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THE UNSTEADY FREE CONVECTION FLOW OF ROTATING SECOND GRADE FLUID OVER AN OSCILLATING VERTICAL PLATE

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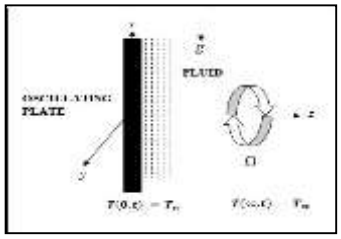
Graphical abstract



Abstract

In this paper, the exact solutions for unsteady free convection flow of rotating second grade fluid over an isothermal oscillating vertical plate are investigated. This phenomenon is modeled in the form of partial differential equations with initial and boundary conditions. Some suitable non dimensional variables are introduced. The corresponding non-dimensional equations with conditions are solved using Laplace transform technique. Exact solutions for velocity and energy profiles are obtained. They are expressed in simple forms in terms of exponential and complementary error functions of Gauss. It is found that they satisfy governing equations and conditions imposed. Computations are carried out and the results are analyzed for various emerging parameters.

Keywords: Second grade fluid; rotating; free convection; oscillating; Laplace transform

Abstrak

Dalam kertas kerja ini, penyelesaian tepat bagi aliran olakan bebas tak mantap bagi bendalir gred kedua yang berputar di atas sesuhu plat menegak yang dikaji. Model untuk fenomena ini dibina dalam bentuk persamaan terbitan separa beserta syarat awal dan syarat sempadan. Beberapa pembolehubah tak bermatra yang bersesuaian diperkenalkan. Persamaan tak bermatra beserta syarat yang berkaitan diselesaikan dengan menggunakan kaedah penjelmaan Laplace. Penyelesaian tepat bagi profil halaju dan tenaga telah diperoleh dan diungkapkan secara mudah dalam bentuk sebutan eksponen dan fungsi ralat pelengkap Gauss. Didapati bahawa penyelesaian yang diperoleh ini memenuhi persamaan menakluk dan semua syarat yang dikenakan. Pengiraan telah dijalankan dan semua keputusan dianalisis untuk pelbagai parameter yang terlibat.

Kata kunci: Bendalir gred kedua; berputar; aliran olakan bebas; berayun; penjelmaan Laplace

1.0 INTRODUCTION

The study of fluid flow on rotating plate has drawn the interest of researchers in fluid studies such as Manna et al [1]. They studied an exact solution for the unsteady rotating flow of a viscous fluid. Laplace and inverse Laplace transforms have been used to obtain an exact solution of the problem. In 2008, Hayat et al [2] investigated the rotating flow of a second grade fluid. The effect of magnetohydrodynamics (MHD) flow over

a porous half space was also considered in this problem. But, they used a different analytical method to solve the governing equation that is using Fourier Sine transform. The derivation of the governing equation for the rotating MHD second grade fluid over a porous half space was discussed in this study. The existing solution for Newtonian fluid has been also deduced as the limiting cases. Meanwhile, Tiwari and Ravi [3] studied on rotating incompressible second grade fluid over a porous medium without considering the effect of MHD flow. The analytical solution was obtained using Laplace transform technique for the two main cases, which were sudden started and constant acceleration flow. Hayat et al [4] also worked on the analytical solution for shrinking flow of second grade fluid in a rotating frame by using Homotopy Analysis Method (HAM). In this problem, they considered the fluid to be bounded between two porous walls. Comparison between viscous fluid and second grade fluid was also discussed as the limiting cases. Khan et al [5] investigated the exact solution of a rotating second grade fluid over an accelerated plate, while considering the effect of MHD flow in a porous medium. Laplace transform method has been used to solve the governing equation for the cases of constant and variable accelerations.

The study of free convection flow has been discussed by several authors in their research. Lahurikar [6] studied the free convection flow of rotating viscous fluid over an infinite vertical isothermal plate. The exact solution was obtained using the Laplace transform method. Vijayalakshmi [7] continued the study made by Lahurikar [6] but in the presence of thermal radiation effect. Laplace transform method has also been used to obtain an exact solution as well as skin friction. It was found that, the skin friction profiles increased with decreasing radiation parameter. In 2014, Samiulhaq et al [8] discussed the free convection flow of second grade fluid with the effect of ramped wall temperature. The dimensionless governing equation has been solved analytically by using Laplace transform method. They also made the comparison between isothermal and ramped wall temperatures where the velocity of fluid is greater in the case of isothermal temperature compared to ramped wall temperature. In the same year, Samiulhaq et al [9] extended the same problem by considering the effect of MHD flow in a porous medium. Besides that, the study on oscillating plate has attracted many researchers such as Khan et al [10] and Mohamad et al [11] in presenting their new exact solutions. Khan et al [12] studied the exact solution of unsteady hydromagnetic flow of viscous fluid in a rotating frame. The porous medium was also considered in this problem. Two methods, namely Laplace transform and Fourier Sine transform have been used to solve the dimensionless governing equation of motion. The velocity profiles are plotted in real and imaginary parts. Recently, Farhad et al [13] produced a new result on closed form solution for unsteady free convection flow of a second grade fluid over an oscillating vertical plate. The governing equation for cases of cosine and sine oscillations were solved by using Laplace transform method. The comparison between present problem solutions with published solution was also presented graphically by considering the case of zero Grashof number.

To the best of author's knowledge, no study has been conducted in analyzing the unsteady free convection flow of rotating second grade fluids over an oscillating plate so far. Therefore, this present investigation is attempting to study on the said topic. In this problem, the exact solutions are obtained by using Laplace transform technique. The exact solutions obtained for the velocity and temperature profiles satisfy the governing equations and all the imposed boundary conditions. The obtained results are plotted to see the effects of indispensable flow parameters.

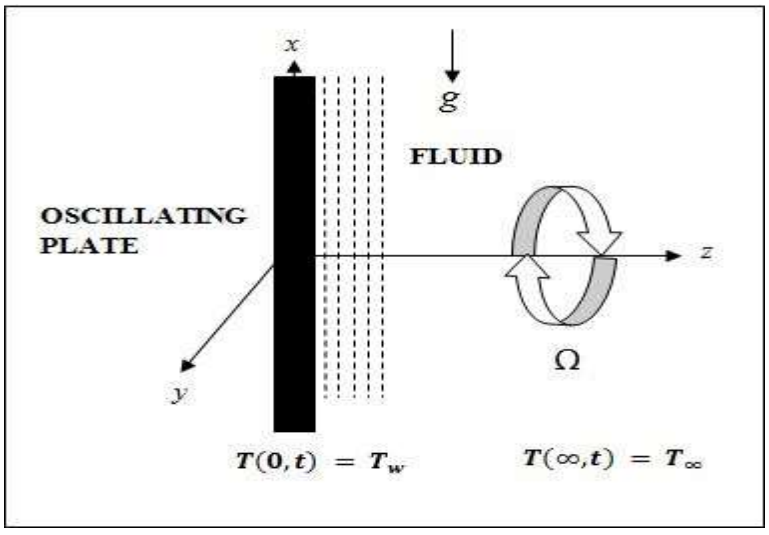


Figure 1 Physical problem of the study

2.0 MATHEMATICAL FORMULATION

Consider the unsteady free convection flows of a rotating second grade fluid passing through an isothermal vertical plate. The z-axis is taken normal to the plate. Initially, both the plate and fluid are at rest with constant temperature T_∞. At time $t = 0^+$, the plate starts motion in its plane with oscillating velocity and the fluid starts solid body rotation with constant angular velocity Ω parallel to z-axis. The appropriate governing equations are given as

$$\frac{\partial F}{\partial t}+2i\Omega F=\upsilon\frac{\partial^2 F}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2\partial t}+gB\left(T-T_\infty\right), \tag{1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}, \tag{2}$$

in which $F = u + iv$ is the complex velocity where u and v are its real and imaginary parts, ρ designates the density of the fluid, υ is the kinematic viscosity, α_1 is the second grade parameter, g is the acceleration due to gravity, B is the volumetric coefficient of thermal expansion, T is the temperature of the fluid, k is the thermal conductivity and c_p is the specific heat capacity of the fluid at constant pressure. The appropriate initial and boundary conditions are

$$F\left(0,t\right)=UH\left(t\right)\cos\left(\omega_1 t\right) \text{ or } F\left(0,t\right)=U\sin\left(\omega_1 t\right), \tag{3}$$

$$F\left(z,t\right)=0 \text{ as } z\to\infty\ ;\ t>0, \tag{4}$$

$$F\left(z,0\right)=0\ ;\ z>0, \tag{5}$$

and

$$T\left(0,t\right)=T_w,\quad T\left(z,t\right)=T_\infty \text{ as } z\to\infty\ ;\ t>0, \tag{6}$$

$$T\left(z,0\right)=T_\infty\ ;\ z>0. \tag{7}$$

Introducing the following dimensionless variables

$$F^*=\frac{F}{U},\quad z^*=\frac{U}{\upsilon}z,\quad t^*=\frac{U^2 t}{\upsilon},\quad \omega_1^*=\frac{\upsilon\omega_1}{U^2},\quad T^*=\frac{T-T_\infty}{T_w-T_\infty}. \tag{8}$$

By using equations in (8), the dimensional equations (1)-(7) reduce to dimensionless equations (* notations are dropped for the sake of simplicity)

$$\frac{\partial F}{\partial t}+2ibF=\frac{\partial^2 F}{\partial z^2}+\alpha\frac{\partial^3 F}{\partial z^2\partial t}+GrT, \tag{9}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial z^2}, \tag{10}$$

subjected to the initial and boundary conditions

$$F\left(0,t\right)=H\left(t\right)\cos\left(\omega_1 t\right) \text{ or } F\left(0,t\right)=\sin\left(\omega_1 t\right), \tag{11}$$

$$F\left(z,t\right)=0 \text{ as } z\to\infty\ ;\ t>0, \tag{12}$$

$$F\left(z,0\right)=0\ ;\ z>0, \tag{13}$$

and

$$T\left(0,t\right)=1,\quad T\left(z,t\right)=0 \text{ as } z\to\infty\ ;\ t>0, \tag{14}$$

$$T\left(z,0\right)=0\ ;\ z>0, \tag{15}$$

where

$$b=\frac{\Omega\upsilon}{U^2},\quad \alpha=\frac{\alpha_1 U^2}{\rho\upsilon^2},\quad Gr=\frac{\upsilon gB\left(T_w-T_\infty\right)}{U^3},\quad \Pr=\frac{\mu c_p}{k}.$$

where b is rotation parameter, α is second grade parameter, Gr is Grashof number, Pr is Prandtl number and ω_1 is frequency of oscillation. The Laplace transform method has been used to solve the dimensionless governing equations (9) and (10) in q-domain. Subjected to the initial conditions (13) and (15), we have

$$\frac{d^2\bar{F}}{dz^2}-\left(\frac{q+2ib}{\alpha q+1}\right)\bar{F}=-\frac{Gr\bar{T}}{\alpha q+1}, \tag{16}$$

$$\bar{F}\left(0,q\right)=\frac{q}{q^2+\omega^2} \text{ or } \bar{F}\left(0,q\right)=\frac{\omega}{q^2+\omega^2}, \tag{17}$$

$$\bar{F}\left(\infty,q\right)=0, \tag{18}$$

and

$$\frac{d^2\bar{T}}{dz^2}-\Pr q\bar{T}=0, \tag{19}$$

$$\bar{T}\left(0,q\right)=\frac{1}{q},\quad \bar{T}\left(\infty,q\right)=0, \tag{20}$$

where q is the Laplace transform parameter. Here, equation (16) is a non-homogenous differential equation, which has the solution

$$\bar{F}\left(z,q\right)=\bar{F}_h\left(z,q\right)+\bar{F}_p\left(z,q\right), \tag{21}$$

and by using characteristics equation, the solution of energy equation (10) subjected to the boundary conditions (14) is obtained as

$$\bar{T}\left(z,q\right)=\frac{1}{q}\exp\left(-z\sqrt{\Pr q}\right). \tag{22}$$

The functions of $\bar{F}_h(z,q)$ and $\bar{F}_p\left(z,q\right)$ are defined as

$$\bar{F}_h\left(z,q\right)=c_1\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)+c_2\exp\left(\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right) \tag{23}$$

and

$$\bar{F}_p(z,q)=-\frac{Gr}{q\left[\alpha\Pr q^2+\left(\Pr-1\right)q-a_1\right]}\exp\left(-z\sqrt{\Pr q}\right). \tag{24}$$

Therefore equation (21) can be written as

$$\bar{F}(z,q)=c_1\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)+c_2\exp\left(\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)-\frac{Gr}{q\left[\alpha\,\mathrm{Pr}\,q^2+(\mathrm{Pr}-1)q-a_1\right]}\exp\left(-z\sqrt{\mathrm{Pr}\,q}\right) \tag{25}$$

where $a_1=2ib$ and $\beta=\frac{1}{\alpha}$. In order to find the constants c_1 and c_2, boundary conditions (11) and (12) are applied into equation (25), hence we have

$$\bar{F}_c(z,q)=\frac{q}{q^2+\omega^2}\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)+\frac{Gr}{\left[\alpha\,\mathrm{Pr}\,q^2+(\mathrm{Pr}-1)q-a_1\right]}\times\frac{1}{q}\cdot\left[\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)-\exp\left(-z\sqrt{\mathrm{Pr}\,q}\right)\right] \tag{26}$$

and

$$\bar{F}_s(z,q)=\frac{\omega}{q^2+\omega^2}\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)+\frac{Gr}{\left[\alpha\,\mathrm{Pr}\,q^2+(\mathrm{Pr}-1)q-a_1\right]}\times\frac{1}{q}\cdot\left[\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)-\exp\left(-z\sqrt{\mathrm{Pr}\,q}\right)\right]. \tag{27}$$

Note that, the subscripts c and s in equations (26) and (27) referred to cosine and sine oscillations of the plate and equations (26) and (27) can be expressed as

$$\bar{F}_c(z,q)=\bar{F}_1(z,q)+\bar{F}_2(z,q) \tag{28}$$

and

$$\bar{F}_s(z,q)=\bar{F}_3(z,q)+\bar{F}_2(z,q). \tag{29}$$

Here, we have

$$\bar{F}_1(z,q)=\frac{q}{q^2+\omega^2}\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right), \tag{30}$$

$$\bar{F}_3(z,q)=\frac{\omega}{q^2+\omega^2}\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right) \tag{31}$$

and

$$\bar{F}_2(z,q)=\frac{Gr}{\left[\alpha\,\mathrm{Pr}\,q^2+(\mathrm{Pr}-1)q-a_1\right]}\times\frac{1}{q}\cdot\left[\exp\left(-\frac{z}{\alpha}\sqrt{\frac{q+a_1}{q+\beta}}\right)-\exp\left(-z\sqrt{\mathrm{Pr}\,q}\right)\right]. \tag{32}$$

Hence, the inverse Laplace transform of equations (28) and (29) are obtained as

$$F_c(z,t)=F_1(z,t)+F_2(z,t) \tag{33}$$

and

$$F_s(z,t)=F_3(z,t)+F_2(z,t). \tag{34}$$

In order to find the inverse Laplace transform of function $F_1(z,t)$ and $F_3(z,t)$ in equations (33) and (34), we need to use a convolution theorem that can be defined as

$$F_1(z,t)=\int_0^t F_{11}(t-s)F_{12}(z,s)ds \tag{35}$$

and

$$F_3(z,t)=\int_0^t F_{31}(t-s)F_{12}(z,s)ds \tag{36}$$

where

$$F_{11}(t)=H(t)\cos(\omega_1 t) \tag{37}$$

and

$$F_{31}(t)=\sin(\omega_1 t). \tag{38}$$

The inverse Laplace transform of $F_{12}(z,t)$ can be obtained by using formula of compound function[5]. Therefore, we get

$$F_{12}(z,t)=\frac{\delta(t)z}{2\sqrt{\pi\alpha}}\int_0^\infty\frac{\exp\left(-\frac{z^2}{4\alpha u}-u\right)}{u\sqrt{u}}du+\frac{z\sqrt{a_2}\exp(-\beta t)}{2\sqrt{\pi\alpha t}}\int_0^\infty\frac{1}{u}\exp\left(-\frac{z^2}{4\alpha u}-u\right)I_1\left(2\sqrt{a_2 ut}\right)du. \tag{39}$$

Then, the inverse Laplace transform of $F_2(z,t)$ in equations (33) and (34) is

$$F_2(z,t)=\int_0^t F_{21}(t-p)F_{22}(z,p)dp \tag{40}$$

where

$$F_{21}(t)=\frac{Gr}{\alpha\,\mathrm{Pr}\,m_2}\sinh(m_2 t)\exp(-m_1 t), \tag{41}$$

and

$$F_{22}(z,t)=\frac{z}{2\sqrt{\alpha\pi}}\int_0^\infty \frac{1}{u\sqrt{u}}\exp\left(-\frac{z^2}{4\alpha u}-u\right)du+\frac{z\sqrt{a_2}}{2\sqrt{\alpha\pi}}\int_0^t\int_0^\infty \frac{1}{u\sqrt{s}}\exp\left(-\frac{z^2}{4\alpha u}-\beta s-u\right)\times I_1\left(2\sqrt{a_2 us}\right)duds-\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{\mathrm{Pr}}{t}}z\right). \quad (42)$$

Hence, the solutions for equations (33) and (34) can be obtained by substituting equations (37), (38) and (39) into equations (35), (36) and (40), therefore we have

$$F_C(z,t)=H(t)\cos(\omega_1 t)\exp\left(-\frac{z}{\sqrt{\alpha}}\right)+\frac{zH(t)\sqrt{a_2}}{2\sqrt{\alpha\pi}}\int_0^t\int_0^\infty \frac{1}{u\sqrt{s}}\cos(\omega_1 t-\omega_1 s)\times\exp\left(-\frac{z^2}{4\alpha u}-\beta s-u\right)\times I_1\left(2\sqrt{a_2 us}\right)dsdu+\frac{Gr}{\alpha\,\mathrm{Pr}\,m_2}\exp\left(-\frac{z}{\sqrt{\alpha}}\right)\int_0^t \sinh\left(m_2(t-p)\right)\times\exp\left(-m_1(t-p)\right)dp+\frac{zGr\sqrt{a_2}}{2\alpha\,\mathrm{Pr}\,m_2\sqrt{\alpha\pi}}\int_0^t\int_0^p\int_0^\infty \frac{1}{u\sqrt{s}}\sinh\left(m_2(t-p)\right)\times\exp\left(-m_1(t-p)-\frac{z^2}{4\alpha u}\right)\times\exp(-\beta s-u)\times I_1\left(2\sqrt{a_2 us}\right)duds\,dp-\frac{Gr}{\alpha\,\mathrm{Pr}\,m_2}\int_0^t \sinh\left(m_2(t-p)\right)\exp\left(-m_1(t-p)\right)\times\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{\mathrm{Pr}}{t}}z\right)dp \quad (43)$$

and

$$F_S(z,t)=\sin(\omega_1 t)\exp\left(-\frac{z}{\sqrt{\alpha}}\right)+\frac{z\sqrt{a_2}}{2\sqrt{\alpha\pi}}\int_0^t\int_0^\infty \frac{1}{u\sqrt{s}}\sin(\omega_1 t-\omega_1 s)\times\exp\left(-\frac{z^2}{4\alpha u}-\beta s-u\right)I_1\left(2\sqrt{a_2 us}\right)dsdu+\frac{Gr}{\alpha\,\mathrm{Pr}\,m_2}\exp\left(-\frac{z}{\sqrt{\alpha}}\right)\int_0^t \sinh\left(m_2(t-p)\right)\times\exp\left(-m_1(t-p)\right)dp+\frac{zGr\sqrt{a_2}}{2\alpha\,\mathrm{Pr}\,m_2\sqrt{\alpha\pi}}\int_0^t\int_0^p\int_0^\infty \frac{1}{u\sqrt{s}}\sinh\left(m_2(t-p)\right)\times\exp\left(-m_1(t-p)-\frac{z^2}{4\alpha u}\right)\times\exp(-\beta s-u)\times I_1\left(2\sqrt{a_2 us}\right)duds\,dp-\frac{Gr}{\alpha\,\mathrm{Pr}\,m_2}\int_0^t \sinh\left(m_2(t-p)\right)\exp\left(-m_1(t-p)\right)\times\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{\mathrm{Pr}}{t}}z\right)dp \quad (44)$$

where

$$a_2=\beta-a_1,\ a_3=\frac{\mathrm{Pr}-1}{\alpha\,\mathrm{Pr}},\ a_4=\frac{a_1}{\alpha\,\mathrm{Pr}},\ m_1=\frac{a_3}{2},\ m_2=\sqrt{\frac{a_3^2+4a_4}{4}}.$$

The Laplace inverse of energy equation (22) can be written as

$$T(z,t)=\operatorname{erfc}\left(\frac{z\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right). \quad (45)$$

3.0 RESULTS AND DISCUSSION

In this section, the solution of equation (44) is discussed numerically in order to investigate the effects of parameters such as *a*, *b,* Pr and *Gr*. In this problem, the velocity profiles are presented graphically by using Mathcad in real part of F_s in Figures 2-5 whereas the temperature profiles of equation (45) are shown in Figures 6-7 for values of Pr and time parameters *t*.

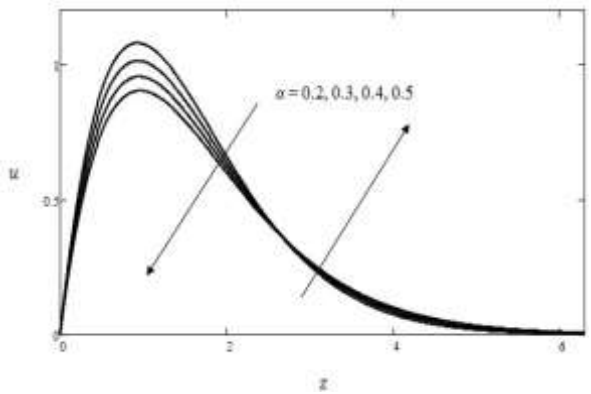


Figure 2 Real velocity profiles for different values of a with Pr = 0.71, ω_1 = 0.2, Gr = 5.0, b = 0.1, $\omega_1 t = \pi$ and t = 1.0.

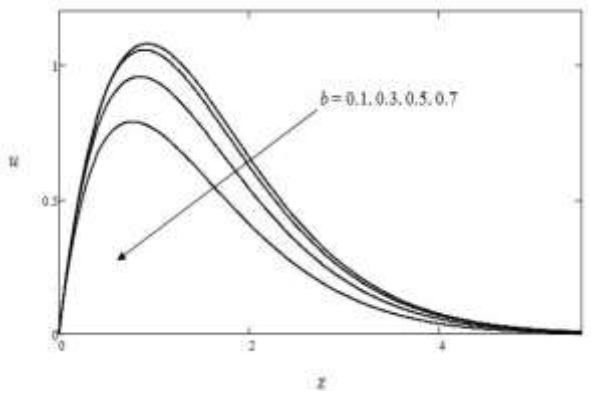


Figure 3 Real velocity profiles for different values of b with Pr = 0.71, ω_1 = 0.2, Gr = 5.0, a = 0.2, $\omega_1 t = \pi$ and t = 1.0.

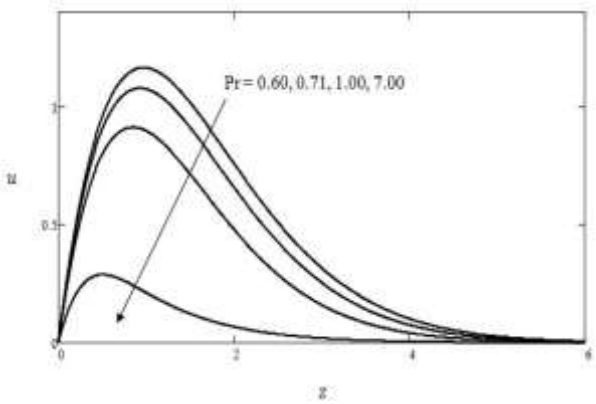


Figure 4 Real velocity profiles for different values of Pr with b = 0.1, ω_1 = 0.2, Gr = 5.0, a = 0.2, $\omega_1 t = \pi$ and t = 1.0.

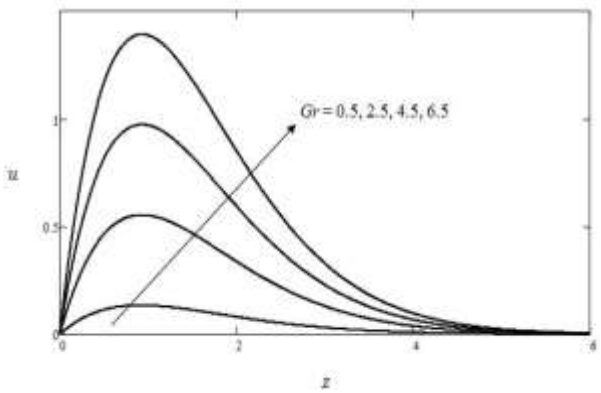


Figure 5 Real velocity profiles for different values of Gr with b = 0.1, ω_1 = 0.2, Pr = 0.71, a = 0.2, $\omega_1 t = \pi$ and t = 1.0.

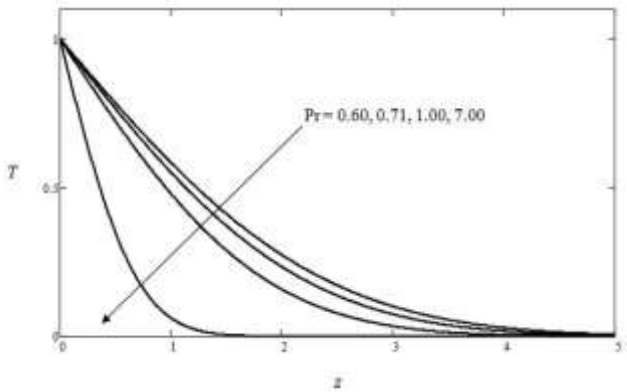


Figure 6 Temperature profiles for different values of Pr with t = 1.0.

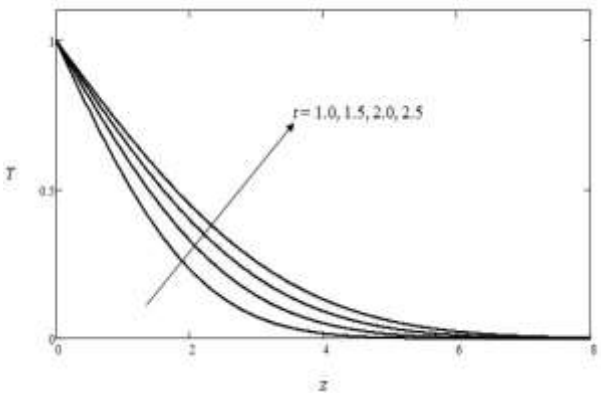


Figure 7 Temperature profiles for different values of t with Pr = 0.71.

Figure 2 shows the effect of second grade parameter a on the real part of velocity. It is found that, the velocity decreases before increasing when the value of a increases. The behavior of rotation parameter b can be observed in Figure 3. It is shown that, when b is increasing, the velocity will be decreasing. Figure 4 discusses the behavior of velocity profiles with the effect of Pr. When the values of Pr increase, the

velocity will decrease. In Figure 5, it is noted that an increase in values of *Gr* will increase the velocity profiles. Physically, this scenario is important due to the fact that, an increase in *Gr* will increase the buoyancy effect which results in more induced flow. The behavior of Prandtl number, Pr, on the temperature profile is showed in Figure 6. It is found that the temperature decreases with increasing values of Pr. It is possible because with the increase in Pr the thermal boundary layer thickness will decrease, and hence the heat transfers slowly. Lastly, Figure 7 shows the variation of temperature for different values of *t* which reveals that the temperature is increasing when *t* increases.

4.0 CONCLUSION

In this paper, a mathematical model is presented to investigate the free convection flow of rotating second grade fluid over an oscillating vertical plate. The momentum and energy equations are reduced to become dimensionless equations by using non dimensional variables. After that, the Laplace transform method is used to obtain the exact solutions for the problem. The graphical results are prepared to observe the effects of various parameters such as second grade parameter *a*, rotation parameter *b*, Prandtl number parameter Pr, Grashof number parameter *Gr* and time parameter *t*. From the graphical results shown in Figures 2 to 7, it can be concluded that, the velocity will increase when the values of parameters *b* and *Gr* increase, while the velocity will decrease with the increase in parameter Pr.

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