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VERIFICATION OF WEIBULL'S THEORY OF BRITTLE FRACTURE TO MERANTI'S TIMBER LOADED IN TENSION PARALLEL TO THE GRAIN

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Abstract. Timber is a well-known brittle material under tensile load and is also an inhomogeneous material. Although Weibull's theory of brittle fracture is well established for softwood timber, the verification of this theory to hardwood species, particularly to the timber of local species is very limited. This paper presents the verification of Weibull's theory to local timber of Meranti species loaded in tension parallel to the grain. The theoretical prediction is compared to the test results. It is found that the variation of tensile strength at several probabilities is in close agreement with theoretical prediction. Two values of Weibull's parameters were proposed in the formulation to predict the tensile strength for local timber. It is also found that the tensile strength of local timber is affected by its volume.

Keywords: Weibull's theory, brittle fracture, probability of failure, timber tension tests, Meranti species

Abstrak. Kayu sudah dikenali ramai sebagai bahan yang rapuh di bawah pembebanan tegangan dan mempunyai sifat tidak homogen. Sungguhpun teori rapuh musnah Weibull sudah disahkan untuk kayu lampung, pengesahan teori ini terhadap spesis kayu keras terutamanya daripada kalangan kayu spesis tempatan adalah sangat terhad. Kertas kerja ini mengemukakan pengesahan teori Weibull terhadap kayu tempatan untuk spesis Meranti yang dibebani secara tegangan selari dengan ira. Penganggaran teori dibandingkan dengan keputusan ujian. Taburan kekuatan tegangan pada beberapa kebarangkalian ditemui hampir sama dengan penganggaran teori. Dua nilai parameter Weibull dicadangkan dalam formulasi penganggaran kekuatan tegangan untuk kayu tempatan. Juga ditemui, kekuatan tegangan bagi kayu tempatan dipengaruhi oleh isipadu kayu tersebut.

Kata kunci: Teori Weibull, musnah rapuh, kebarangkalian gagal, ujian tegangan kayu, spesis Meranti

1.0 INTRODUCTION

Conventional brittle fracture theory (or statistical strength theory) has been developed on the basis of the weakest link concept proposed by Pierce, who studied cotton yarns, and Tucker, who studied concrete [1]. Major developments of the theory were made by Weibull [2], who verified his results with tests on many different brittle materials, but apparently not timber. Weibull showed how the strength of a weakest link system can be explained by a cumulative distribution of the exponential

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type, and how the strength depends on the volume of the test specimen for uniform or varying distributions of stress within the specimen.

The first study in which the Weibull's brittle fracture theory was applied to timber, was reported by Bohannan [3]. He studied clear timber beams and found that for geometrically similar beams, the strength was proportional to the depth of the beam to the power 1/9, this being the result of a depth effect and a length effect of equal importance. He found that strength was not affected by beam breadth.

Barrett [4] used the weakest link theory in tension perpendicular to grain studies, which were subsequently used to explain the cracking phenomenon that occurred in pitched cambered beams, a design method based on that work is included in Canadian Standards Association 1980, 1984 (CSA-086).

Foschi and Barrett [5] applied the weakest link theory to the shear strength of glulam beams and developed a design formula, which was incorporated into Canadian Standards Association 1980, 1984 (CSA-086).

Buchanan [6] used brittle fracture theory to relate the strength of timber in bending to the strength in axial tension. For clear timber, he found that the effect of varying length was approximately the same for the two testing modes, but that the effect of varying depth was much greater for tension members than for bending members.

Most studies about the size effect and Weibull's theory of brittle fracture were made to softwood timber but limited to local timber. The two-parameter model and the Weibull's coefficient for softwood timber have been established, however, the verification of Weibull's theory to hardwood timber or local timber is still not available.

2.0 CONCEPT OF BRITTLE AND DUCTILE MATERIALS

Most materials can be classified as being either "brittle" or "ductile", depending on the type of failure they exhibit. Brittle materials fail very suddenly, without warning signs such as large increases in deflections. Glass and ceramics are typical brittle materials. Ductile materials (including many metals and plastics) fail in a more gradual manner, with a large amount of yielding before final failure.

To further examine the difference between brittle and ductile materials, it is useful to consider a member that conceptually consists of a large amount of small elements. The strength of the individual elements varies according to a strength distribution.

If the member is subjected to a uniform stress (as occurs in, for instance, tension members) and the stress is increased gradually, failure for any one of the elements will cause a redistribution of stresses within the member. If the member is made of brittle material and the failed element loses all of its strength suddenly, there will be an instantaneous increase in stresses in the adjacent elements. There may also be further stress increases due to developing stress concentrations in the vicinity of the failed element. These stress increases make it highly probable that the strength of adjacent elements will be exceeded, in which case the fracture will propagate suddenly through the member, causing immediate and total failure. A material of this type is called "perfectly brittle" material, and its strength is governed by the strength of its weakest element.

In a ductile material, on the other hand, an element will yield and still carry a portion of the load so there is no sudden increase in the stress in the adjacent elements. A "perfectly ductile" material has unlimited capacity to undergo deformations after the maximum strength has been reached.

Most real materials exhibit behaviour somewhere between the two extremes of perfectly brittle and perfectly ductile behaviour. Timber is interesting because it is somewhat ductile in compression but exhibits brittle behaviour in tension.

3.0 WEIBULL'S THEORY OF BRITTLE FRACTURE

Briefly, consider a volume V of timber under a distribution of tensile stress σ. Weibull's theory allows the computation of the probability of failure F_v of the volume V when the stresses are known. This probability is given by:

$$F_v = 1 - e^{-\frac{1}{V^*}\int_V \left(\frac{\sigma - \sigma_o}{m}\right)^k dV} \tag{1}$$

where m, k and σ_o are material constants, V^* is the reference volume, and σ_o corresponds to the minimum strength of the material. Since three material constants are involved, Equation (1) is referred to as a 'three-parameter' Weibull model. A simpler, 'two-parameter' model may be used by assuming $\sigma_o = 0$. The assumption of zero minimum strength may appear to be unrealistic, but for practical purposes, as shown by Foschi and Barrett [5], both models give approximately the same results at probabilities of failure larger than or equal to 0.05. The parameter k is related to the coefficient of variation of the material for a given geometric and loading configuration. For coefficients of variation in the order of 0.20 commonly encountered with timber, k is in the order of 5.

Considering the structural element subjected to uniform stresses and assuming 'two-parameter' model with reference volume V^* equal to 1 m^3, Equation (1) becomes:

$$F_v = 1 - e^{-V\left(\frac{\sigma}{m}\right)^k} \tag{2}$$

or

$$\ln(1 - F_v) = -V\left(\frac{\sigma}{m}\right)^k \tag{3}$$

Applying Equation (3) and considering two volumes, V_1 and V_2, with respective strengths of σ_1 and σ_2, and assuming volumes V_1 and V_2 to have the same probability of failure in tension, then:

$$\ln(1-F_v) = -\left(\frac{\sigma_1}{m}\right)^k V_1 \tag{4}$$

$$\ln(1-F_v) = -\left(\frac{\sigma_2}{m}\right)^k V_2 \tag{5}$$

Combining Equations (4) and (5) give:

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{1/k} \tag{6}$$

Equation (6) shows the relationship between two strengths and its respective volumes with the same probability of failure.

4.0 EXPERIMENTAL DATA FOR TENSILE TEST

The experimental data for tensile tests is based on Khairul Salleh [7]. The type of timber is Dark Red Meranti and the dimensions of the specimens are as shown in Figure 1. A total of 145 specimens were tested in tension parallel to the grain in accordance to BS 373: 1957.

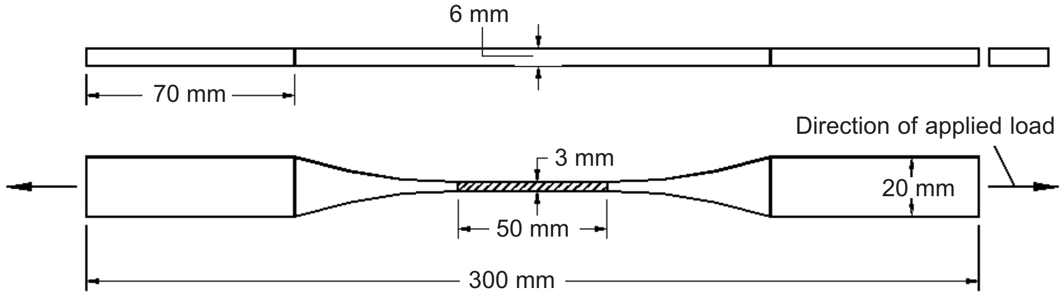

Figure 1 Test piece for tension parallel to grain test

Based on experimental data given by Khairul Salleh [7], the number of specimens at every 5 kN interval of tensile strength is counted and given in Table 1. The cumulative number of samples and cumulative probability were calculated and tabulated in Table 1. Based on Table 1, the cumulative probability versus tensile strength is plotted and shown in Figure 2.

Table 1 Cumulative probability of failure for Meranti's specimen loaded in tension

Tensile strength (MPa)	Average tensile strength (MPa)	Number of samples	Cumulative number	Cumulative probability
0	0	0	0	0
30-34	32	10	10	0.07
35-39	37	10	20	0.14
40-44	42	7	27	0.19
45-49	47	7	34	0.23
50-54	52	10	44	0.30
55-59	57	7	51	0.35
60-64	62	11	62	0.43
65-69	67	6	68	0.47
70-74	72	16	84	0.58
75-79	77	10	94	0.65
80-84	82	12	106	0.73
85-89	87	8	114	0.79
90-94	92	4	118	0.81
95-99	97	9	127	0.88
100-104	102	4	131	0.90
105-109	107	7	138	0.95
110-114	112	4	142	0.98
115-119	117	1	143	0.99
120-124	122	1	144	0.99
125-129	127	0	144	0.99
130-134	132	1	145	1

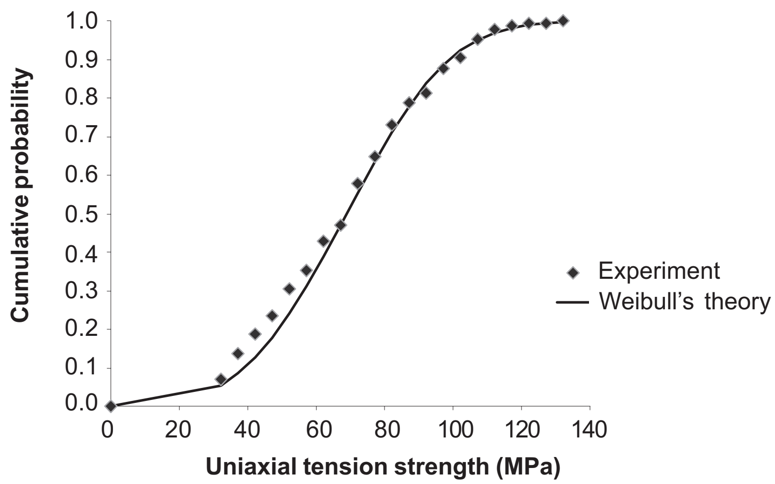

Figure 2 Idealisation of cumulative probability for local timber with Weibull's theory of brittle fracture

5.0 DETERMINATION OF WEIBULL'S PARAMETERS FOR LOCAL TIMBER

The determination of parameters k and m is based on two cumulative probabilities, 0.47 and 0.95, which are given in Table 1. Substituting $F_v = 0.47$ and $\sigma = 67$ MPa into Equation (3) gives:

$$0.635 = \left(\frac{67}{m}\right)^k V \tag{7}$$

Similarly, substituting $F_v = 0.95$ and $\sigma = 107$ MPa into Equation (3), gives:

$$2.996 = \left(\frac{107}{m}\right)^k V \tag{8}$$

It is assumed that the tensile stress is uniform within the shaded area of tension specimen shown in Figure 1. Then, the volume V (the place where the stresses are uniform) is calculated and equal to 9×10^{-7} m^3. Substituting this value into Equations (7) and (8) and solving these equations simultaneously will give $m = 1.15$ MN/m^2 and $k = 3.313$.

These values of m and k are the proposed values of Weibull's parameters for local timber. Based on these values, the Weibull's cumulative curve is plotted using Equation (2) and shown in Figure 2.

6.0 DISCUSSION OF THE FINDING

From Figure 2, it is found that the cumulative probability curve for local timber can be idealised as Weibull's cumulative curve. The experimental data is scattered around the predicted curve. The values $m = 1.15$ MN/m^2 and $k = 3.313$ for Meranti timber are found to be reasonable. The Weibull's theory of brittle fracture is verified and can be used for local or hardwood timber.

The values of Weibull's parameters, $m = 1.15$ MN/m^2 and $k = 3.313$, for a local timber are found to be lower than those of softwood timber. For instance, the values of m and k for Aspen timber are 8.13 MN/m^2 and 5.7, respectively [8].

For the same volume of timber structure, it is found that the tensile strength of timber varies from 32 to 132 MPa. This result shows that the property of timber is highly inhomogeneous in tension.

For other volume of timber structure, the tensile strength can be predicted using Equation (6). For example, if volume 9×10^{-7} m^3 produce the tensile strength equals to 32 MPa at 5% probability (refer Figure 2), then by using Equation (6) with $k = 3.313$, the tensile strength for volume 18×10^{-7} m^3 (at the same probability) is predicted and equals to 3.22 MPa. This result shows that the increase in volume will decrease the tensile strength, thus shows that the volume of timber structure will affect the tensile strength.

7.0 CONCLUDING REMARKS

The following remarks are noted:

- (i) The cumulative probability curve for a Meranti timber can be idealised as Weibull's cumulative curve.
- (ii) The Weibull's theory of brittle fracture is verified and can be applied to local or hardwood timber.
- (iii) The proposed values of m and k for Meranti timber are 1.15 MN/m^2 and 3.313, respectively. These values are found to be lower than the m and k values for softwood timber.
- (iv) The property of timber is found to be inhomogeneous in tension.
- (v) The tensile strength of timber structure is affected by its volume.

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