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EFFECTS OF NEWTONIAN HEATING AND MASS DIFFUSION ON MHD FREE CONVECTION FLOW OVER VERTICAL PLATE WITH SHEAR STRESS AT THE WALL

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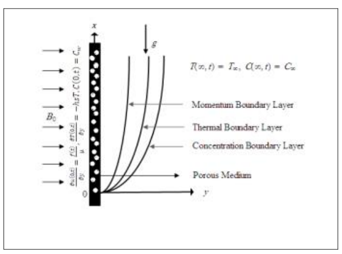
Graphical abstract



Abstract

Effects of Newtonian heating and mass diffusion on magnetohydrodynamic free convection flow over a vertical plate that applies arbitrary shear stress to the fluid is studied. The fluid is considered electrically conducting and passing through a porous medium. The influence of thermal radiation in the energy equations is also considered. General solutions of the problem are obtained in closed form using the Laplace transform technique. They satisfy the governing equations, initial and boundary conditions and can set up a huge number of exact solutions correlatives to various fluid motions. The effects of various parameters on velocity profiles are shown graphically and discussed in details.

Keywords: Free convection; mass diffusion; Newtonian heating; MHD; shear stress; laplace transform

Abstrak

Kesan pemanasan Newtonan dan resapan jisim pada aliran olakan bebas hidrodinamik magnet ke atas plat menegak di dalam bendalir yang dikenakan tegasan ricih sembarangan dikaji. Bendalir ini dipertimbangkan sebagai pengalir elektrik dan melintasi suatu bahantara berliang. Pengaruh sinaran haba di dalam persamaan tenaga juga dipertimbangkan. Penyelesaian am dalam bentuk tertutup bagi masalah ini diperoleh dengan menggunakan kaedah penjelmaan Laplace. Penyelesaian ini telah memenuhi persamaan tertakluk, syarat awal dan syarat sempadan dan boleh menyediakan sebilangan besar penyelesaian bagi pelbagai gerakan bendalir yang berkaitan. Kesan dari pelbagai parameter ke atas profil halaju dipaparkan secara graf dan dibincangkan dengan terperinci.

Kata kunci: Olakan bebas; resapan jisim; pemanasan Newtonan; hidrodinamik magnet; tegasan ricih; penjelmaan laplace

1.0 INTRODUCTION

Generally, the heat and mass transfer together with free convection flows are gradually becoming the main focus of attention not only in the field of fluid dynamics but also in several other disciplines. Perhaps, it is due to their several important applications in various branches of engineering and industrial activities such as food processing and polymer production, fiber and granular insulation, geothermal systems etc [1-3].

However, recently the attention has been diverted mostly towards the conjugate boundary condition also knows as the Newtonian heating condition [4] Soon after the pioneering work of Merkin the Newtonian heating condition has been used by several researchers including Pop et al. [5], Lesnic et al. [6]

On the other hand, it is important to bear in mind that the no slip boundary condition may not be necessarily applicable to flows of polymeric fluids that can slip or slide on the boundary. Having in mind such motivation, Fetecau et al. [7], for the first time investigated free convection flow near a vertical plate that applies arbitrary shear stress to the fluid when the thermal radiation and porosity effects are taken into consideration.

However, so far no study has been reported in the literature which focuses on the free convection flow with Newtonian heating past a vertical plate that applies arbitrary shear stress to the fluid. Even such studies are not available for viscous fluids. Therefore, in the present investigation, we study this problem for viscous fluid. However, for future research this problem can be also extended to other non-Newtonian fluids.

In fact the main purpose of this paper is to investigate the effects of Newtonian heating and mass diffusion on MHD free convection flow over a vertical plate that applies arbitrary shear stress to the fluid passing through a porous medium. General solutions of the problem are obtained using the Laplace transform technique. Some special cases are extracted from the general solutions. The results for velocity profiles are plotted graphically and discussed for the embedded flow parameters.

2.0 FORMULATION OF THE PROBLEM

Let us consider the unsteady MHD free convection flow of an incompressible viscous fluid over an infinite vertical plate. The axis is taken along the vertical plate and the $x-$axis is taken normal to the plate. Initially, both the plate and fluid are at stationary condition with the constant temperature T_∞. After time $t \geq 0$, the plate applies a time dependent shear stress $f(t)$ to the fluid along the $x-$axis. Meanwhile, the temperature of the plate is raised to T_w . The radiation term is considered in the energy equation. However, the radiative heat flux is considered to be negligible in the $x-$direction in comparison to the $y-$direction. We assume that the flow is laminar and the fluid is grey absorbing-emitting radiation but no scattering medium. Under the usual Boussinesq's approximation and neglecting the viscous dissipation, the unsteady free convection flow is governed by the following equations of momentum, energy and concentration:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T \left(T - T_\infty\right) + g\beta_C \left(C - C_\infty\right) - \frac{\nu}{K}u - \frac{\sigma B_0^2}{\rho}u, \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad y,\ t > 0, \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \quad y,\ t > 0, \tag{3}$$

where $u, T, C, \nu, \rho, g, \beta_T, \beta_C, K, \sigma, B_0, C_p, k, q_r$ and D are velocity of the fluid in $x-$direction, its temperature and concentration, the kinematic viscosity, the constant density, the gravitational acceleration, the heat transfer coefficient, the mass transfer coefficient, the permeability of the porous medium, the electric conductivity of the fluid, the applied magnetic field, the heat capacity at the constant pressure, the thermal conductivity, the radiative heat flux and the mass diffusivity.

The corresponding initial and boundary conditions are

$$u(y,0) = 0,\ T(y,0) = T_\infty,$$
$$C(y,0) = C_\infty \qquad \forall\ y \geq 0,$$
$$\frac{\partial u(0,t)}{\partial y} = \frac{f(t)}{\mu},$$
$$\frac{\partial T(0,t)}{\partial y} = -hsT,$$
$$C(0,t) = C_w,\ u(\infty,t) = 0,$$
$$T(\infty,t) = T_\infty,\ C(\infty,t) = C_\infty;\quad t > 0, \tag{4}$$

where μ, hs are the coefficient of viscosity, the heat transfer parameter for Newtonian heating and the function $f(t)$ satisfies the condition $f(0) = 0.$ The radiation heat flux under Rosseland approximation[8] is given by

$$q_r = -\frac{4\sigma^*}{3k_R}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ^* and k_R are the Stefan-Boltzmann constant and the mean spectral absorption coefficient, respectively. Here we limit our analysis to optically thick fluids while using Rosseland approximation. It is supposed that the temperature difference within the flow are sufficiently small, then Equation (5) can be linearized by expanding T^4 into Taylor series about T_∞, and neglecting higher order terms, we find that

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

Introducing Equation (6) into Equation (5) and putting the obtained result in Equation (2), we get

$$\Pr\frac{\partial T}{\partial t}=\nu\left(1+Nr\right)\frac{\partial^2 T}{\partial y^2};\quad y,t>0, \tag{7}$$

where $\Pr$, ν and Nr are defined by

$$\Pr=\frac{\mu C_p}{k},\ \nu=\frac{\mu}{\rho},\ N_r=\frac{16\sigma T_\infty^3}{3kk_R}. \tag{8}$$

In order to reduce Equations (1), (3) and (7) into their non-dimensional forms, we introduce the following dimensionless variables

$$u^*=\frac{u}{U},\ T^*=\frac{T-T_\infty}{T_\infty},\ C^*=\frac{C-C_\infty}{C_w-C_\infty},\ y^*=\frac{U}{\nu}y,$$
$$t^*=\frac{U^2}{\nu}t,\quad f^*\left(t^*\right)=\frac{1}{\rho U^2}f\left(\frac{\nu}{U^2}t^*\right), \tag{9}$$

By using (9) into Equations (1) and (7) and (3) and dropping out the " $*$ " notation, it yields

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+GrT+GmC-K_p u-Mu, \tag{10}$$

$$\Pr_{eff}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{12}$$

where $\Pr_{eff}=\frac{\Pr}{1+Nr}$ is the effective Prandtl number[8] and

$$Gr=\frac{g\beta_T\nu T_\infty}{U^3},\ Gm=\frac{g\beta_c\left(C_w-C_\infty\right)\nu}{U^3},$$
$$M=\frac{\sigma B_0^2\nu}{\rho U^2},\ Sc=\frac{\nu}{D},\ \ K_p=\frac{\nu^2}{U^2}\frac{1}{K}, \tag{13}$$

are the Grashof number, modified Grashof number, magnetic parameter, Schmidt number and the inverse permeability parameter for the porous medium respectively.

The corresponding dimensionless initial and boundary conditions are

$$u\left(y,0\right)=0,\ T\left(y,0\right)=0\ \ C(y,0)=0,$$
$$\frac{\partial u}{\partial y}|_{y=0}=f\left(t\right),\ \frac{\partial T}{\partial y}|_{y=0}=-\gamma\left(1+T\right),\ C\left(0,t\right)=1, \tag{14}$$
$$C\left(\infty,t\right)=0,\ T\left(\infty,t\right)=0,\ u\left(\infty,t\right)=0,$$

where $\gamma=\frac{\nu}{U}hs$ is the Newtonian heating parameter.

3.0 SOLUTION OF THE PROBLEM

Applying Laplace transform to Equations (10), (11) and (12) and using the initial and boundary conditions from Equations (14) and then by taking inverse Laplace transform the equations for temperature, concentration and velocity are obtained as

$$T\left(y,t\right)=e^{b_1^2t-b_1y\sqrt{\Pr_{eff}}}\operatorname{erf}c\left(\frac{y\sqrt{\Pr_{eff}}}{2\sqrt{t}}-b_1\sqrt{t}\right)-\operatorname{erf}c\left(\frac{y\sqrt{\Pr_{eff}}}{2\sqrt{t}}\right). \tag{15}$$

Here $\operatorname{erf}(.)$ and $\operatorname{erf}c(.)$ denote the error function and complementary error function of Gauss [7]

$$C\left(y,t\right)=\operatorname{erf}c\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right). \tag{16}$$

$$u\left(y,t\right)=u_c\left(y,t\right)+u_m\left(y,t\right), \tag{17}$$

where

$$u_c\left(y,t\right)=\frac{b_4b_5}{b_8\sqrt{\pi}}\int_0^t e^{b_1^2(t-s)}\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$-\frac{b_1b_4b_5}{b_8\sqrt{\pi}}\int_0^t e^{b_1^2(t-s)}\operatorname{erf}\left(b_1\sqrt{t-s}\right)\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$-\frac{\sqrt{Sc}Gm}{a_2\sqrt{a_1\pi}}\int_0^t\frac{\operatorname{erf}\left(\sqrt{a_1}\sqrt{t-s}\right)e^{a_1(t-s)-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$+\frac{\sqrt{b_2}b_7b_4b_5}{b_8\sqrt{\pi b_2}}\int_0^t e^{b_2(t-s)}\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$+\frac{b_1b_4b_5}{b_8\sqrt{\pi b_2}}\int_0^t\left[e^{b_2(t-s)}\operatorname{erf}\left(\sqrt{b_2}\sqrt{t-s}\right)\right]\frac{e^{-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds$$
$$-\frac{Gm}{a_2a_1}\operatorname{erf}c\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)+\frac{2b_4b_5}{b_2\sqrt{\pi}}\int_0^t\frac{e^{b_2(t-s)-\frac{y^2}{4s}-b_6s}}{\sqrt{s}}ds,$$

$$+\frac{1}{2b_1}\int_0^t e^{b_2 s-y\sqrt{\Pr_{eff}}\sqrt{b_2}}\left[-1+e^{b_1^2(t-s)}\left(1+\text{erf}\left(b_1\sqrt{t-s}\right)\right)\right]$$

$$\text{erf}\,c\left(\frac{y\sqrt{\Pr_{eff}}}{2\sqrt{s}}-\sqrt{b_2 s}\right)ds$$

$$+\frac{Gme^{a_1 t}}{2a_2 a_1}\left[\begin{array}{l} e^{y\sqrt{Sc}\sqrt{a_1}}\,\text{erf}\,c\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{a_1 t}\right)+e^{-y\sqrt{Sc}\sqrt{a_1}} \\ \text{erf}\,c\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{a_1 t}\right)\end{array}\right]$$

and

$$u_m(y,t)=-\frac{1}{\sqrt{\pi}}\int_0^t f(t-s)\frac{e^{-\frac{y^2}{4s}-b_6 s}}{\sqrt{s}}ds,$$

correspond to the convective and mechanical parts of velocity.

4.0 SPECIAL CASE

The solutions of velocity obtained in Section 3, are more general. Hence, in this section we intend to discuss some special cases of the present solutions together with some limiting solutions in order to gain more about the physical insight of the problem. So, we discuss the following important special cases whose technical relevance is well-known in the literature.

In this first case we take the arbitrary function $f(t)=fH(t),$ where f is a dimensionless constant and $H(\cdot)$ denotes the unit step function. After time $t=0,$ the infinite vertical plate applies a constant shear stress to the fluid. The convective part of the velocity remains unchanged while the mechanical part takes the following form

$$u_m(y,t)=-\frac{f}{\sqrt{\pi}}\int_0^t\frac{e^{-\frac{y^2}{4s}-b_6 s}}{\sqrt{s}}ds, \quad (18)$$

or

$$u_m(y,t)=-\frac{f}{\sqrt{b_6}}e^{-y\sqrt{b_6}}+\frac{2f}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty}e^{-\frac{y^2}{4z^2}-b_6 z^2}dz, \quad (19)$$

for $K_p\neq 0, M\neq 0$. Moreover, if we take $M=0$, Equations (19), reduces to

$$u_m(y,t)=-\frac{f}{\sqrt{K_p}}e^{-y\sqrt{K_p}}+\frac{2f}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty}e^{-\frac{y^2}{4z^2}-K_p z^2}dz, \quad (20)$$

which is equivalent to[7], in equation (29)] with the correction of $\sqrt{K_p}$. Here,

$$b_1=\frac{\gamma}{\sqrt{\Pr_{eff}}},\ b_2=\frac{b_6}{b_3},b_3=\Pr_{eff}-1,\ b_4=\frac{-Grb_1}{b_3},$$

$$b_5=\sqrt{\Pr_{eff}},\ b_6=K_p+M,$$

$$b_7=1+\frac{b_1\sqrt{b_2}}{b_2}-\frac{b_1}{\sqrt{b_2}},\ b_8=b_1^3-b_1 b_2,$$

$$a_1=\frac{b_6}{a_2},\ a_2=Sc-1.$$

5.0 RESULTS AND DISCUSSION

In this paper exact analysis of heat and mass transfer past an infinite vertical plate that applies arbitrary shear stress to the fluid with Newtonian heating is investigated. More general solutions of the problem are obtained using the Laplace transform technique. The graphical results of velocity profiles for various flow parameters such as magnetic parameter M, wall shear stress f and Newtonian heating parameter γ are analyzed. The non-dimensional velocity profiles for different values of magnetic parameter M are shown in Figure 1.

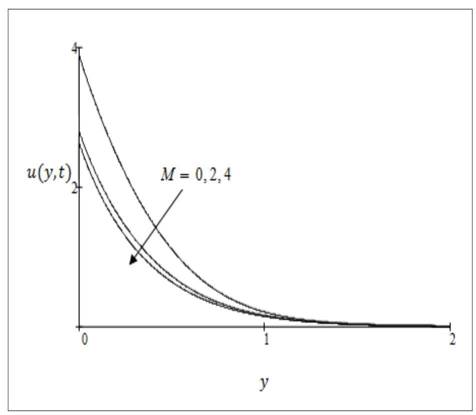


Figure 1 Velocity profiles for different values of M

It is found that the velocity is decreasing with increasing values of M. Physically, it is true as increasing values of M increase the resistance force. As a result fluid flow tends to resist and reducing its velocity. This result agrees well with that resulting from [7, Figure. 33]. The influence of the wall shear stress f induced by the bounding plate on the non- dimensional velocity profiles is shown in Figure 2.

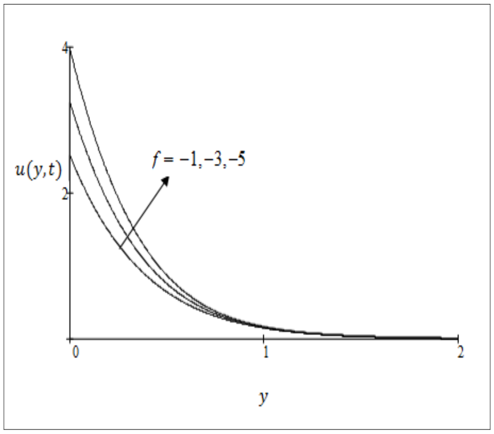


Figure 2 Velocity profiles for for different values of f.

As expected the velocity of fluid decreases with increasing f. Newtonian heating parameter on velocity is studied in Figure 3. It is observed that an increase in the Newtonian heating parameter increase the fluid velocity.

6.0 CONCLUSIONS

The effects of Newtonian heating an unsteady MHD free convection flow past a vertical plate that applies arbitrary shear stress to the fluid through a porous medium is presented. The Laplace transform method is used to obtain the exact solutions. The effects of various parameters on velocity are graphically studied.

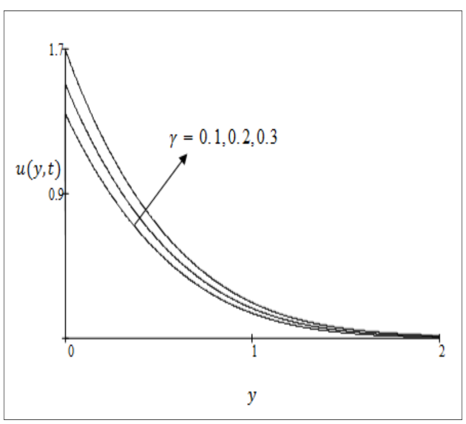


Figure 3 Velocity profiles for different values of γ.

The fluid velocity is presented as a sum of mechanical and convection parts. One of the special case is obtained from general solutions. We concluded that fluid velocity is decreasing function with respect to the magnetic parameter M and wall shear stress f while increasing function with respect to the Newtonian heating parameter γ.

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