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NEW RESULTS ON PARTIAL FRACTION BASED FREQUENCY WEIGHTED BALANCED TRUNCATION TECHNIQUE FOR DISCRETE-TIME SYSTEM

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Abstract. In this paper, we present some new results on frequency weighted model reduction technique based on partial fraction expansion idea in discrete-time system. The reduced order models of the newly proposed method obtained by direct truncation, produces lower errors when compared to existing techniques. The new method is guaranteed to be stable even for double sided weightings. A simple and easily computable *a priori* error bound is also derived. Numerical examples with comparisons to the existing techniques show the effectiveness of the proposed method.

Keywords: Model order reductions; error bounds; partial fraction expansion

Abstrak. Artikel in membentangkan keputusan baru bagi kaedah pengurangan model frekuensi tertimbang berdasarkan kaedah pengembangan fraksi separa didalam masa diskrit. Rangka pengurangan model bagi kaedah baru yang dicadangkan diperolehi melalui pemotongan langsung, menghasilkan kesalahan yang lebih rendah berbanding kaedah-kaedah lain yang sudah ada. Kaedah baru ini dijamin akan stabil bahkan untuk tertimbang bersisi ganda. Sebuah batas kesalahan *a priori* yang mudah dan senang dihitung juga diperoleh. Contoh berangka dengan perbandingan dengan teknik yang ada menunjukkan keberkesanan kaedah yang dicadangkan

Keywords: Rangka pengurangan model; batas kesalahan; kaedah pengembangan fraksi spara

1.0 INTRODUCTION

The concept of approximating a linear system into a more manageable order without jeopardizing the properties of the original dynamical system has attracted

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many researchers [9], [16], [1]. Moore [9] in particular has introduced the well-known technique, balanced truncation technique. Enns [3] then presented a scheme for reducing a stable high order model with frequency weighting based on [9]. In Enns' method, when using input or output weighting, the reduced order system will yield stable reduced order model. However, when both weightings are present, the stability of the reduced order system is not guaranteed. Lin and Chiu [8] has since proposed a different method to guarantee stability even when both weightings are present under certain assumptions i.e. using strictly proper functions and no occurrence of pole-zero cancellations when forming the augmented system. Wang et al [15] has also solved the stability problem of Enns' for two-sided case by introducing fictitious input and output matrices

Another group of methods which is based on partial fraction expansion was originally proposed by [7]. Inspired by this, Al-Saggaf and Fanklin [1] proposed a technique for frequency weighted model reduction. In their technique, the numerator of the reduced order model is calculated by forcing the reduction error to have zeros at the poles of the weighting function. Several limitations on [1] restrict its application. Sreeram and Anderson [12] generalized [1] by including double-sided weightings but only capable in handling strictly proper weighting functions. Ghafoor and Sreeram [6] then proposed a parameterized method which combines the advantages of the un-weighted balancing technique [9] and Sreeram and Anderson's [12] method. This newly proposed method can conveniently handle both proper and strictly proper weighting functions but it is an ad-hoc method. To overcome this, [10] has proposed a new improved frequency weighted balanced truncation technique based on partial fraction expansion which offers the same advantages of [6] but with clear theoretical justification and yield better results when compared to Enns' [3] and Ghafoor and Sreeram's [6] method.

In this paper, we present the modification of [10] in discrete-time domain. The proposed method has the following advantages: (i) guaranteed stability of models in case of double-sided weightings (ii) simple, elegant and easily computable error bound (iii) extendable to frequency weighted optimal Hankel norm approximations (iv) choice of free parameters to reduce the weighted error and error bounds and (v) easily applicable to controller reduction problem. Numerical example shows that by properly choosing the variable parameters of α_D and β_D, a lower error can be obtained.

2.0 FREQUENCY WEIGHTED MODEL REDUCTION IN DISCRETE-TIME

To have a better understanding of frequency weighted model reduction, let us first consider a minimal realization of $G_D(z) = \{A_D, B_D, C_D, D_D\}$ as shown below:

$$G_D(z) = \left[\begin{array}{c|c} A_D & B_D \\ \hline C_D & D_D \end{array}\right]$$

where z is the complex Z-transform variable. The controllability and observability Gramians P_D and Q_D of the original system satisfy the following Lyapunov equations:

$$\begin{aligned} A_D P_D A_D^T - P_D + B_D B_D^T &= 0 \\ A_D^T Q_D A_D - Q_D + D_D^T D_D &= 0 \end{aligned} \tag{1}$$

Given the transfer functions of the input weight $V_D(z) = \left[\begin{array}{c|c} A_{VD} & B_{VD} \\ \hline C_{VD} & D_{VD} \end{array}\right]$ and the output weight $W_D(z) = \left[\begin{array}{c|c} A_{WD} & B_{WD} \\ \hline C_{WD} & D_{WD} \end{array}\right]$, the augmented system of $\overline{G}_D(z) = W_D(z) G_D(z) V_D(z)$ is then given as following:

$$\overline{G}_D(z) = \left[\begin{array}{c|c} \overline{A}_D & \overline{B}_D \\ \hline \overline{C}_D & \overline{D}_D \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} A_{WD} & B_{WD} C_D & B_{WD} D_D C_{VD} & B_{WD} D_D D_{VD} \\ 0 & A_D & B_D C_{VD} & B_D D_{VD} \\ 0 & 0 & A_{VD} & B_{VD} \\ \hline C_{WD} & D_{WD} C_D & D_{WD} D_D C_{VD} & D_{WD} D_D D_{VD} \end{array}\right] \tag{2}$$

The controllability and observability Gramians of the augmented system are then given by $\overline{P}_D$ and $\overline{Q}_D$ as following:

$$\overline{P}_D = \begin{bmatrix} P_W & P_{12} & P_{13} \\ P_{12}^T & P_D & P_{23} \\ P_{13}^T & P_{23}^T & P_V \end{bmatrix}, \qquad \overline{Q}_D = \begin{bmatrix} Q_W & Q_{12} & Q_{13} \\ Q_{12}^T & Q_D & Q_{23} \\ Q_{13}^T & Q_{23}^T & Q_V \end{bmatrix} \tag{3}$$

which satisfy the Lyapunov equations below:

$$\overline{A}_D \overline{P}_D \overline{A}_D^T - \overline{P}_D + \overline{B}_D \overline{B}_D^T = 0 \tag{4}$$

$$\overline{A}_D^T \overline{Q}_D \overline{A}_D - \overline{Q}_D + \overline{C}_D^T \overline{C}_D = 0 \tag{5}$$

2.1 ENNS' TECHNIQUE

Enns was the first to introduce frequency weighted model reduction. In Enns [3], the technique was first utilized by expanding the (*2,2*) block of equations (4) and (5) for controllability and observability Gramians respectively. This will yield the following pair of equations for $-P_{DE}$ and $-Q_{DE}$.

$$\begin{aligned} A_D P_D A_D{}^T - P_D &= -B_D C_{VD} P_{D12}^T A_D^T - A P_{D12} C_{VD}^T B_D^T \\ &\quad - B_D C_{VD} P_{VD} C_{VD}^T B_D{}^T - B_D D_{VD} D_{VD}^T B_D^T \\ &\equiv -P_{DE} \end{aligned} \tag{6}$$

$$\begin{aligned} A_D{}^T Q_D A_D - Q_D &= -C_D^T B_{WD}^T Q_{12}^T A_D - A_D^T Q_{12} B_{WD} C_D \\ &\quad - C_D^T B_{WD}^T Q_{12}^T - C_D^T D_{WD}^T D_{WD} C_D \\ &\equiv -Q_{DE} \end{aligned} \tag{7}$$

The matrices P_D and Q_D in equations (6) and (7) are then diagonalize simultaneously to obtain

$$T^{-1} P_D T^{-T} = T^T Q_D T = diag(\sigma_1, \sigma_2, ..., \sigma_r, \sigma_{r+1}, ...\sigma_n)$$

where $\sigma_1 > \sigma_2 > \ldots > \sigma_n > 0$. The original system is then transformed using the similarity transformation T and partitioned as below:

$$\left[\begin{array}{c|c} T^{-1}A_D T & T^{-1}B_D \\ \hline C_D T & D_D \end{array}\right] = \left[\begin{array}{cc|c} A_r & A_{12} & B_r \\ A_{21} & A_{22} & B_2 \\ \hline C_r & C_2 & D \end{array}\right]$$

and the dimension of A_r is equal to the dimension of $diag(\sigma_1, \sigma_2, \ldots, \sigma_r)$.

Applying Enn's method, the reduced order model G_r is then given by $G_{Dr}(z) = \left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}\right]$. Essentially, Enns' method is based on diagonalizing simultaneously the solutions of Lyapunov equations as given in equations (4) and (5). However, Enns' method cannot guarantee the stability of reduced order models as P_{DE} and Q_{ED} may be indefinite.

2.2 LIN & CHIU'S TECHNIQUE [8]

A subsequent technique following Enns' is Lin and Chiu's [8] and the commonly referred technique, its generalization in [13]. The generalized Lin and Chiu's technique [13] differs from Enns' technique as it simultaneously diagonalizes the Gramians P_{LCD} and Q_{LCD} instead of diagonalizing P_D and Q_D as given below:

$$\begin{aligned} P_{LCD} &= P_D - P_{23} P_V^{-1} P_{23}^T \\ Q_{LCD} &= Q_D - Q_{12}^T Q_W^{-1} Q_{12} \end{aligned} \tag{8}$$

The new Gramians now satisfy the following pair of Lyapunov equations

$$\begin{aligned} A_D P_{LCD} A_D^T - P_{LCD} + B_{LCD} B_{LCD}^T &= 0 \\ A_D^T Q_{LCD} A_D - Q_{LCD} + C_{LCD}^T C_{LCD} &= 0 \end{aligned}$$

where B_{LCD} and C_{LCD} are given as

$$B_{LCD} = \begin{bmatrix} (A_D P_{12} P_V^{-1} + B_D C_{VD} - P_{12} P_V^{-1} A_{VD}) P_V^{1/2} \\ B_D D_{VD} - P_{12} P_V^{-1} B_{VD} \end{bmatrix}$$

$$C_{LCD} = \begin{bmatrix} Q_W^{1/2} (Q_W^{-1} Q_{12} A_D + B_{WD} C_D - A_{WD} Q_W^{-1} Q_{12}) \\ D_{WD} C_D - C_{WD} Q_V^{-1} Q_{12} \end{bmatrix}$$

Assuming that there are no pole-zero cancellations between the weights and the original system, the realization is minimal and Lin and Chiu's technique yields stable models for two-sided frequency weighting system.

2.3 WANG ET AL'S TECHNIQUE [8]

Another commonly referred method following Enns' is proposed by Wang et al [15]. In this technique, the stability of the reduced order model is guaranteed by making the matrices P_{DE} and Q_{DE} positive (semi) definite. [9] proposes that the new controllability (P_{DW}) and observability (Q_{DW}) Gramians are diagonalized, as obtained from the solution of the following pair of Lyapunov equations:

$$\begin{aligned} A_D P_{DW} A_D^T - P_{DW} + B_{DWE} B_{DWE}^T &= 0 \\ A_D^T Q_{DW} A_D - Q_{DW} + C_{DWE}^T C_{DWE} &= 0 \end{aligned} \tag{9}$$

The matrices B_{DWE} and C_{DWE} in the above Lyapunov equations are fictitious input and output matrices which are determined from $B_{DWE} = U|S_{WE}|^{1/2}$ and $C_{DWE} = |R_{WE}|^{1/2} V^T$ where U, S_{WE}, R_{WE} and V^T are obtained from the singular value decomposition of matrices, $P_{DW} = US_{WE}U^T$ and $Q_{DW} = VR_{WE}V^T$. Since

$$P_{DW} \le B_{DWE} B_{DWE}^T \ge 0, \qquad Q_{DW} \le C_{DWE}^T C_{DWE} \ge 0$$

and $\{A_D, B_{DWE}, C_{DWE}\}$ is minimal, stability of the reduced order model in case of two-sided frequency weighting is guaranteed.

2.4 PARTIAL FRACTION EXPANSION TECHNIQUE

In the partial fraction expansion technique of Sreeram and Anderson's [12], instead of diagonalizing the controllability Gramians P_D and the observability Gramian Q_D, they diagonalized P_{PFD} and Q_{PFD} as shown below:

$$T_{PF}^{-1}P_{PFD}T_{PF}^{-T} = T_{PF}^{T}Q_{PFD}T_{PF} = \Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_n)$$

where $\sigma_1 > \sigma_2 > \ldots > \sigma_n$ and

$$\begin{aligned} P_{PF} &= P_D - P_{23}X_{PF}^T - X_{PF}P_{23}^T + X_{PF}P_{VD}X_{PF}^T \\ Q_{PF} &= Q_D - Q_{12}Y_{PF} - Y_{PF}^T Q_{12}^T + Y_{PF}^T Q_{WD}Y_{PF} \end{aligned} \quad (10)$$

The matrices X_{PF} and Y_{PF} satisfy the following equations

$$\begin{aligned} A_D X_{PF} - X_{PF}A_{VD} + B_D C_{VD} &= 0 \\ Y_{PF}A_D - A_{WD}Y_{PF} + B_{WD}C_D &= 0 \end{aligned}$$

and the Gramians P_{PF} and Q_{PF} satisfy the following Lyapunov equations

$$\begin{aligned} A_D P_{PF} A_D^T - P_{PF} + B_{PFD}B_{PFD}^T &= 0 \\ A_D^T Q_{PF} A_D - Q_{PF} + C_{PFD}^T C_{PFD} &= 0 \end{aligned}$$

where B_{PFD} and C_{PFD} are given as shown below

$$\begin{aligned} B_{PFD} &= B_D D_{VD} - X_{PF}B_{VD} \\ C_{PFD} &= D_{WD}C_D - C_{WD}Y_{PF} \end{aligned}$$

Since the realization $\{A_D, B_{PFD}, C_{PFD}\}$ is minimal and the Gramians diagonalized satisfy the Lyapunov equations, the partial fraction technique yields stable model in the case of double-sided weightings.

2.5 GHAFOOR AND SREERAM'S PARAMETERIZED PARTIAL FRACTION EXPANSION METHOD

Ghafoor and Sreeram [6] then proposed a new frequency weighted balanced reduction technique which is based on parameterized combination of the un-weighted technique [9] and the partial fraction expansion technique [12]. In this method, instead of simultaneously diagonalizing P_{PF} and Q_{PF}, P_{XD} and Q_{YD} are simultaneously diagonalized

$$P_{XD} = P_D + \alpha^2 P_{PF}$$
$$Q_{YD} = Q_D + \beta^2 Q_{PF}$$

where α and β are real constants. The Gramians P_{PF} and Q_{PF} satisfy equation (15) respectively while Gramians P_{XD} and Q_{YD} satisfy the following equations:

$$A_D P_{XD} A_D^T - P_{XD} + B_{XD} B_{XD}^T = 0$$
$$A_D^T Q_{YD} A_D - P_{YD} + C_{YD}^T C_{YD} \quad = 0$$

where B_{XD} and C_{YD} are fictitious input and output matrices and are given as below:

$$B_{XD} = \begin{bmatrix} \alpha B_D & (B_D D_{VD} - X_{PF} B_{VD}) \end{bmatrix}$$

$$C_{YD} = \begin{bmatrix} \beta C_{VD} \\ (D_{WD} C_D - C_{WD} Y_{PF}) \end{bmatrix}$$

$\alpha > 0$ and $\beta > 0$.

Although the results yield better than Enn's technique, it is an ad-hoc method without a proper theoretical explanation on how the new system $\{A_D, B_{XD}, C_{YD}\}$ is obtained.

3.0 RESULTS AND DISCUSSIONS

This section explores the new proposed frequency weighted balanced truncation technique which is based on the un-weighted balanced truncation technique [9] and the partial fraction expansion technique [6]. In [10], a frequency weighted model

Let us assume the system described in equations (2), (3) and (4). Let $\overline{T}_D$ be the transformation matrix as defined below: $\overline{T}_D = \begin{bmatrix} I & -\overline{Y}_D & 0 \\ 0 & I & \overline{X}_D \\ 0 & 0 & I \end{bmatrix}$

Let $\overline{X}_D$ and $\overline{Y}_D$ be defined from the following equation

$$\begin{aligned} A_D\overline{X}_D - \overline{X}_D A_{VD} + B_D C_{VD} &= 0 \\ \overline{Y}_D A_D - A_{WD}\overline{Y}_D + B_{WD}C_D &= 0 \end{aligned}$$

Transforming the augmented realization of the system $\overline{G}_D(z)$ using the transformation matrix $\overline{T}_D$, will yield the following:

$$\overline{G}_{PFD}(z) = \left[\begin{array}{c|c} \overline{T}_D^{-1}\overline{A}_D\overline{T}_D & \overline{T}_D^{-1}\overline{B}_D \\ \hline \overline{C}_D\overline{T}_D & \overline{D}_D \end{array}\right] = \left[\begin{array}{c|c} \overline{A}_{PFD} & \overline{B}_{PFD} \\ \hline \overline{C}_{PFD} & \overline{D}_{PFD} \end{array}\right]$$
$$= \left[\begin{array}{ccc|c} A_{WD} & X_{D12} & X_{D13} & X_{D1} \\ 0 & A_D & X_{D23} & X_{D2} \\ 0 & 0 & A_{VD} & B_{VD} \\ \hline C_{WD} & Y_{D1} & Y_{D2} & D_{WD}D_D D_{VD} \end{array}\right] \qquad (12)$$

where the system matrix yield

$$\begin{aligned} X_{D12} &= A_D\overline{X}_D - \overline{X}_D A_{VD} + B_D C_{VD} \\ X_{D13} &= \overline{Y}_D A_D - A_{WD}\overline{Y}_D + B_{WD}C_D \\ X_{D23} &= B_{WD}C_D\overline{X}_D - \overline{Y}_D A_D\overline{X}_D + B_{WD}D_D C_{VD} + \overline{Y}_D B_D C_{VD} - \overline{Y}_D\overline{X}_D A_{VD} \end{aligned}$$

and from the input and output matrix, yield

$$\begin{aligned} X_{D1} &= B_{WD}D_D D_{VD} + \overline{Y}_D B_D D_{VD} + \overline{Y}_D\overline{X}_D B_{VD} \\ X_{D2} &= B_D D_{VD} - \overline{X}_D B_{VD} \\ Y_{D1} &= D_{WD}C_D - C_{WD}\overline{Y}_D \\ Y_{D2} &= D_{WD}C_D\overline{X}_D + D_{WD}D_D C_{VD} \end{aligned}$$

Note that the system matrix is block diagonalized by the similarity transformation $\overline{T}_D$.

Consider the discrete original signal $G_D(z)$ and the discrete input and output weights $V_D(z)$ and $W_D(z)$, the augmented system is then given by $\overline{G}_D(z)$ as shown in equation (2). In the proposed method, the matrices $X_{D12}, X_{D13}, X_{D23}, Y_{D1}, Y_{D2}, X_{D1}$ and X_{D2} of equation (12) are referred. The matrices $\overline{X}_D$ and $\overline{Y}_D$ from the transformation matrix $\overline{T}_D$ above are obtained by solving the following equations:

$$X_{D12} = \overline{Y}_D A_D - A_{WD}\overline{Y}_D + B_{WD}C_D = 0$$
$$X_{D23} = A_D\overline{X}_D - \overline{X}_D A_{VD} + B_D C_{VD} = 0$$

By factorizing the matrices X_{D12}, Y_{D1}, X_{D23} and X_{D2} as shown below,

$$X_{D12} = \begin{bmatrix} B_{WD} & A_{WD} & I \end{bmatrix} \begin{bmatrix} C_D \\ -\overline{Y}_D \\ \overline{Y}_D A_D \end{bmatrix} = \overline{B}_{WD}\overline{C}_D$$

$$Y_{D1} = \begin{bmatrix} D_{WD} & C_{WD} & 0 \end{bmatrix} \begin{bmatrix} C_D \\ -\overline{Y}_D \\ \overline{Y}_D A_D \end{bmatrix} = \overline{D}_{WD}\overline{C}_D$$

$$X_{D23} = \begin{bmatrix} B_D & -\overline{X}_D & A_D\overline{X}_D \end{bmatrix} \begin{bmatrix} C_{VD} \\ A_{VD} \\ I \end{bmatrix} = \overline{B}_D\overline{C}_{VD}$$

$$X_{D2} = \begin{bmatrix} B_D & -\overline{X}_D & A_D\overline{X}_D \end{bmatrix} \begin{bmatrix} D_{VD} \\ B_{VD} \\ 0 \end{bmatrix} = \overline{B}_D\overline{D}_{VD}$$

the new fictitious input and output matrices of the new original system $\overline{B}_D$ and $\overline{C}_D$ respectively, as well as the new fictitious matrices for the new weights, $\overline{C}_{VD}, \overline{D}_{VD}$ and $\overline{B}_{WD}, \overline{D}_{WD}$, are defined.

Algorithm The algorithm for the proposed method is as given below:

1. Given a stable, original minimal realization $G_D(z) = \left[\begin{array}{c|c} A_D & B_D \\ \hline C_D & D_D \end{array}\right]$ and minimal realizations of the weights $V_D(z) = \left[\begin{array}{c|c} A_{VD} & B_{VD} \\ \hline C_{VD} & D_{VD} \end{array}\right]$ and $W_D(z) = \left[\begin{array}{c|c} A_{WD} & B_{WD} \\ \hline C_{WD} & D_{WD} \end{array}\right]$, compute $\overline{X}_D$ and $\overline{Y}_D$

   $\overline{Y}_D A_D - A_{WD}\overline{Y}_D + B_{WD}C_D = 0$

   $A_D\overline{X}_D - \overline{X}_D A_{VD} + B_D C_{VC} = 0$

2. Compute the fictitious input and output matrices $\overline{B}_D$ and $\overline{C}_D$

   $\overline{B}_D = \begin{bmatrix} B_D & -\overline{X}_D & A_D\overline{X}_D \end{bmatrix}$ and $\overline{C}_D = \begin{bmatrix} C_D \\ -\overline{Y}_D \\ \overline{Y}_D A_D \end{bmatrix}$

3. Solve the Lyapunov equations for $\overline{P}_{D1}$ and $\overline{Q}_{D1}$

   $A_D\overline{P}_{D1}A_D^T - \overline{P}_{D1} + \overline{B}_D\overline{B}_D^T = 0$

   $A_D^T\overline{Q}_{D1}A_D - \overline{Q}_{D1} + \overline{C}_D^T\overline{C}_D = 0$

4. Calculate the transformation matrix T_D which balanced $\{A_D, \overline{B}_D, \overline{C}_D\}$

   $$T_D^{-1}\overline{P}_{D1}T_D^{-T} = T_D^T\overline{Q}_{D1}T_D = diag(\sigma_1, \sigma_2, ..., \sigma_n)$$

   where $\sigma_i \geq \sigma_{i+1}, i = 1,2,...,n-1$.

5. Compute the frequency weighted balanced realization and the reduced order model is given as $G_{RD}(z) = \left[\begin{array}{c|c} A_{D11} & B_{D1} \\ \hline C_{D1} & D \end{array}\right]$.

Remark *Since the realization $G_D(z) = \{A_D, B_D, C_D, D\}$ is minimal, then the reduced order model $G_{RD}(z) = \{A_{D11}, B_{D1}, C_{D1}, D\}$ is stable for both single-sided and double-sided weightings.*

Theorem Let $G_D(z)$ be a proper, stable transfer function of order *n* and $W_D(z)$ and $V_D(z)$ be the proper weighting functions. If $G_{RD}(z)$ is a proper, stable reduced-order model obtained using the proposed technique, then the following error bound holds:

$$\|W_D(z)[G_D(z) - G_{RD}(z)]V_D(z)\|_\infty \leq \gamma_D \sum_{i=r+1}^{n} \sigma_i$$

where $\gamma_D = 2\|V_D(z)\|_\infty \|W_D(z)\|_\infty$

Proof The proof is similar to [10] hence omitted.

3.0 LIMITATIONS

Even though the frequency weighted errors obtained using the new methods are generally lower than Enns' and Ghafoor and Sreeram's method, the technique is realization dependant. For different realization of input and output weights, different reduced order models and weighted approximation errors are obtained. Hence, to obtain the optimum weighted errors, simple transformation $\alpha_D I$ for the input and $\beta_D I$ for the output weight are utilized. By varying the scalars α_D and β_D and, one can easily reduce the weighted approximation errors

4.0 EXAMPLE

In this section, an example of a stable 5^{th} order system extracted from [5] is utilized. In [5], it was shown that Ghafoor and Sreeram's method [6] compares well to Enns' technique [3]. Here the author compares the newly proposed method to [6] as well as [3].

Consider the stable system

$$A_D = \begin{bmatrix} 0.0010 & -0.6334 & 0.0015 & -0.0557 & 0.0001 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad B_D = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_D = \begin{bmatrix} 4.951 & 9.1676 & 9.7045 & 4.7473 & 0.951 \end{bmatrix}, \qquad D_D = \begin{bmatrix} 1 \end{bmatrix}$$

with stable input and output weights as given below:

$$V_D(z) = W_D(z) = \left[\begin{array}{ccc|c} -1.1619 & -0.6959 & -0.1378 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1.8081 & 2.2444 & 0.8325 & 1 \end{array}\right]$$

Table 1 Error and error bounds of the proposed technique compared to [3] and [6]

Order	Enns	Ghafoor and Sreeram's Method				Proposed Method			
		α	β	*Error*	*Error Bound*	α_D	β_D	*Error*	*Error Bound*
1	119.0175	6	6	118.8405	244.2878	200	200	117.0057	219.4063
		9	9	117.9744	230.5783	100	200	117.0082	219.4876
		1	10	121.7365	461.6540	5	100	118.0022	234.3603
2	43.8173	4	4	41.7730	83.2099	3.2	3.2	39.8043	125.5738
		6	6	41.4428	74.4676	36.8	3.2	40.4129	92.0167
		3	3	42.9521	94.5191	19.5	19.5	41.4418	68.6829
3	6.7124	3	3	5.7510	15.0555	17	3.1	5.8853	15.0387
		4	4	5.7352	13.2190	16.9	3.2	5.8894	14.8213
		2	3	5.9047	17.2472	26.9	26.9	5.9400	10.7979
4	0.3706	1	2	0.3411	1.7425	1.2	18.9	0.3193	1.8770
		2	2	0.3348	1.2368	8.8	1.3	0.3201	1.7557
		5	1	0.3219	1.3938	38.9	1.1	0.3210	2.0027

From Table 1, by varying the free parameters α_D and β_D, it is possible to obtain consistently lower weighted errors for the proposed method compared to Enns's technique [3], while it compares well to Ghafoor and Sreeram's technique [6].

5.0 CONCLUSION

An improved frequency weighted balanced truncation based on partial fraction expansion in discrete-time domain is presented. The method has the following advantages: (i) Guaranteed stability in case of double-sided weightings (ii) two sets of easily computable *a priori* error bounds. The only disadvantage of the technique is it is dependant on the realization of the weights. However, this property can be manipulated to obtain the lowest error of the reduced order model. This is done by varying the realization of the weights. The proposed method can be easily extended to optimal Hankel norm approximations.

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