

PREDICTION OF DISC BRAKE CONTACT PRESSURE DISTRIBUTIONS BY FINITE ELEMENT ANALYSIS

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Abstract. In recent years, prediction of contact pressure distributions is regarded as an important step in studying disc brake squeal noise. Contact analysis forms part of the whole procedure in the complex eigenvalue method. The essence of such a method lies in the asymmetric stiffness matrix derived from the contact stiffness and the friction coefficient at the disc/pads interfaces. This paper presents the analysis of the contact pressure distributions at the disc/pad interfaces using a detailed 3-dimensional finite element model of a real car disc brake. A general-purpose commercial software package was utilised and assessed. The paper also investigates different levels in modeling a disc brake and simulating contact pressure distributions. Having obtained the suitable model, prediction of interface pressure distributions on an original disc brake was carried out. Finally, modifications on the geometry and/or materials of disc brake components were performed to search for a more uniform contact pressure distribution. It is believed that a uniform contact pressure distribution could prevent excessive tapered wear on the pads and subsequently could prolong the life of pads.

Keywords: Finite element method, disc brake, eigenvalues method, contact pressure distribution, wear

Abstrak. Pada masa kini, penentuan taburan tekanan sentuhan dianggap penting dalam kajian terhadap hingar *squeal* bagi sistem brek cakera. Ini kerana analisis sentuhan merupakan sebahagian daripada prosedur dalam penggunaan kaedah nilai-eigen kompleks. Kaedah ini memerlukan matrik kekakuan yang tidak simetri di mana ia diperolehi hasil daripada kekakuan sentuhan dan juga pekali geseran pada permukaan cakera dan pelapik brek. Kertas kerja ini menganalisis taburan tekanan sentuhan pada permukaan brek cakera dan pelapik brek menggunakan model terperinci 3 dimensi yang dibina. Bagi tujuan tersebut, satu program komputer komersial digunakan secara meluas. Kajian ke atas tekanan sentuhan terhadap beberapa kategori model unsur terhingga dilakukan. Model yang memberi hasil keputusan yang praktikal akan dipilih untuk kajian seterusnya. Akhir sekali, beberapa model yang diubahsuai bentuk/struktur dan bahan disimulasikan bagi mendapatkan taburan tekanan sentuhan yang lebih sekata. Ini penting bagi sistem brek kerana taburan yang sekata boleh mengelakkan kehausan pada bahagian tertentu dan seterusnya memanjangkan jangka hayat pelapik brek.

Kata kunci: Kaedah unsur terhingga, brek cakera, kaedah nilai-eigen, taburan tekanan sentuhan, kehausan

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1.0 INTRODUCTION

In general, there are three main functions of a brake system, i.e., to maintain a vehicle's speed when driving downhill, to reduce a vehicle's speed when necessary and to hold a vehicle when in parking. Today, most passenger vehicles are fitted with disc brake systems. A disc brake of floating caliper design typically consists of two pads, a caliper, a disc, a piston, a carrier bracket and two guide pins. One of the major requirements of the caliper is to press pads against the disc and it should ideally achieve as uniform interface pressure as possible. Limpert [1] stated that uniform pad wear and brake temperature, and more even friction coefficient could only be achieved when pressure distributions between the pads and disc are uniform. In addition, unevenness of the pressure distribution causes uneven wear and consequently shortens the life of pads. This might lead to dissatisfaction to the customers who need to visit their garage more frequently in order to replace tapered wear pads.

In the brake research community, it has been speculated that a non-uniform pressure may promote disc brake squeal. Bergman *et al.* [2], by experimental means, showed that a smaller apparent pad surface reduced the likelihood of squeal. They suggested that one of the reasons for the improvement was the change of the interface pressure distribution. Recently, Lee *et al.* [3] stated that the uniformity of pressure distributions could affect the squeal occurrence. This was suggested by the results that the higher contact area of the pads the lower the squeal index. In recent studies of disc brake squeal, contact analysis forms part of the whole procedure of complex eigenvalue method. The main idea of the complex eigenvalue method is the incorporation of asymmetric part in the stiffness matrix, whose elements may be derived from the contact pressure analysis. Some previous studies assumed full contacts at the pads and disc interfaces [4,5]. However, previous works on brake contact pressure analysis [6-11] has shown that the contact pressure distributions at the pads/disc interfaces are not uniform and there exists partial contact over the disc surfaces.

The subject of interface pressure distributions has been studied by a number of people. Unfortunately, there is no experimental method available to measure contact pressure distributions when a torque is applied to the disc. Tumbrink [12] attempted to measure static pressure distribution using a ball pressure method. Contact pressure prediction by means of numerical methods was studied in [6-11]. There are a number of approaches employed to predict contact pressure distributions through numerical methods. Ripin [7] used a rigid surface for the disc when studying the effect of magnitude of pressure loading, the effect of the pad abutment constraint and the effect of Young's modulus of the friction material on contact pressure distributions. While Lee *et al.* [8] adopted a deformable disc surface which, their model did not include the caliper and the carrier. They used gap elements to represent the contact effect at the pad/disc interfaces. Tirovic and Day [6] investigated the influence of

component geometry, material properties and contact characteristics on the pressure distributions. Hohmann *et al.* [9] carried out simulation of pressure distributions for the drum and disc brakes using ADINA software package. They included a deformable disc with more disc brake components than those in [7,8]. Tamari *et al.* [10] presented a method of predicting the disc brake contact pressure for certain operating conditions by means of experimental and numerical methods. They developed a quite detailed model and validated the model by fitting the numerical deformations of the disc brake with experimental data. Rumold and Swift [11] studied the contact pressure distributions by shifting different positions of the fingers and pistons of a twin piston disc brake of a medium truck. They employed a multibody code with flexible super elements. Among others, only the authors of [10,11] considered all disc brake components and used deformable-to-deformable surface of the disc and pads. Even though various models are available, contact pressure analysis should be carried out carefully in order to obtain more accurate results. Therefore, comparison of models at different levels of details afore-mentioned should be made.

It has been known that more wear appears on the leading side than the trailing side [1]. This agrees with the result [6] that higher pressure occurred on the leading side when the disc starts to slide. There are several solutions that have been implemented to the disc brake in order to minimize and/or eliminate tapered wear in pads. Amongst them are changing friction material compressibility and back plate stiffness as proposed by Tirovic and Day [6], off-centre braking pressure application to the back plate, locating the piston towards the trailing edge, using “hammerhead back plate design” as patented by ITT-Teves and using opposed pistons [1], and modifying shim and offsetting piston position by Tamari *et al.* [10]. Fieldhouse [13] placed a thin wire between the piston head and the back plate to offset the pressure distribution towards the trailing side. While Lee *et al.* [3] stated that reducing back plate thickness could reduce uniformity of pressure distribution.

A good pad design should produce more uniform pressure distributions and therefore, lead to more even pad wear. This paper examines pressure distributions at both interfaces between the disc and the pads at different disc speeds by using a validated and detailed three-dimensional finite element model. Prior to structural modifications, several contact pressure distribution models are constructed and simulations were carried out and compared. The best model will be used for the subsequent work. The paper also investigates several modifications on the pad, the disc and the caliper. From those simulated modifications that produce favourable contact pressure distributions, physical modifications may be made and tested to establish a good design.

2.0 FINITE ELEMENT MODEL

The finite element model of the disc brake of floating caliper design being studied consists of a disc, two pads, a caliper, a carrier, a piston and two guide pins, as shown in Figure 1. The model uses up to about 8000 solid elements and a total of approximately 70 000 degrees-of-freedom (DOFs). Before contact analysis is simulated, normal mode analysis is firstly performed on the model of the disc. By adjusting the Young's modulus and the density of the disc, the numerical and experimental frequencies of the free-free disc become very close and are listed in Table 1. The material data of the other brake components comes from an industrial source and has been validated and given in Table 2. Spring elements were used to make connections between the disc brake components. A very stiff spring could cause conditioning problems in a solution while a very soft spring could make a large penetration between the components. Appropriate spring stiffness should be determined in order to avoid such a problem and a value of 10^6 is usually sufficient [14].

With the validated model, contact analysis was carried out to obtain the pressure distribution between the disc and the piston pad, and between the disc and the finger pad. For the contact interface between the pads and the disc, a friction coefficient of $\mu = 0.6$ was prescribed. The structure is loaded in two steps. First, a uniform pressure of 8.0 MPa was applied on the top of the piston and on top of caliper housing. In the second step, the disc was rotated about the central axis at an angular velocity of 6 rad/s or equivalent to 5.4 km/h of vehicle speed.

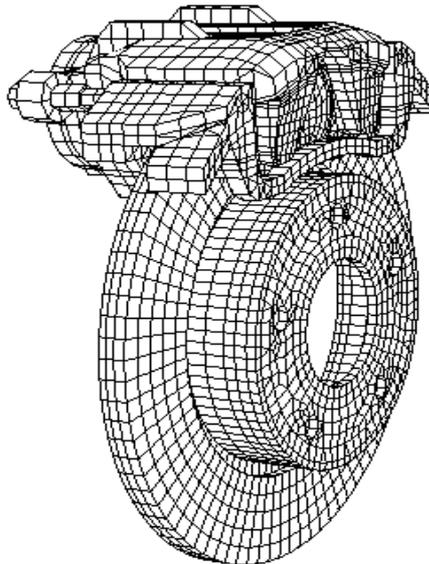


Figure 1 Finite element model of the disc brake

Table 1 Modal results of the disc

Mode	2ND*	3ND	4ND	5ND	6ND	7ND
Test (Hz)	937	1809	2942	4371	6064	7961
FEA (Hz)	960	1820	2939	4365	6062	7964
Error (%)	2.4	0.6	-0.1	-0.1	-0.0	-0.0

*ND is the Nodal Diameters

Table 2 Geometric and material data of disc brake

	Lining	Back plate	Disc	Piston	Carrier	Caliper	Bolt	Guide pin
Length/Diameter	0.095	0.118	0.13	0.057	-	-	-	-
Width (m)	0.036	0.045	-	-	-	-	-	-
Thickness (m)	0.0095	0.005	0.012	0.0035	-	-	-	-
Density (kg.m ⁻³)	2798	7850	6505	7918	6997	7545	9720	9320
Young's modulus (GPa)	Orthotropic	210	99	210	170	187	52	700
Poisson's ratio	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3

3.0 CONTACT ANALYSIS

In this paper, contact problem is modelled by using element-based surfaces and surface-based contact, and is analysed using small sliding interaction. The element-based surfaces are used owing to its advantages over the node-based surfaces, i.e. more accurate results in contact pressure and contact stresses [15]. For surface-based contact, a master and a slave surface are required to form a contact pair. Details on these parameters are discussed in the next section. Meanwhile for small sliding, the contact formulation assumes that the contact surfaces may undergo arbitrarily large rotations but that a slave node will interact with the same local area of the master surface throughout the analysis. This is suitable for the case of disc brake where the pads will interact with the same profile of the rotating disc surface. Convergence could also be easily obtained, compared with the finite sliding formulation.

ABAQUS/Standard defines the contact pressure between the surfaces at a point, p , as a function of the over-closure, h , of the surfaces. In this work, a hard contact model is considered, where the pad and disc surfaces will separate (or contact constraint is removed) when the contact pressure between them becomes zero or

negative and on the other hand, the pad and disc surfaces will interact (or contact constraint is applied) when the contact pressure between them is larger than zero. Two regimes for $p = f(h)$ are given in the formulations below [16],

$$\begin{cases} \mathbf{p} = 0 \text{ for } \mathbf{h} < 0 \text{ (open)} \\ \mathbf{h} = 0 \text{ for } \mathbf{p} > 0 \text{ (closed)} \end{cases} \quad (1)$$

3.1 Modeling and Simulation

Four levels in modeling and simulation of contact pressure distributions were carried out and they are referred to as follows:

- (1) Model A1: Rigid disc surface with piston and piston pad
- (2) Model A2: Rigid disc surface with all disc brake components
- (3) Model A3: Deformable disc surface with major disc brake components
- (4) Model A4: Deformable disc surface with all disc brake components

Model A1 is shown in Figure 2. There are a number of spring elements between the piston and the pad back plate with appropriate stiffness values. The outer wall of the piston is rigidly constrained in the radial direction. At the trailing abutment, the pad is rigidly constrained in the radial and circumferential directions while at the leading side, only the circumferential direction is rigidly constrained. Contact pressure distributions at speed $\Omega = 0$ rad/s and $\Omega = 6$ rad/s are given in Figure 3. For Model A2, all the disc brake components are included except that the disc was replaced by two rigid surfaces, as shown in Figure 4. The interface pressure distribution is given

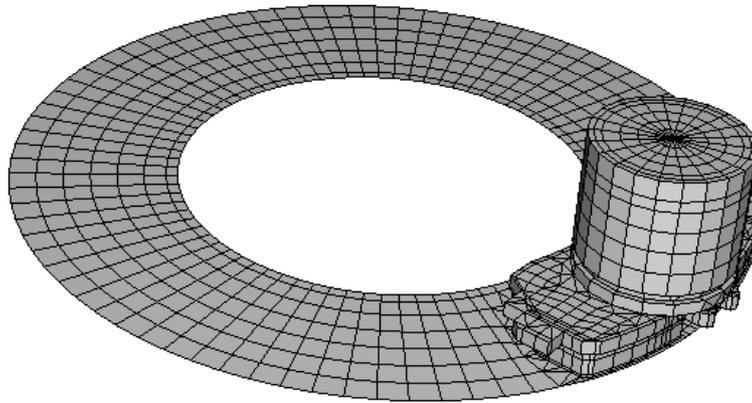


Figure 2 FE model with pad, piston and rigid surface

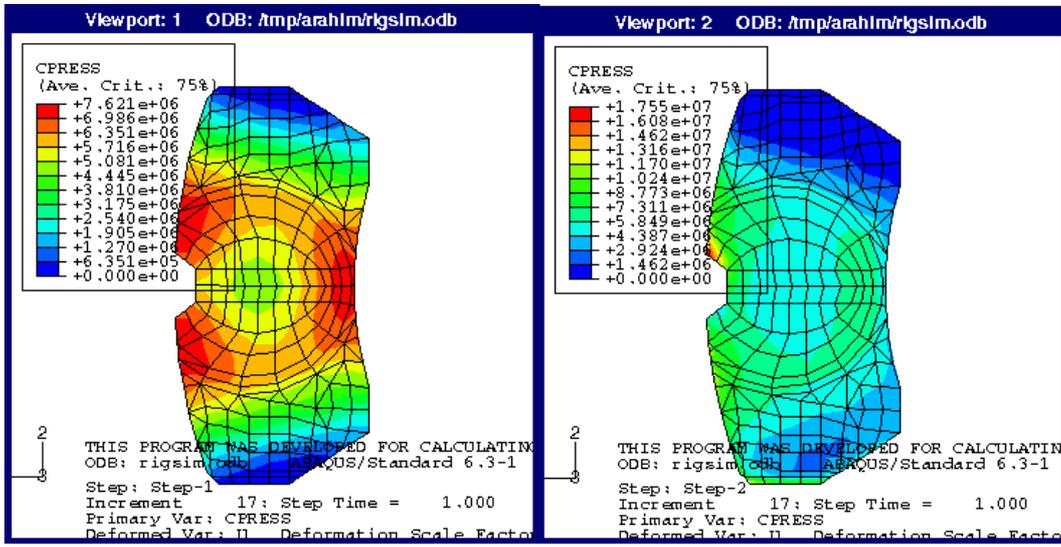


Figure 3 Interface pressure distribution for model A1 at speeds of $\Omega = 0$ rad/s (left) and $\Omega = 6$ rad/s (right)

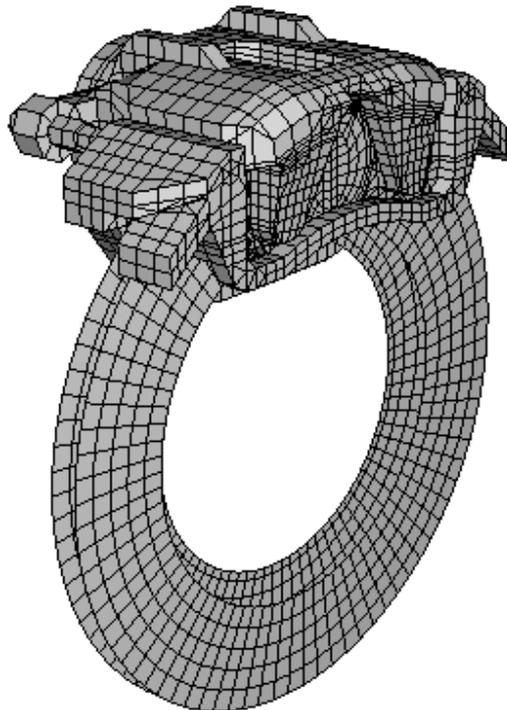


Figure 4 Rigid surface with complete disc brake model

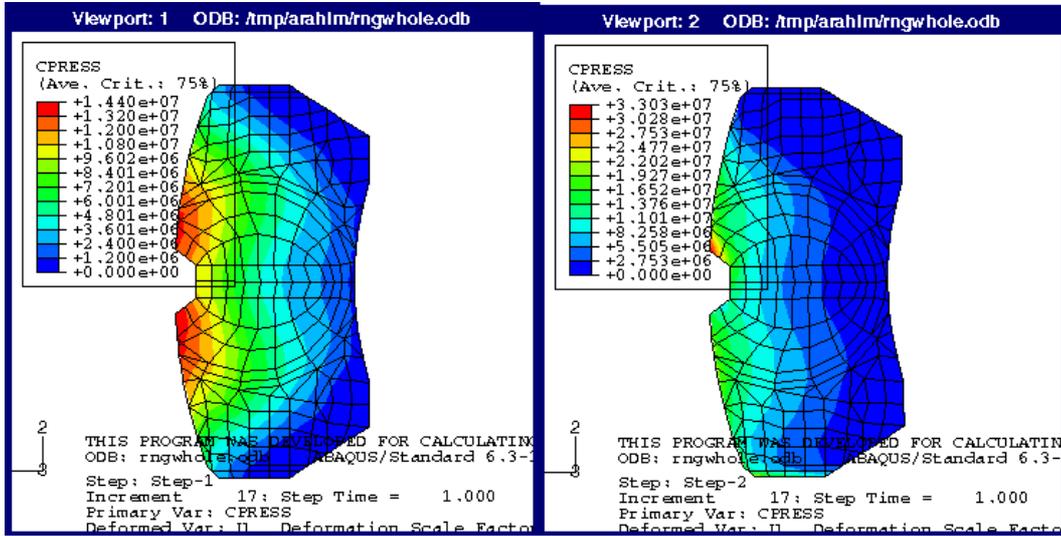


Figure 5 Interface pressure distribution for model A2 at speeds of $\Omega = 0$ rad/s (left) and $\Omega = 6$ rad/s (right)

in Figure 5. While in model A3, a deformable disc is adopted. However, the carrier and guide pins are not presented in the model. Therefore, connections are imposed by rigid constraints in the radial and circumferential directions at the trailing abutment and only the circumferential direction at the leading abutment. The contact pressure distributions are shown in Figure 6. In model A4, all the contacts between components

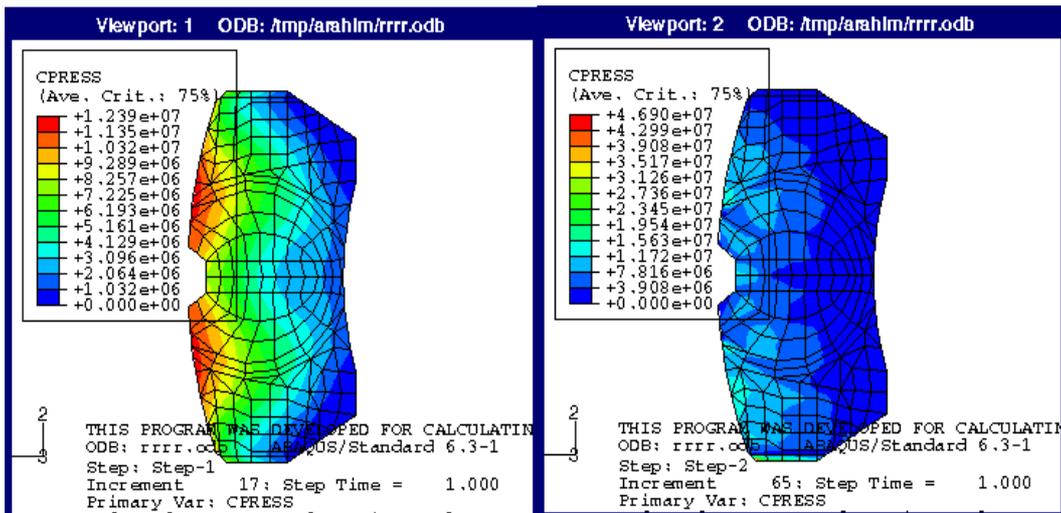


Figure 6 Contact pressure for model A3 at speeds of $\Omega = 0$ rad/s (left) and $\Omega = 6$ rad/s (right)

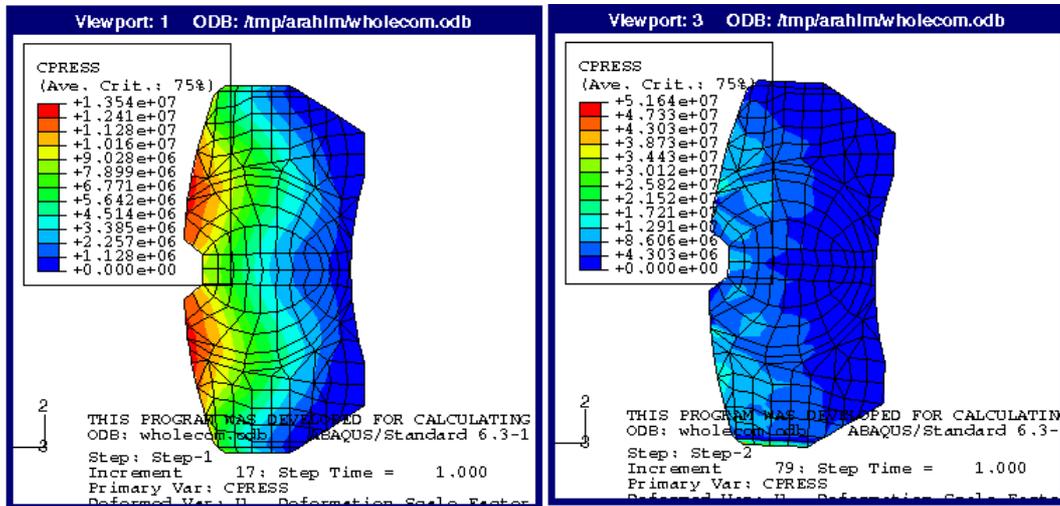


Figure 7 Contact pressure distribution of piston pad for A4 model at speeds of $\Omega = 0$ rad/s (left) and $\Omega = 6$ rad/s (right)

are represented by spring elements with appropriate stiffness. Figure 7 shows the interface pressure distributions at different speeds. Note that the pressure distributions illustrated in the above figures are all for the piston pad.

3.2 Discussion on Contact Analysis Models

For comparison, model A4 is used as the benchmark as it bears more resemblance to a real disc brake. It can be seen in Figure 3 that using model A1, the contact pressure distribution is completely different from that of model A4. This is mainly due to the assumptions of a rigid disc. For model A2, the interface pressure distribution at $\Omega = 0$ rad/s exhibits a similar pattern to that shown in model A4. However, during sliding operation the pressure distributions are dissimilar. The contact area and the contact pressure are also different from those of model A4, as shown in Table 3.

Table 3 Comparison between contact analysis models

Models	Contact area (m ²)		Highest pressure (MPa)		Time required (s)
	Piston	Finger	Piston	Finger	
A1	0.00346	-	17.6	-	374
A2	0.00296	0.00219	33.0	69.9	1453
A3	0.00352	0.00287	46.9	73.1	21686
A4	0.00333	0.00296	51.6	69.8	25540

This model requires less computational time. However, there is 11 ~ 26 percent difference in the contact areas between the two pads. The contact pressure of model A3 shown in Figure 6 gives almost similar distributions to those in model A4 (see Figure 7). The contact area and contact pressure are quite similar (3 ~ 10 percent difference). The computational time is also quite close. There is a minor difference between the two sets of result from models A3 and A4. It can be seen that various models give quite different contact pressure distributions. Therefore, careful consideration should be taken when conducting contact analysis, especially when using a simplified model. Furthermore, contact pressure may be an important parameter for studying squeal propensity. For subsequent work, the authors will use model A4 as it provides a more realistic representation of a disc brake than the other three models.

4.0 STRUCTURAL MODIFICATIONS

The aim of modifications being made is to obtain more uniform pressure distributions by seeking a greater contact area but lower pressures. It is also desirable if both piston pad and finger pad can produce similar pressures at the leading edge. These criteria are considered when evaluating the merit and drawback of a modification. Any modification that meets these three criteria will be considered as a plausible one.

First, contact analysis is carried out for an original disc brake. A commercial software package ABAQUS was employed. A surface-based element provided in this software was utilised. In order to conduct contact analysis between two deformable components with large rotation of the disc, the master-slave surface approach is suitable. Since the disc is much stiffer and has coarser mesh, it was chosen as the master surface and on the other hand, the pads as slave surface. Convergence problem could occur if there are large over-closures during the first step, i.e., during the application of the brake line pressure. Therefore, it is necessary to make sure any nodes at the slave surface do not initially penetrate the master surface. Then, the interface pressure distributions of the real pads under normal (centralized) piston line pressure at certain rotational speeds W are computed as a benchmark. The results on the piston pad and the finger pad both at speeds of $\Omega = 0$ rad/s and $\Omega = 6$ rad/s are shown in Figures 7 and 8 respectively. These figures indicate that when the disc is at rest, the pressure distribution is symmetric about the geometric centre line of the pad. When the disc slides, the pressure distributions are no longer symmetric and the highest pressure occurs at the leading side of the pads. This is consistent with previous findings [6-11]. The predicted contact area and the highest pressure of the pads can be found in Table 3 (see the last row). The apparent contact area (geometric area) of the pad is 0.0039 m^2 . The bottom part in the diagrams is the leading edge of the pad.

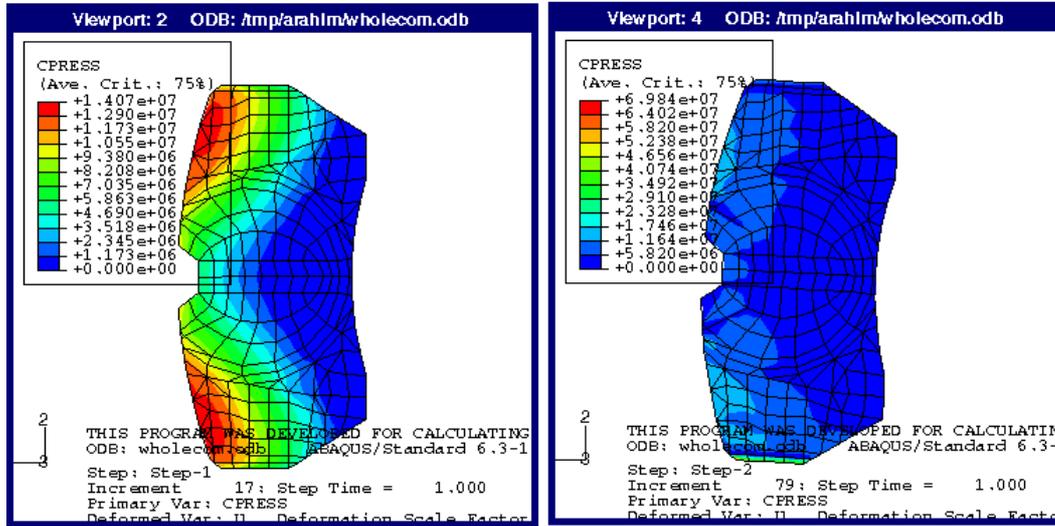


Figure 8 Contact pressure distribution at finger pad for the original disc brake at speeds of $\Omega = 0$ rad/s (left) and $\Omega = 6$ rad/s (right)

5.0 RESULTS AND DISCUSSION

It is the authors’ intention to know how the interface pressure distributions vary as a result of structural modifications. The modifications studied in this paper are summarized in Table 4. The contact area ratio is defined as the ratio of computed contact area to the apparent contact area and the pressure ratio as the computed highest contact pressure of that particular modification to that of the original brake

Table 4 Structural modifications

Modifications	Descriptions	Changes
A	Stiffer back plate	$E = 331$ GPa
B	Stiffer caliper	$E = 700$ GPa
C	Partial connection for piston and finger pad	Figure 9
D	Caliper modification	Figure 10
E	Slotted pad	
F	Stiffer disc	$E = 130$ GPa
G	Vented disc (8 slots)	
H	Increased pad geometric area	$A = 0.00396$ m ²

Table 5 Contact pressure distribution results

Modification	Contact ratio		Pressure ratio	
	Piston pad	Finger pad	Piston pad	Finger pad
Original	85.3	75.8	100.0	100.0
A	83.3	75.2	103.8	100.5
B	91.6	91.3	83.8	75.7
C	93.2	91.1	85.4	80.1
D	84.9	78.5	97.8	84.2
E	82.4	72.9	101.5	103.6
F	85.0	72.0	93.8	99.4
G	82.7	75.3	100.4	100.9
H	85.9	76.2	98.9	99.6

system. It can be seen from Table 5 that stiffening back plate produces worse results both in the contact area and in the pressure. While neither modification E nor modification G could produce better results in either contact or pressure ratio. The caliper modification, as shown in Figure 10, does not promise any significant improvement in the contact ratio although the pressure ratio is reduced. Increasing the area of pad interface, such as in modification H, seems to produce slightly better contact ratio and pressure ratio but the pressure ratio is still quite high. In modification, F where the disc becomes stiffer, the pressure ratio slightly reduces, however, the contact ratio also drops. All the above modifications do not meet the three aforementioned criteria and therefore, they are rejected.

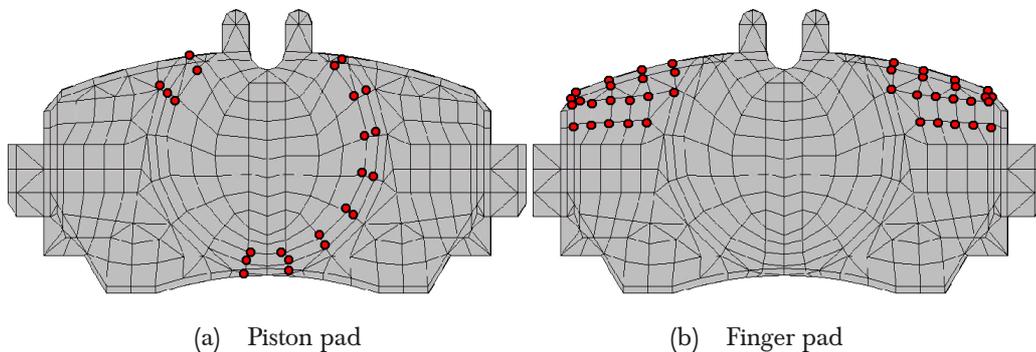


Figure 9 Partial connections in the axial direction (the red dot represents removal of one axial connection)

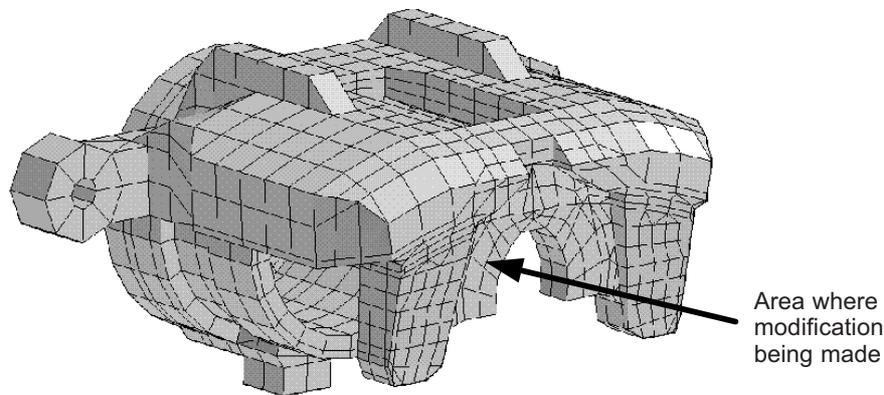


Figure 10 Caliper modification (added material between the two fingers)

Stiffening the caliper seems to produce good results of the contact area and pressure. The contact pressure of this modification is shown in Figure 11(a). The contact area ratio increases by about 6 ~ 15 percent and the contact pressure ratio decreases by about 16 ~ 24 percent. However, to make caliper even stiffer an extra process in manufacturing is required, for instance, heat treatment to harden the material. In this case, extra cost could be involved. Nevertheless, this modification could be considered but on the condition that there are significant improvements in braking and noise performance. The other good results in the contact area and pressure seem to come from modification C, where the connections between the pistons head ring and the back plate, between caliper finger and the back plate are partially connected. The highest pressures at piston pad and the finger pad are quite close. Therefore, this modification meets the three criteria as stated before. At these contact interfaces, there are a number of rigid springs that simulate the contact in the axial direction. If these springs are present over the whole interface, in a dense and symmetric manner, they represent a full contact. If, on the other hand, some of the springs are removed, a partial contact is represented. By choosing different combinations of the locations of the retained springs and/or spring constants, favourable contact pressure distributions at the pad and disc interfaces can be achieved. This has the effect of a well-designed piston adapter and affords an opportunity for an improved disc brake design. The contact pressure distributions at two different rotating speeds of the disc for one particular partial, rigid connection, as shown in Figure 9, are presented in Figure 11(b). The significant advantage of modification C over stiffer caliper is that it only requires inserting another component (the adapter) between the piston and the pad back plate and hence, this does not affect modal behaviour of other individual disc brake components. However, the disadvantages of this modification are that it needs more assembly processes to fit the adapters and should need extra costs to design and fabricate those adapters.

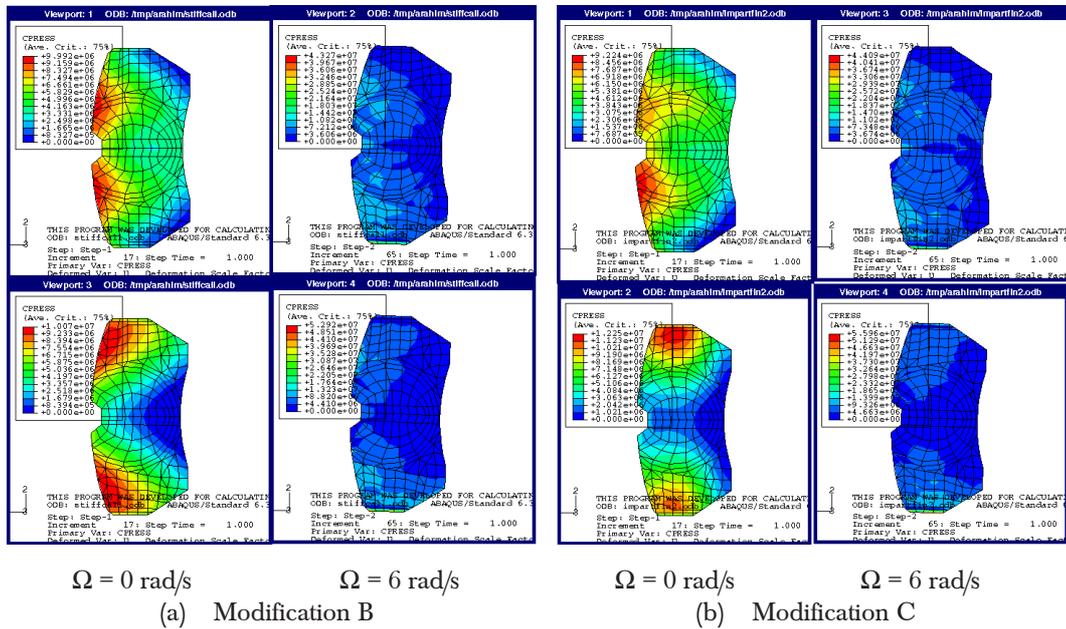


Figure 11 Interface pressure distribution at the piston pad (upper) and the finger pad (bottom)

6.0 CONCLUSIONS

This paper studies the contact pressure distribution of a solid disc brake as a result of structural modifications. Before modifications are simulated, four different models of different degrees of complexity for contact analysis are investigated. It is shown that the contact pressure distributions obtained from these four models are quite different. This suggests that one should be careful in modeling disc brakes in order to obtain correct contact pressure distributions. Using the most refined model, a number of modification ideas have been simulated. From this study, it is found that there are two ideas that meet the three criteria for plausible modifications. However, modification C of the partial connections between the piston head and the pad back plate, and between the fingers and the pad back plate is the better one due to the closeness of the highest pressure of the piston pad to that of the finger pad. The advantages and disadvantages of both modifications in terms of manufacturing issues are also discussed briefly. This work could help design engineers to obtain a more uniform pressure distribution and subsequently satisfy customers' needs by making pad life longer.

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