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EMPIRICAL FORMULAE TO PREDICT PRESSURE AND IMPULSIVE ASYMPTOTES FOR P-I DIAGRAMS OF RC COLUMNS STRENGTHENED WITH FRP

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Abstract. There are only limited studies that directly correlate the increase in structural capacities in resisting the blast loads with the fiber reinforced polymer (FRP) strengthenin.. In this paper, numerical analyses of dynamic response and damage of reinforced concrete (RC) columns strengthened with FRP to blast loads are carried out using the commercial software LS-DYNA. A series of simulations are performed to predict the blast response and damage of columns with different FRP type. The simulations also involved parametric studies by varying the FRP thickness, configuration, different column dimension, concrete strength, and longitudinal and transverse reinforcement ratio. The numerical results are used to develop pressure-impulse (P-I) diagrams of FRP strengthened RC columns. Based on the numerical results, the empirical formulae are derived to calculate the pressure and impulse asymptotes of the P-I diagrams of RC columns strengthened with FRP

Keywords: Strengthening; blast loads; FRP; P-I diagrams

Abstrak. Kajian terhadap keupayaan struktur dalam menahan beban letupan menggunakan Fiber Reinforced Polymer (FRP) adalah sangat terhad. Dalam kajian ini, satu analisis terhadap keupayaan FRP bagi menahan beban letupan dilakukan. Tujuan analisis ini adalah untuk memperolehi hubungan antara kekuatan FRP, bilangan lapisan ketebalan FRP dan susunatur FRP bagi menahan kekuatan sesuatu beban letupan. Kajian ini dilakukan mengunakan model tiang diperkukuh dengan FRP yang dibina menggunakan perisian LS-DYNA. Ia melibatkan beberapa siri simulasi untuk meramalkan tindakbalas letupan dan kerosakkan pada tiang sekiranya sesuatu beban letupan dikenakan. Melalui simulasi ini, kekuatan FRP, bilangan lapisan ketebalan FRP dan susunatur FRP dapat ditentukan. melalui keputusan-keputusan yang diperolehi, pressure-impulse diagram (P-I) bagi tiang yang diperkukuhkan dengan FRP dapat dibentuk

Kata kunci: Pengukuhan; beban letupan; FRP; P-I diagrams

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1.0 INTRODUCTION

Since the early 1990s, the FRP composites have been widely used to strengthen existing concrete and other structures in resisting the blast and impact loads. It has been proven that FRP strengthening is highly effective at preventing injuries from explosive bombs [1,2]. However, despite of a number of studies that demonstrated the effectiveness of FRP strengthening, no systematic study that quantifies the blast resistance capacities of RC columns with various FRP strengthening measures has been reported. The P-I curves can be utilized to quantify the column capacities in blast loading resistance.

This paper performs numerical simulations of responses of RC columns with or without FRP strengthening measures to blast loads. The numerical models are developed in LS-DYNA and are verified with results presented by other researchers. The numerical results are used to formulate empirical formulae to predict pressure asymptote, P_o and impulsive asymptote, I_o to generate P-I diagrams for RC columns strengthened with FRP.

2.0 METHOD TO DEVELOP P-I DIAGRAMS USING EMPIRICAL FORMULAE

P-I Diagrams can be developed based on a Single Degree of freedom Model (SDOF) model, simplified numerical method and using experiment results [3, 4, 5, 6, 7]. On the subject of RC columns, the SDOF method may not very accurately reflect the true behavior of a structure due to the rigid plastic material idealization, negligence of strain rate effects and axial force effects in the analysis, and incapable of predicting local failure. The explosion test is good to derive the P-I diagrams however it needs a very large amount of data therefore it is very expensive. With the development of computer technology [8], numerical method is determined to be reliable to analyze the RC column damage to blast loads.

Shi *et al.* [6] proposed the simplified numerical technique to develop the P-I diagrams for RC columns. The method proposed in [6] consists of four stages to estimate the damage index, D as illustrated in Figure 1.

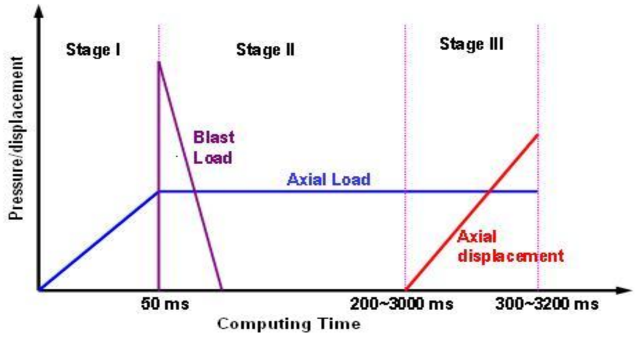

Figure1 Loading procedures to determine the damage index (Shi *et al.* 2008)

They define D as Eq. (1), in which $P_{Residual}$ is the residual axial load-carrying capacity of the damaged RC column and P_{Design} is the maximum axial load carrying capacity of the undamaged RC column [9,10].

$$D = 1 - \frac{P_{residual}}{P_{Design}} \tag{1}$$

The different values of D are correlated to different damage degrees; in particular D=0 - 0.2, low damage; 0.2 - 0.5, medium damage; 0.5 - 0.8 high damage and 0.8 - 1 collapse. An examination of fitted P-I diagrams in [6] finds that P-I diagram for RC columns can be expressed analytically as

$$(\boldsymbol{P} - \boldsymbol{P_O})(\boldsymbol{I} - \boldsymbol{I_O}) = 12\left(\frac{\boldsymbol{P_O}}{2} + \frac{\boldsymbol{I_O}}{2}\right)^{1.5} \tag{2}$$

where P_o and I_o are the pressure and impulse asymptotes respectively. Figure 2 shows typical P-I diagrams with P_O and I_O of different *D*.

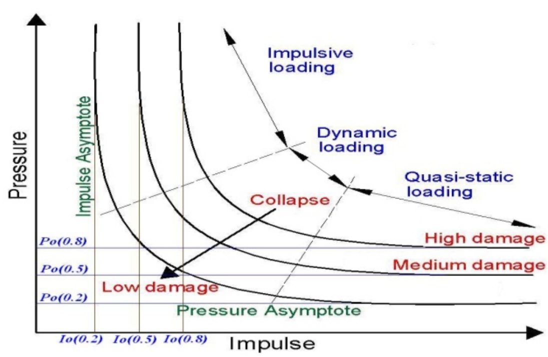

Figure 2 P-I diagrams with different damage index, *D*

In this study, empirical formulae to predict P_0 and I_0 are derived from a series of numerical results for RC columns with and without FRP strengthening using the least squares-fitting method. They are expressed as a function of transverse reinforcement ratio ρ_s, longitudinal reinforcement ratio ρ, concrete strength f_c, column height H, column depth h, column width b, FRP wrap and strip strength f_{wrap} and f_{strip} and FRP wrap thicknesses $t_{wrap.}$

2.0 VERIFICATION OF NUMERICAL MODEL

The commercial software LS-DYNA is employed for numerical modelling throughout this research to calculate responses of the non-retrofitted and FRP strengthened RC columns under blast loading. Material model 72Rel3 (MAT _CONCRETE_DAMAGE_REL3) is chosen to model the concrete, both longitudinal reinforcements and cross-ties reinforcement are modelled with material model MAT_PIECEWISE_LINEAR_ PLASTICITY and the material model MAT_ENHANCED-COMPOSITE_DAMAGE_TITLE (Material model 54) is used to model FRP composite [11]. In this study, the dynamic increase factor (DIF) of the tensile strength of concrete is determined with the empirical formulae proposed by [12], strain rate enhancement of concrete in compression is given by [13] and the strain rate effect for steel is based on model given in [14]. Bond slip between the concrete and steel rebar is accounted for by using the contact function CONTACT 1D in LS-DYNA and the AUTOMATIC_SURFACE_TO_SURFACE_ TIEBREAK contact option in LS-DYNA is employed to model the adhesive contact between the concrete column and FRP [11].

In order to validate the present model, numerical analysis of unstrengthened and FRP strengthened RC column are compared with the field test and previous numerical results. Baylot and Bevins [15] conducted field test of a quarter scale unstrengthened RC column under blast loads and the results are calibrated with their numerical results. The column dimensions are 85 x 85 mm cross section and 935 mm height. The diameters of longitudinal reinforcements and the cross tie reinforcements are 7.1 mm and 3.85 mm respectively. The peak pressure and impulse of the blast load are 7000 kPa and 1100 kPa ms as measured in the test. Table 1 gives the material properties of the concrete and steel reinforcement of the scaled column. This scaled column is analyzed in this study to verify the present numerical model for unstrengthened RC column. The numerical results are also compared with those given in [6].

Table1 Material properties of concrete and steel reinforcement [15]

Material Properties	
Unconfined concrete strength (MPa)	42
Yield stress of logitudinal steel (MPa)	450
Ultimate stress of longitudinal steel (MPa)	510
Fracture strain of longitudinal steel	18%
Yield stress of cross-tie hoop steel (MPa)	400
Ultimate stress of cross-tie hoop steel (MPa)	610
Fracture strain of cross-tie hoop steel	18%

For FRP strengthened RC column, the numerical model is verified by comparing the simulation results with those by Crawford *et al.* [16]. A RC column of dimension 750x750 mm cross section and 3650 mm height was modeled. The reinforcements are ASTM A615 Grade 60 steel bar with a rupture strain of 13 percent. Eight longitudinal rebar with 32 mm diameter and 10 mm diameters of stirrup at 450 centre to centre are used. The concrete has a nominal strength of 34.5 MPa and 29 GPa modulus of elasticity. The column is fully wrapped with six layers of FRP jackets of 0.5 mm thickness, strength 372 MPa and stiffness 52 GPa.

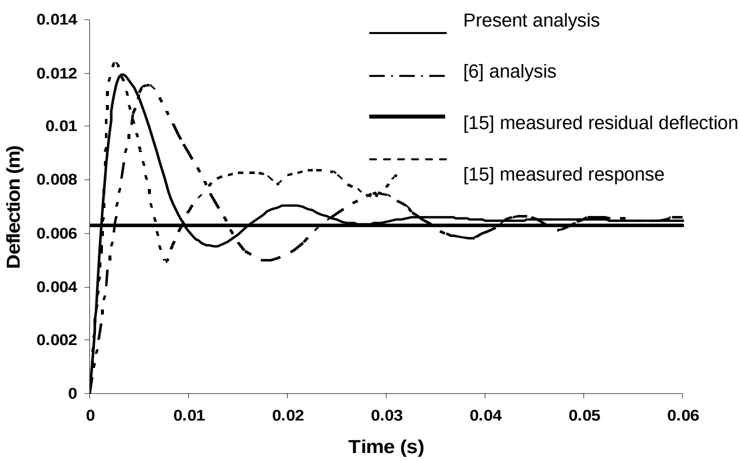

Figure 3 Maximum deflection time-histories

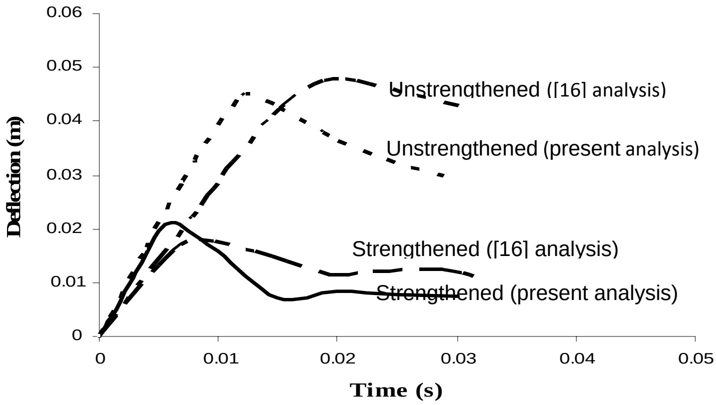

Figure 4. Comparison of responses of FRP strengthened columns

The comparison of the calculated and measured deflection time histories for unstrengthened RC column and FRP strengthened RC column are shown in Figure 3 and 4, respectively. From these figures one can find that the present model gives a reliable prediction of the column response.

3.0 EMPIRICAL FORMULAE DERIVATION

In order to derive the analytical formulae of pressure and impulse asymptotes of P-I diagrams, a series of numerical simulations are carried out. The FRP parameters considered in the simulations include the FRP wrap and strip strength, f_{wrap} and f_{strip}, all in MPa, [17, 18, 19], and thicknesses t_{wrap}, in mm. The RC column parameters considered are concrete strength f_{cu}, in MPa, column height H, column width b, column depth d, all in mm and the longitudinal and transverse reinforcement ratio, ρ and ρ_s, respectively. Figure 5 shows the RC column details and the FRP strengthened RC columns are illustrated in Figure 6. The empirical formulae are derived using curve fitting method.

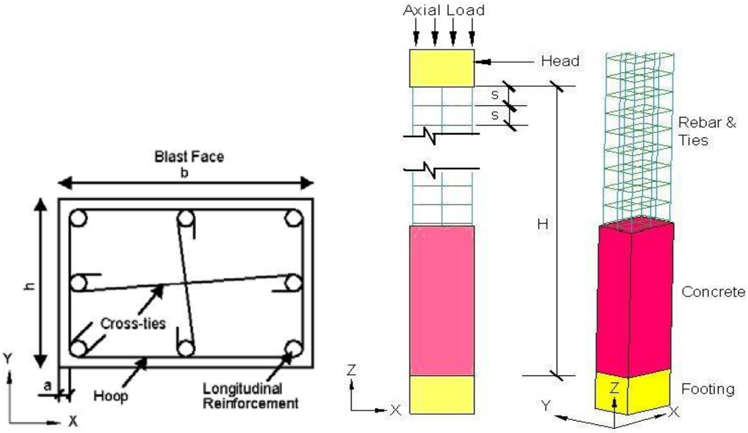

Figure 5 Details of RC column

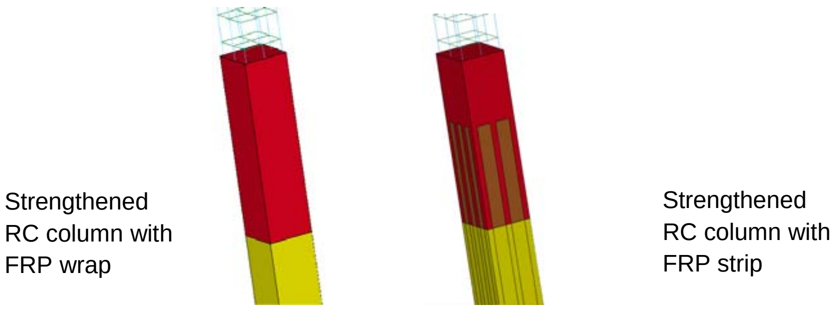

Figure 6 FRP strengthened RC column configurations

The derived empirical formulae for the impulsive asymptote (I_o) and pressure asymptote (P_o) at different damage level are given below, in which I_o is in kPa.ms and P_o is in kPa,

$$\boldsymbol{P_O}(0.2) = 7.25\boldsymbol{f_{cu}} + 2.37\boldsymbol{d} - 0.147\boldsymbol{H} - 0.414\boldsymbol{b} + 7342.47\rho + 10073.44\rho_s + \alpha_1 \quad (3)$$

$$\boldsymbol{I_O}(0.2) = 25\boldsymbol{f_{cu}} + 7.289\boldsymbol{d} - 0.158\boldsymbol{H} - 0.168\boldsymbol{b} + 19261.3\rho + 44864.881\rho_s - 2398.62 + \alpha_2 \quad (4)$$

$$\boldsymbol{P_O}(0.5) = 2\boldsymbol{f_{cu}} + 3.174\boldsymbol{d} - 0.217\boldsymbol{H} - 0.445\boldsymbol{b} + 15786.72\rho + 18137.95\rho_s + 210 + \alpha_3 \quad (5)$$

$$I_O(0.5) = 27.5 f_{cu} + 9.75d - 0.168H - 1.776b + 13121.77\rho + 29433.94\rho_s - 1848.178 + \alpha_4 \quad (6)$$

$$P_O(0.8) = 11 f_{cu} + 3.456d - 0.268H - 1.552b + 14753.44\rho + 8924.068\rho_s + 851.90 + \alpha_5 \quad (7)$$

$$I_O(0.8) = 59 f_{cu} + 13.16d - 0.43H - 0.26b + 1091.78\rho + 489.97\rho_s - 3302.33 + \alpha_6 \quad (8)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 = 0$ for non-retrofitted RC columns, while for FRP strengthened RC columns,

$$\alpha_1 = \exp(0.000169 f_{strip} + 0.000423 f_{wrap} + 0.252 t_{wrap} + 3.114) \quad (9)$$

$$\alpha_2 = \exp(0.000163 f_{strip} - 0.000132 f_{wrap} + 0.307 t_{wrap} + 5.09) \quad (10)$$

$$\alpha_3 = 0.0539 f_{strip} - 0.00909 f_{wrap} + 54.53 t_{wrap} + 32.302 \quad (11)$$

$$\alpha_4 = \exp(-0.00000295 f_{strip} + 0.00124 f_{wrap} + 0.382 t_{wrap} + 2.524) \quad (12)$$

$$\alpha_5 = \exp(0.000189 f_{strip} + 0.0000795 f_{wrap} 0.16 t_{wrap} + 4.286) \quad (13)$$

$$\alpha_6 = \exp(0.0000868 f_{strip} + 0.0012 f_{wrap} + 0.549 t_{wrap} + 2.068) \quad (14)$$

The above empirical formulae are valid for reinforcement steel strength 550 MPa. For reinforcements with other strengths, the equivalent longitudinal and transverse steel area A_{se} should be used when calculating the respective reinforcement ratio.

$$A_{se} = \frac{f_y}{550} A_s \quad (15)$$

4.0 DEVELOPMENT OF P-I DIAGRAMS USING EMPIRICAL FORMULAE

To develop the P-I diagram, initially P_O and I_O of different damage level are calculated using Eq. (3)-(8) as derived in the foregoing. Afterward, Eq. (2) is employed to plot the P-I diagrams. Figure 7-9 show P-I diagrams for unstrengthened RC column, strengthened RC column with FRP wrap and strengthened RC column with FRP wrap and strips respectively. Figure. 7 shows the calculated **P-I** curve for unstrengthened **RC** column in comparison with numerical calculation.

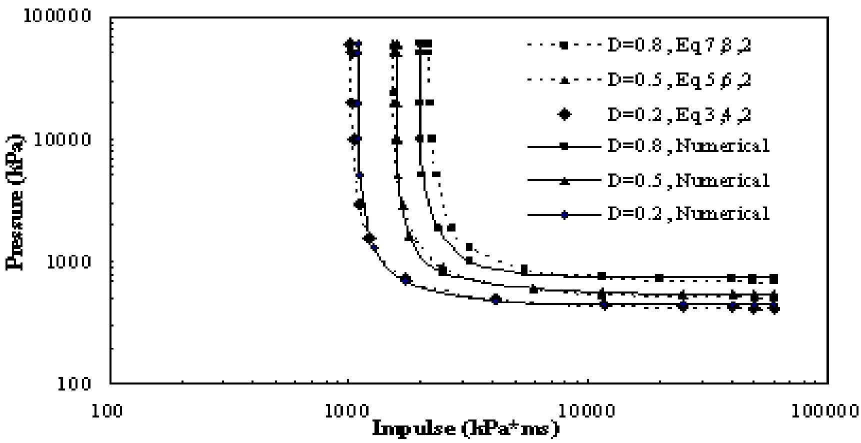

Figure7 Comparison of P-I curves from Equation 2 and the fitted numerical data for unstrengthened RC column (h=600mm, d=400mm, H=4600mm, f_{cu}=40MPa, ρ=0.01, $ρ_s$=0.006)

Based on the **P-I** curved derived from the empirical formulae as shown in Figure 7, it is seen that the P_0 of light, medium and severe damage for unstrengthened **RC** column are 440 kPa, 550 kPa and 730 kPa respectively. While I_0 are 1100 kPa.ms, 1600 kPa.ms and 2000 kPa.ms for light, medium and severe damage respectively.

The **P-I** diagrams for **RC** column strengthened with **FRP** wrap shown in Figure 8 reveals the improvement in blast resistant capacity of the **RC** column especially at impulsive region since the **FRP** wrap provides an addition tensile hoop stress. The **FRP** wrap increases the P_0 of light damage 25% to 550 kPa, 22% of medium damage to 670 kPa and 12% of light damage to 820 kPa. While I_0 are 1430 kPa.ms, 2200 kPa.ms and 2800 kPa.ms for light, medium and severe damage increased to 30%, 38% and 40% respectively.

Figure 8 shows the **P-I** curve for **RC** column strengthened with **FRP** wrap and strip. The strips are mainly acting as additional flexural reinforcements; hence improve the flexural resistant capacity of **RC** column. The P_0 of light, medium and severe damage for unstrengthened **RC** column are 640 kPa, 825 kPa and 910 kPa respectively. While I_0 are 1450 kPa.ms, 2240 kPa.ms and 3100 kPa.ms for light, medium and severe damage respectively. It is evidenced that the FRP strengthening scheme increased the P_o up to 50% (medium damage) and I_o up to 50% (severe damage) compared to unstrengthen RC column capacity.

Based on the results, it is evidenced that the derived P-I diagrams using empirical formulae are nearly the same as the P-I diagrams obtained from LS-DYNA simulations. Thus the developed empirical formulae can be considered as reasonable and reliable. More over, the results also show that FRP is able to provide extra blast resistance to RC column especially when FRP WRAP and strip are applied.

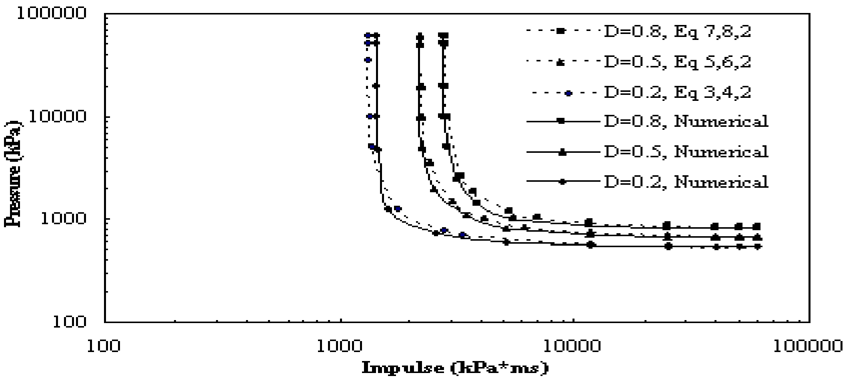

Figure 8 Comparison of P-I curves from Equation 2 and the fitted numerical data for RC column strengthened with FRP wrap (h=600mm, d=400mm, H=4600mm, f_{cu}=40MPa, ρ=0.01, $ρ_s$=0.006,f_{wrap}=2080MPa and $_{wrap}$=3mm)

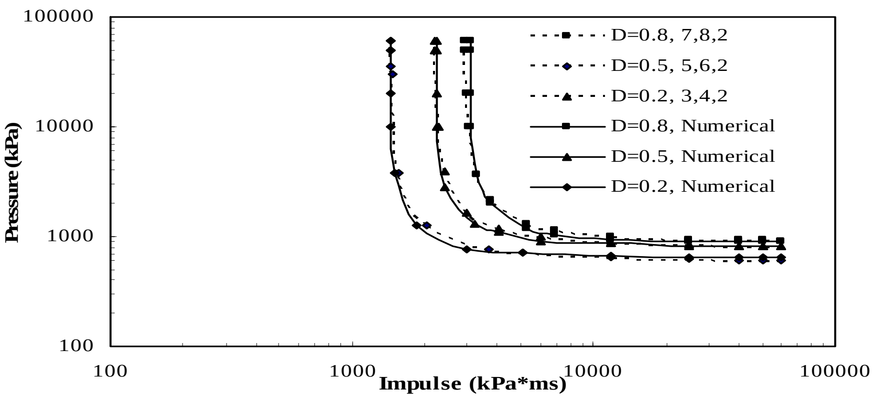

Figure 9 Comparison of P-I curves from Equation 2 and the fitted numerical data for RC column strengthened with FRP wrap and strips (h=600mm, d=400mm, H=4600mm, f_{cu}=40MPa, ρ=0.01, $ρ_s$=0.006,f_{strip}=2080MPa, f_{wrap}=2080MPa, t_{wrap}=3mm and t_{strip}=1mm)

5.0 CONCLUSIONS

In this paper, the numerical results are used to derive empirical formulae to predict pressure and impulse asymptotes of P-I diagrams as functions of RC column and FRP properties. The P-I curve for RC column in unstrengthen condition and strengthen condition are calculated and compared with numerical examples. Based on the results, it can be concluded that the empirical formulae can be easily used to construct pressure-impulse diagrams of RC columns with or without FRP strengthening.

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