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NUMERICAL ANALYSIS ALGORITHM FOR INSPECTION POLICY PROCEDURE

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Abstract

In this paper, we present some tools helping us to make decision i.e., designing an optimal inspection measure. The approach is developed in search proposed by Hemodialysis center health, an implementation and computation program is developed. This problem has also applications in the theories of reliability and can be modeled by a Markov chain structure; this can be done by parallel series or series parallel coherent structures. The algorithms are encouraging and measure can be found.

Keywords: Reliability theory, failure, renewal, optimal inspection policy, computational algorithms

1.0 INTRODUCTION

In this paper we are concerned with the problem of inspection policy. Such actions are necessary for certain complex systems in order to detect failures that would otherwise not be apparent.

Let " w "be a nonnegative random variable with probability distribution $F(x)=P(w\leq x)$, - ∞ < x < + ∞.

It will be interpreted as the life length of the system. For convenience and without loss of generality, we assume that all random variables introduced here (included. w) are defined on the same probability space (Ω,F,P).

The reliability (or survival probability) of the system corresponding to a mission of duration x is by definition $R(x)=1-F(x)$. If moreover, " w" has a density $f(x)=F'(x)$, then one can define the conditional failure rate at time t : $r(t)=f(t)/R(t)$, or

$$r(t)=\lim_{x\to o}\frac{1}{x}\frac{F(t+x)-F(t)}{1-F(t)} \quad (1)$$

if $R(t)>0$

The quantity $r(t)dt+0(dt)$ has a meaningful interpretation: it's the conditional probability of failure in the interval $(t+tdt)$ given no failure had occurred before. t . If the conditional probability of failure during
the next interval of duration x of a system at age . t .
then :

$$F(x/t)=\frac{F(t+x)-F(t)}{1-F(t)} \quad (2)$$
$$=1\text{-}R(x/t)$$

Is such that $R(x/t)=R(x)$ for all $x,t\geq 0$ then $R(x)=e^{-\lambda x}$, the failure rate $r(t)$ =λ>0, and the MTBF (mean time before failure) $t_o=1/\lambda$. It's the well known "memory less property". Certain systems improve their performance in time, in the sense that $R(x/t)$ is increasing in $-\infty\prec t\leq+\infty$ for each $x\geq 0$
(3)

If the density exists, then (3) holds if and only if $r(t)$ is decreasing in $t \geq 0$. We are concerned here by systems with an increasing failure rate. The state of such systems often can be known only by using some inspection policy.

2.0 METHODOLOGY OF APPROACH

The problem has been widely studied, the illustration of the system using the form of the parallel- series (or series-parallel) structure is always possible with the Barlow-Proshan coherent structures [1]. The current tools for this type of representation require first the construction of the tree of errors, and the search for minimal ways and cuts.

In the case whereby the laws of reliability and repair can be numerically expressed, we propose a research algorithm for the optimal policy of inspection. This problem was first studies by Barlow, Hunter and Proshan. Some problems are discussed in the book of Beichelt and Franken [2]. Munford [3] proposed a new cost model which is more adapted to the situation described above.

We derive an algorithm for the computation of the optimal inspection policy relatively to the generalized Munford's cost model over an infinite time span. Such algorithm is not stable numerically. Some models are discussed in [5,6].

3.0 OPTIMAL INSPECTION PROCEDURE

An inspection policy is a random (or deterministic) sequence: $S = \{x_n \ , \ n = 0,1,.....\}$. taking values in $\Re^+$, and such that

$$0 \leq x_0 < x_1 < x_2 < ....< x_n < x_{n+1} < ......$$

We denote by Φ={S} the space of such elements S .

Let C_1 be the cost of an inspection, and assume that the system cannot fails during this inspection. In the other hand, let C_2 be the cost per unit of sejourn time of the system in the state "undetected failure". One can now express the system's life length in terms of the inspection policy.

$$w = \sum_{i=1}^{N(t)-1} \delta_i + X_{N(t)} - t \tag{4}$$

where $N(t)$ = max $\{ j : x_j < t; t \geq 0 \}$ is the number of inspections up to time t of failure, and

$\delta_i = x_j - x_{j-1}$, j=1,2,..... (Duration between two inspections)

In this paper we consider some inspection problems connected specifically with renewal after detection of failure. Let " w' " be the duration of such renewal with probability distribution. $G(t) = P(w' \leq t)$, and C_3 , the cost of such operation.

The theoretical developments presented here are based on the paper of Barlow-Hunter-Proshan . In this work the authors assume that:

H1. " w' " ≡ 1 almost surely ;

H2. The inspection policy stops at time of failure.

Arguing in a manner similar to that of Barlow-Hunter-Proshan [1], one can prove that the use of a deterministic policy is better to a random policy, relatively to the objective functions considered here.

Denote $D(t;S)$ the total cost involving by using the inspection policy $S = \{x_n\}$ given that the failure occurs at time t.

The inspection policy $S^* = \{x_n^*, n = 0,1,...\}$ is optimal (OIP) if :

$$\min_{S \in \Phi} E\{D(t;S)\} = E\{D(t;S^*)\} \tag{5}$$

4.0 INSPECTION WITHOUT RENEWAL

Under the BHP's assumptions H1, the total costs may be written under the form:

$$D_1(t;S) = C_1 N(t) + C_2 (x_{N(t)} - t) \tag{6}$$

Thus the OIP, S^* is such that (5) holds for the functional (6). If $f(x)$ exists, then using the Kuhn-Tucker recurrent equation.

$$x_{n+1} = x_n + \frac{F(x_n) - F(x_{n-1})}{f(x_n)} - \frac{C_1}{C_2} \tag{7}$$

The BHP's theorem states that if $f(x)$ is Polya-Function II (5) then $\{\delta_n < 0\}$ is a non decreasing sequence. Moreover, for each policy $S = \{\delta_n\}$, for some $n > 0$ $\delta_n > \delta_{n-1}$ if $x_1 > x_1^*$, and $\delta_n < 0$ if $x_1 < x_1^*$. This theorem gives the well known algorithm for computation of the OIP.

1. Choose the initial value x_1 ;
2. Compute x_2 , x_3,.......using (7);
3. If $\delta_n > \delta_{n-1}$ for some n, then reduce x_1; return to step 2;

4. If $\delta_n < 0$ for some n, then augment x_1; return to step 2;
5. Repeat the process until the determination of the sequence $S^* = \{x_0 < x_1 < ....\}$ with the chosen precision. $|S_i - S_{i-1}| < \varepsilon$

5.0 GENERALIZATION OF THE MODEL

In the general case, the time of failure cannot be known exactly; so that the penality C_2 is in fact extended to all the current interval between adjacent inspections (x_{n-1}, x_n). Munford [3] propose the following model of cost:

$$D_2\ (t;S) = C_1\ N(t) + C_2\ (x_{N(t)} - x_{N(t)-1}) \tag{8}$$

Note that if "w" is exponentially distributed, then the optimal policy is of the form $x_n^* = n x_1^*$, n = 1,2,.....
Substituting this expression in (5.1) we obtain that for the BHP's model, x_1^* is given by the equation $\exp(\lambda x_1^*) = \lambda x_1^* - 1 - \tau$, where $\tau = \lambda C_1 / C_2$.

6.0 OPTIMAL POLICY WITH RENEWAL

Let us now assume that:
H1. $P(w' > 0)$, so that $G(t)$ is a non degenerate distribution;
H2. The renewal cost is C_3. After renewal, the system becomes as new, and the inspection process begins at first.

The generalization of the BHP's model under these assumptions was first studied by Brender [4] with the objective function.

$$D_3\ (t;S) = C_1\ N(t) + C_2\ (x_{N(t)} - t) + C_3 \tag{9}$$

Which represents the total cost during a cycle.
The duration of such elementary cycle for a policy $S = \{x_n\}$ is:

$$T_1(t;.S) = w + x_{N(t)-1} - t + w' \tag{10}$$

The Brender's algorithm gives the optimal policy S^* minimizing the expected total cost per unit of time in an infinite span.

$$R_1(S^*) = \min_{S \in \Phi} R_1(S) \tag{11}$$

Where

$$R_1(S) = \frac{E\{D_3(t;S)\}}{E\{T_1(t;S)\}} \tag{12}$$

Now according to the Munford's considerations the renewal seem possible, we have to consider the objective function.(repair at date of inspection).

$$R_2(S) = \frac{E\{D_4(t;S)\}}{E\{T_2(t;S)\}} \tag{13}$$

Where now

$$D_4(t;S) = C_1 N(t) + C_2 (x_{N(t)} - x_{N(t)-1}) + C_3 \tag{14}$$

the cycle duration is :

$$T_2(t;S) = w + x_{N(t)} - x_{N(t)-1} + w' \tag{15}$$

The existence of the optimal policy can be proved in the usual way. In particular, it's sufficient that $F(x)$ is a continuous function and $\min\{w, w'\} \prec \infty$
Now, assume that $f(x)$ is Polya Function2, and denote.

$$H(y;S) = E\{D_4(t;S)\} - yE\{T_2(t;S)\} \tag{16}$$

Then there exists y^* such that $H(y^*;S(y^*)) = 0$, and $S(y^*)$ minimize the objective function $R_2(S)$.

The proof is similar to the Brender's proof with slight modifications. We can now derive the algorithm for the construction of the optimal inspection in view of (14) and (15), the functional (16) may be written as

$$H(y;S) = \sum_{n=1}^{\infty} \int_{x_{n-1}}^{x_n} [C_1 n + (C_2 - y)(x_n - x_{n-1})] \tag{17}$$

$$dF(t) + C_3 - y\left\{ \int_{\Re^+} t dF(t) + \int_{\Re^+} t dG(t) \right\}$$

Or equivalently,

$$H(y;S) = C_1 \sum_{n=0}^{\infty} R(x_n) + (C_2 - y) \tag{18}$$

$$\sum_{n=1}^{\infty} (x_n - x_{n-1}).(R(x_{n-1}) - R(x_n)) + C_3 - y(t_0 + t_0')$$

Where

$$t_0 = \int_0^\infty (1 - F(t))dt \quad , \qquad t'_0 = \int_0^\infty (1 - G(t))dt$$

The necessary condition of extremum gives the recurrent equation

$$X_{n+1} - 2X_n + X_n = \frac{R(X_{n+1}) - 2R(X_n) + R(X_{n-1})}{f(x_n)} \qquad (19)$$

$$-\frac{c_1}{c_2 - y}$$

7.0 PROPOSED ALGORITHM

1. Choose ' y ' as "near" the optimal value y^* as possible (for example according to physical considerations).

2. Choose the initial value x_1.

3. Compute $x_2, x_3, \ldots\ldots$ in a manner similar to that of the BHP's algorithm by using the recurrent formula (6.9).

If $H(y;S(y)) > 0$ then augment y , return to step 2

if $H(y;S(y)) < 0$ then reduce y , return to step 2

if $H(y;S(y)) = 0$ then $y^* = y$, and $S(y^*)$ is the optimal inspection policy. Else, choose another value of y and repeat the procedure from the step 2.

8.0 NUMERICAL RESULTS

A few numerical comparisons are available in this view, here we present some results obtained with algorithm mentioned before, and we used some statistical laws (Normal, Weibull, exponential) to compare the fitness of results.

For inspection procedure, the algorithm gives the sequences for optimal inspection, this policy propose a one week cycle for doing inspection, table 1 show these results after seven iterations.

Table 1 Optimal Inspection policy without renewal (Exponential law)

X_0 =0.000	X_0 =0.000	X_0 =0.000
X_1 =0.131	X_1 =1.125	X_1 =1.0935
	X_2 =2.14572	X_2 =1.5699

X_0 =0.000	X_0 =0.000	X_0 =0.000	X_0 =0.000000
X_1 =9.8415	X_1 =0.8875	X_1 =0.9349	X_1 =0.922640
X_2 =1.141	X_2 =0.9712	X_2 =0.971	X_2 =0.930716

When applying Weibull distribution the algorithm stops after reducing the second value and we obtain after 2 iterations. Results are presented in table 2 below.

Table2 Optimal Inspection policy without Renewal (Weibull law)

X_0 =0.000	X_0 =0.000
X_1 =0.500	X_1 =0.500
X_2 =0.750	

In the other case where the procedure of the inspection is applied with renewal, the sequence is obtained for different criterion y and mentioned below:

This procedure (see table 3) required a cycle of six weeks and 11 iterations, taken into account repairs cost. This is the best policy obtained for exponential law.

Table 3. Optimal inspection policy with renewal for y =1,2,….,10,…..100.

y=1 (31 iterations)	y=2 (31 iterations)	y=10 (10 iterations)	Y=50 (20 iterations)
X_0 =000000	X_0 =0000000	X_0 =000000	X_0 =0000000
X_1 =0.1333	X_1 =0.59985	X_1 =1.01308	X_1 =1.14638
X_1 =0.2666	X_1 =0.61318	X_1 =0.102641	X_1 =1.15971
……	……	……	…………
……	……	……	…………
……	…….	…….	…………
X_{31} =0.41323	X_{31} =0.41323	X_{10} =0.13305	X_{20} =1.39965

Y=70 (34 iterations)	Y=90 (32 iterations)	y=100 (11 iterations)
X_0 =000000	X_0 =000000	X_0 =0000000
X_1 =2.26610001	X_1 =1.41298	X_1 =0.9672489
X_1 =2.27943001	X_1 =1.43964	X_1 =0.9753255
…….	………	………
…….	………	………
…….	………	……….
X_{34} =3.09256004	X_{32} =2.23944001	X_{11} =5.991123

9.0 CONCLUSION

We believe that the main contribution of this work is a fresh way of thinking about the problem that, in our opinion, will bring a much deeper understanding of its combinatorial nature. This paper contributes to a better understanding of the problem in several ways following the steps of algorithms.

For the optimal policy, we have proposed classical models, we propose an extension of the

Munford model by introducing renewal, and the algorithms are encouraging.

We have to improve the numerical instability of algorithms, and to work to propose an extension to other statistical laws. The computer code is available on the platform of Badji Mokhtar University.

Finally building a proposed software package is considered like a view of future. It is capable to recording all the iteration steps.

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