Δf_m is given with the FM noise $\langle f_{m\Omega}^2 \rangle$ by putting $\Omega = 0$ to be

$$\Delta f_m = 4\pi \langle f_{m0}^2 \rangle \quad . \tag{54}$$

Since the FM noise is proportional to Ω^2, the spectrum linewidth never increase by amplification in the SOA and is fixed by the initial condition at $z = 0$.

Measuring optical spectrum is not the modal power $\overline{P}_m$ but a dispersed spectrum. When we suppose the Lorentzian distribution for measuring optical frequency f to each longitudinal mode at f_{mo} with the half linewidth Δf_m, the measurable optical power $P(f)$ is given by

$$P(f) = \sum_m \frac{\Delta f_m \overline{P}_m}{(f - f_{mo})^2 + (\Delta f_m/2)^2} \times \frac{2}{1 + \delta_{m,s}} \quad \text{[W/Hz]}.$$

(55)

In this paper, we denote the signal mode by the input light is to be $m = s$ and the ASE modes to be $m \neq s$. The signal mode is a TE mode (E_x) but the ASE modes are generated for both TE and TM modes (E_x and E_y). $\delta_{m,s}$ is introduced to count this assumption.

3.4 Initial Conditions

To obtain the FM noise $\langle f_{m\Omega}^2 \rangle$ and the spectrum linewidth Δf_m of the output light, we have to trace values of $\overline{n}(z)$, $\overline{P}_m(z)$, $\langle n_\Omega^2(z) \rangle$, $\langle P_{m\Omega}^2(z) \rangle$, $\langle P_{m\Omega}(z)W_\Omega(z) \rangle$ and $\langle P_{m\Omega}(z)F_{m\Omega}(z) \rangle$ from $z = 0$ to $z = L_o$.

The initial conditions at $z = 0$ are,

$$\overline{P}_s(0) = \overline{P}_{in} \quad \text{for } m = s, \tag{56}$$

$$\overline{P}_m(0) = 0 \quad \text{for } m \neq s, \tag{57}$$

$$\langle P_{s\Omega}^2(0) \rangle = \langle P_\Omega^2 \rangle_{in} \quad \text{for } m = s, \tag{58}$$

$$\langle P_{m\Omega}^2(0) \rangle = 0 \quad \text{for } m \neq s, \tag{59}$$

$$\langle f_{s\Omega}^2(0) \rangle = \langle f_{in\Omega}^2 \rangle \quad \text{for m=s}. \tag{60}$$

We should be careful about the initial condition for the FM noise of the ASE mode. At the position $z = 0$, $\partial\theta_m/\partial z = 0$ can be supposed, but $\partial\theta_m/\partial t = 2\pi f_m$ is not zero. From equation (22), the initial condition becomes

$$\langle f_{m\Omega}^2(0) \rangle = \frac{1}{4\pi^2}\left\{ \left(\frac{vg_m'\alpha}{2} \right)^2 \langle n_\Omega^2(0) \rangle + \langle T_{m\Omega}^2(0) \rangle \right\} \quad \text{for } m \neq s$$

(61)

Since $\overline{P}_m(0) = P_{m\Omega}(0) = 0$ at $z = 0$, $\langle T_{m\Omega}^2(0) \rangle = 4v\overline{g}_{em}(0)$ is obtained, which is much larger than the first term in the right side in equation (61). Then equation (61) is rewritten as

$$\langle f_{m\Omega}^2(0) \rangle = \frac{v\overline{g}_{em}(0)}{\pi^2} \quad \text{for } m \neq s. \tag{62}$$

Initial values of the linewidth are

$$\Delta f_s(0) = 4\pi \langle f_{in0}^2 \rangle = \Delta f_{in} \quad \text{for } m = s, \tag{63}$$

$$\Delta f_m(0) = \frac{4v\overline{g}_{em}(0)}{\pi} \quad \text{for } m \neq s. \tag{64}$$

4.0 CALCULATED DATA

As the model used for numerical calculation, an SOA made of an InGaAsP system is supposed. The geometrical and material parameters used in the numerical calculation are listed in Table 1. These numerical parameters were obtained from [14, 19].

Table 1 Numerical Values of Used Parameters [14, 19]

Symbol	Parameter	Value	Unit
w	active region width	2×10^{-6}	m
d	active region thickness	4×10^{-8}	m
L_0	amplifier length	1×10^{-3}	m
V_o	volume of active region	8×10^{-17}	m³
λ	optical wavelength	1.55×10^{-6}	m
$\hbar\omega$	photon energy	0.8	eV
n_g	transparent electron density	2×10^{24}	m⁻³
τ	electron lifetime	8.6×10^{-10}	s
n_{eq}	equivalent refractive index	3.5	
κ	guiding loss coefficient	3030	m⁻¹
α	linewidth enhancement factor	3	
ξ	confinement factor	3.5×10^{-2}	
a	coefficient in equation for gain	1.345×10^{-19}	m²
b	coefficient in equation for gain	3.583×10^{-25}	m³
$\Delta\lambda$	Half wavelength of the spontaneous emission	8×10^{-8}	m

The gain coefficient and the spontaneous emission are assumed to be identical for all modes within half wavelength $\Delta\lambda$ of the spontaneous emission, and those for the other modes outside of $\Delta\lambda$ are to be zero in order to simplify the calculation. The number of counted longitudinal modes within $\Delta\lambda$ is

$$M = \frac{2n_{eq}L_f\,\Delta\lambda}{\lambda^2} \ . \tag{65}$$

As the operating situation, the relative intensity noise (RIN), the FM noise and the linewidth for the input signal light are assumed to be $RIN_{in} = \left\langle P_{in\Omega}^2 \right\rangle / \bar{P}_{in}^2 = 10^{-15}\,\mathrm{Hz}^{-1}$, $\left\langle f_{in\Omega}^2 \right\rangle = 100\mathrm{KHz}$ and $\Delta f_{in} = 4\pi\left\langle f_{in0}^2 \right\rangle = 1.26\times10^6\,\mathrm{Hz}$ thorough out this paper .

A calculated example of output spectrum from the SOA has been given in Figure 3. The linewidth of the signal mode is not changed from that of the inputted signal light Δf_{in}. Although we supposed discrete frequency f_{m0} for the ASE mode, obtained ASE spectrum profile becomes continuous as shown in Figure 3 because of sufficiently large value of $\Delta f_m(0)$.

As given in equation (52), main valuables to decide the FM noise are the electron density fluctuation $\left\langle n_\Omega^2 \right\rangle$ and the intrinsic phase fluctuation $\left\langle T_{m\Omega}^2 \right\rangle$. These values are changed with the CW components of the electron density and the optical power.

Variation of the electron density $\bar{n}(z)$ along propagating distance z is shown in Figure 4. When

the input signal power $\bar{P}_{in}$ exceeds 1 mW, saturation or reduction of the electron density becomes large.
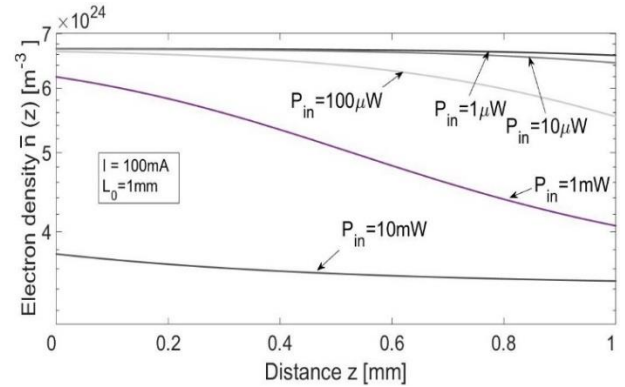

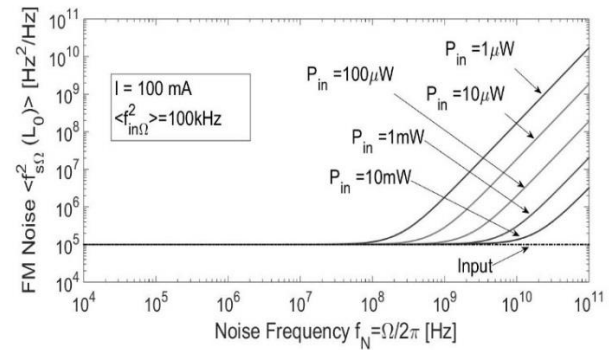**Figure 3** Variation of electron density along propagation

Figure 4 Frequency dispersion of the FM Noise

Frequency dispersion of the FM noise of output signal light $\left\langle f_{s\Omega}^2(L_0) \right\rangle$ is shown in Figure 5. As found from equation (52), the FM noise added in the SOA is proportional to Ω^2, the FM noise hardly increase in the lower frequency region, but rapidly increase in the high frequency region.
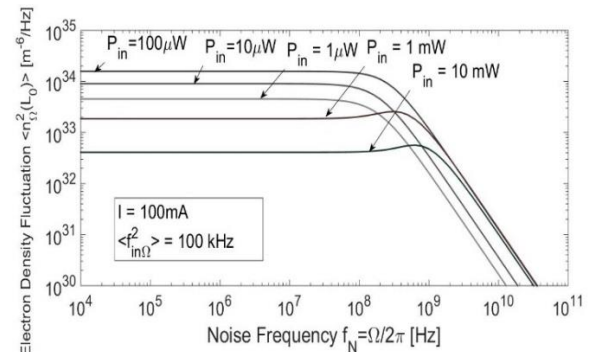

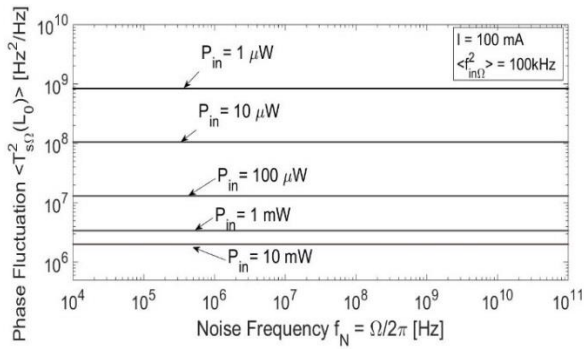**Figure 5** Electron Density Fluctuation

Figure 7 Intrinsic Phase Fluctuation

Figure 6 FM Noise with several linewidth enhancement factor

Variations of the carrier density fluctuation $\left\langle n_{\Omega}^2(L_o) \right\rangle$ and the intrinsic phase fluctuation $\left\langle T_{s\Omega}^2(L_o) \right\rangle$ with the noise frequency f_N are shown in Figures 6 and 7, respectively.

The electron density fluctuation $\left\langle n_{\Omega}^2(L_o) \right\rangle$ reduces in higher frequency region than several 100 MHz as shown in Figure 6. On the other hand, the intrinsic phase fluctuation $\left\langle T_{s\Omega}^2(L_o) \right\rangle$ keeps same value for whole frequency range as in Figure 7. Meanwhile, the FM noise increases in the higher frequency region as has been shown in Figure 5. These facts mean that the higher FM noise in the higher frequency region comes only from the intrinsic phase fluctuation $\left\langle T_{s\Omega}^2(L_o) \right\rangle$.

The effect of the linewidth enhancement factor α on the FM noise is examined as shown in Figure 8 by changing value of the linewidth enhancement factor. We find that the linewidth enhancement factor scarcely affects on the FM noise characteristic, because the FM noise in the low frequency region is low enough and that in the high frequency region does not suffer the electron density fluctuation.

Since we define the longitudinal modes for both the inputted signal mode and the ASE modes holding the orthogonal relations, there is no direct interaction among these modes. Possibility to induce interaction among these modes is through the electron density fluctuation $\left\langle n_{\Omega}^2 \right\rangle$ which is caused by all existing modes. However, the effect from the electron density fluctuation on the FM noise is very weak. Then, we can conclude that the ASE does not affect on the FM noise of the inputted optical signal.
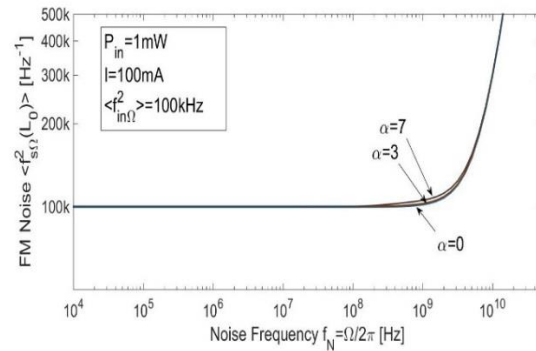
From equation (53), we find that the intrinsic phase fluctuation $\left\langle T_{s\Omega}^2 \right\rangle$ of the signal mode becomes smaller by increasing the optical power $\overline{P}_s$. Then the FM noise given in Figure 5 shows the smaller value for the higher input optical power $\overline{P}_{in}$.

5.0 EXPERIMENTAL VERIFICATION

The setup used for the experimental measurements is illustrated in Figure 9. The FM noise was measured using an interferometer and evaluated electrically through a photodetector (PD) similar with refs. [20, 21].
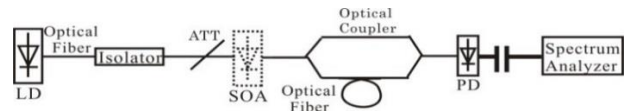

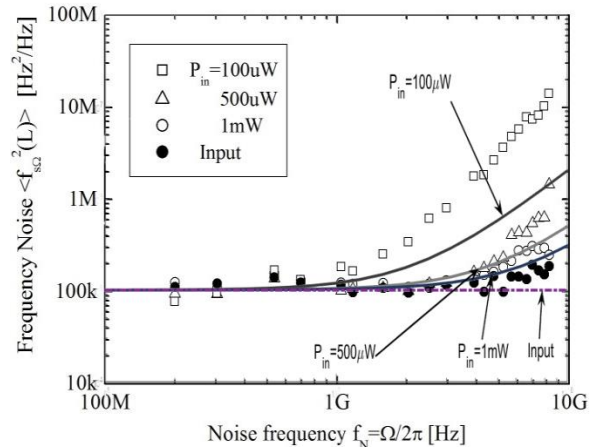**Figure 9** Setup used for experimental measurements

Figure 10 Experimentally measured and theoretically calculated FM noise spectrum. The solid lines indicate calculated theoretical results. The dot-dash line indicates input FM noise
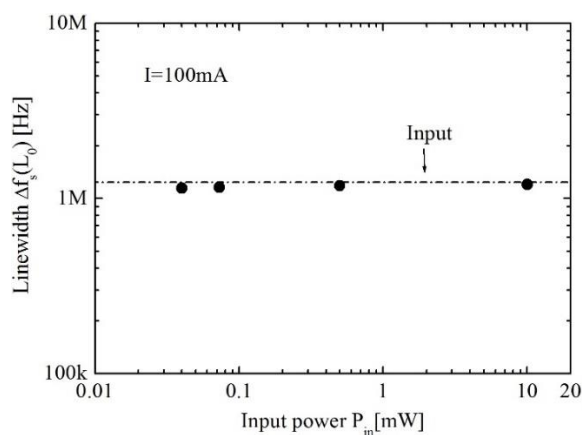
Figure 11 Experimentally measured values for the linewidth. The linewidth of the output light hardly change from that of the input light

We measured the FM noise of the DFB laser (LD) used as the source of the input light for the SOA. The LD oscillation wavelength was 1545.6nm, the driving current was 150mA and output power from LD was 23.7mW. The output power was adjusted to be 10mW by an optical attenuator (ATT).

We inserted the SOA by connecting it to the ATT and measured the FM noise of the output light from the SOA. The FM noise level of the input light was fixed by setting the injection current of the LD to be 100 KHz, while the input power of the SOA was adjusted by the ATT. The driving current of the SOA was I =100mA.

Experimentaly measured and theoretically calculated FM noise spectrum are shown in Figure 10. We found that the characteristics of frequency dependence of the FM noise of output light can be well explained by our theoretical analysis. Slight diffences between the experimental data and theoretical analysis may come from insufficient selection of numerical parameters given in Table 1.

In the low frequency region, the FM noise of the output light is almost same as that of the input light. Meanwhile, the FM noise increases with the noise frequency f_N in the higher frequency region than 1GHz especially for lower input power level.

Figure 11 shows the experimentally measured value of linewidth. The output linewidth is hardly changed from the input light. This condition is just the same with our theoretical analysis.

6.0 CONCLUSION

The characteristics of FM noise and linewidth of semiconductor optical amplifier (SOA) without facet mirrors were theoretically analyzed and experimentally confirmed. The concept of discrete longitudinal mode for the spontaneous emission was introduced as the basis of quantum mechanical characteristics, allowing the quantitative

examination of noise sources for fluctuation of the photon number, the electron density and the optical phase. The continuously broaden output spectrum profile of the amplified spontaneous emission (ASE) was well explained as a spectrum broadening of each longitudinal mode.

Obtained results by theoretical analysis are as followings:

1. The linewidth of the inputted signal light is hardly changed by optical amplification in the SOA.

2. The FM noise increases proportional to the square value of the noise frequency.

3. Fluctuation on the electron density affects on the FM noise in the low frequency region. However, effects by the electron density fluctuation is very weak because the FM noise is low enough in the low frequency region.

4. Therefore, the linewidth enhancement factor scarcely affects on the FM noise in the SOA. This results is very different property from the semiconductor laser.

5. The FM noise in the high frequency region is caused by the intrinsic phase fluctuation on the optical emission, and becomes the stronger for the lower optical power.

6. The amplified spontaneous emission hardly affects on the FM noise of the inputted signal light in the SOA.

We also experimentally confirmed the terms 1 and 2.

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