DIMENSIONAL ANALYSIS RELATIONSHIPS OF STREAMBANK EROSION RATES

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Graphical abstract
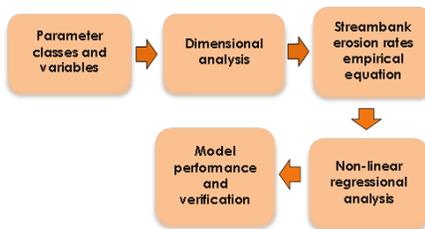
Abstract

Bank erosion is commonly associated with river meandering initiation and development, through width adjustment and planform evolution. It consists of two types of erosion processes; basal erosion due to fluvial hydraulic force and bank failure under the influence of gravity. Most of the studies only focused on one factor rather than integrating both factors. Evidences of previous works have shown integration between both processes of fluvial hydraulic force and bank failure. Bank failure seldom treated as a probabilistic phenomenon without assessing the physical characteristics and the geotechnical aspects of the bank. Thus, the objective of this paper is to investigate factors governing streambank erosion process and to perform a dimensional analysis considering the physical characteristics of both processes namely fluvial erosion and mass failure and their interaction.

Keywords: Streambank erosion, rate of streambank erosion, dimensional analysis

1.0 INTRODUCTION

Streambank erosion is one of the complex problems in river engineering studies that require integration of various field of engineering. It is a key process in fluvial dynamics affecting the physical, ecological and socio-economic. The studies of bank erosion serve as a platform to other various studies in fluvial environment. This includes evolution of meandering [7], meander or channel migration [8; 19; 18], and river bank stability [16; 5].

One of the earliest approaches addressing bank erosion of alluvial channel on rate of erosion has been conducted by [9]. An equation predicting the channel shift has been derived using a simple two-dimensional shallow water flow model. The rate of bank erosion is taken to be proportionally to the excess of near-bank depth-averaged streamwise flow velocity over the cross-sectional mean velocity. Recent studies however shown the contribution of river bank erosion is significant towards the evolution of river and floodplain morphology. It is because the

study of river bank erosion will help to quantify the rate of erosion due to fluvial entrainment or the river bank itself. It was indicated that the bank erosion consist of two processes; basal erosion due to fluvial force and bank failure under the influence of gravity. [6] conducted a study and successfully derived a method for calculating the rate of erosion that integrates both basal erosion and bank failure processes. It includes the effects of hydraulic force, bank geometry, bank materials properties and probability of bank failure, without looking into the physical characteristics of the bank properties. Knowing the effect of such factors to the contribution of the rate of bank erosion, knowledge of the rates, patterns and controls on river bank erosion events is a pre-requisite to a complete understanding of the fluvial system.

Field data and experiment data are very useful in presenting the mechanism with regards to bank erosion. They also serve basic fundamental in developing future predictions by means of simulations in any modeling. However, the basic

principle in any modeling require the identification of the relevant variables and then relating these variables via known physical laws, and one of the most powerful modeling methods is dimensional analysis.

This paper establishes a relationship based on the factors governing streambank erosion. These factors are based on the parameters influencing the rate of streambank erosion. Dimensional analysis is performed to define the independent variables to the dependent variable in order to test the uniformity of the dimension and selection of the most significant parameters with regards to the rate of streambank erosion.

2.0 EMPIRICAL EQUATIONS OF STREAMBANK EROSION

Factors that govern rate of river bank erosion can be categorized into (1) bank geometry; (2) hydraulic; (3) bank properties; and (4) grain resistance. The rate of erosion as functions of porosity, sediment density, static coefficients of Coulomb friction and dynamic coefficients of Coulomb friction [8]. A generalized bank erosion coefficient adopted was derived based on data for channel shift for several rivers in Japan that taken into accounting the types of bank texture (sandy or clayey). The derived bank erosion coefficient has been used by many and amongst the pioneering work in river bank erosion. However, the basis of its formulation in terms of its controlling variables remains uncertain.

The rate of bank erosion from the bank geometry compared to other categories (i.e., hydraulic, bank properties and grain resistance). These factors include bankfull channel width, channel radius of curvature, distance along channel, planform phase lag distance along the channel, bank height, hydraulic depth of the channel and valley slope [19]. The equation was derived based on dimensional analysis of the controlling variables and the use of empirical coefficients using validated data from Hoh River, Washington.

Similar to [1], most of the factors governing rate of bank erosion were derived from the bank geometry. The study explored more on the physical meaning of the bank erosion coefficient used in meander migration modeling. Equation by [8] was employed and the rate of bank erosion relates strongly to the vertically averaged near-bank velocity through the use of a bank erosion coefficient.

The rate of bank erosion is strongly related to the hydraulic, bank properties and grain resistance. The factors under hydraulic categories include the actual shear stress, critical shear stress, coefficient of lift force, and mean velocity [6]. Factors under bank properties include actual concentration of suspended load, equilibrium concentration of suspended load, fluid density and sediment density. Factors relating grain resistance such as roughness coefficient, mean particle size, and roughness height

were also included. The analytical method derived is classified as a complex model compared to other models. The same model was further enhanced into a numerical simulation of meandering evolution [7]. A two-dimensional depth-averaged hydrodynamic model was developed to simulate the evolution of meandering channels from the complex interaction between downstream and secondary flows, bed-load and suspended sediment transport and bank erosion. The bank erosion in this model uses the same approach derived by [6].

Simulation of river meandering processes was conducted using stochastic bank erosion coefficient [7]. This study made comparison of the first order analytical solutions for flow by [9] and [10] statistically. Therefore, the factors that govern the rate of river bank erosion falls in the same category as [1], which is bank geometry, accounting the reach-averaged depth, reach averaged width, local curvature, streamwise distance, and average water slope. The list of governing equations in the form of dimensionless independent variables pertaining river bank erosion by the previous investigators is tabulated in Figure 1.

No.	Investigator (Year)	Dependent variable	Independent variables
			Selected variables
1.	Hasegawa (1989)	$\dfrac{\xi}{U}$	$\left(\sqrt{\dfrac{\tau_{bc}}{\mu_s \mu_k \tau_{bo}}}\right), \left(\dfrac{\tau_{bo}}{\tau_{bo}-\tau_{bc}}\right), \left(\dfrac{C_f U^2}{\left(\dfrac{\rho_s}{\rho}-1\right)g\,d_{50}}\right),$ $(1-p), \left(\dfrac{\rho_s}{\rho}-1\right), (S_b), (\theta_k), \left(\dfrac{U^2}{g\,d_{50}}\right)$
2.	Randle (2004; 2006)		$C_s\left(\dfrac{B}{R_c}\right), r_y\left(\dfrac{r_d}{h_b}\right), LWD\left(\dfrac{d_w}{D}\right), (PI), d_c\left(\dfrac{h_b}{D}\right)$
3.	Duan (2006)		$\left(1-\dfrac{\tau_{bc}}{\tau_{bo}}\right), \left(1-\dfrac{C}{C_*}\cos\beta\right), \left(\dfrac{d_{50}}{k_s}\right), \left(\dfrac{D}{d_{50}}\right), \left(\dfrac{1}{\kappa^2}\right)$
4.	Constantine et. al (2006)		$\left(\dfrac{\tau_o}{\rho U^2}\right)$
5.	Duan and Julien (2010)		$\left(1-\dfrac{\tau_{bc}}{\tau_{bo}}\right), \left(1-\dfrac{C}{C_*}\cos\beta\right), \left(\dfrac{D}{d_{50}}\right), \left(\dfrac{d_{50}}{k_s}\right),$ $\left(\dfrac{T^{1.5}}{d_*^{0.3}}\right), \left(\dfrac{\tau_*-\tau_{*c}}{\tau_{*c}}\right), \left(\dfrac{d_{50}}{a}\right),$ $\left(d_{50}\left[\dfrac{(s-1)g}{v^2}\right]\right), \left(\dfrac{\omega}{\kappa\,\beta\,U_*}\right), \left(\dfrac{1}{\kappa^2}\right), \left(\dfrac{\rho_s}{\rho_w}\right)$
6.	Duan and Posner (2012)		$\left(\dfrac{\tau_o}{\rho U^2}\right)$

Figure 1 Dimensionless independent variable by previous investigators

3.0 DIMENSIONAL ANALYSIS

Dimensional analysis is one of the methods determining the relationship between variables. It is used to develop a dimensionally correct equation among certain variables. The objective of dimensional analysis include: i) to reduce the number of variables for subsequent analysis, and ii) to provide dimensionless parameters that numerical value are independent of any system of unit.

Dimensional analysis provides a similarity law for the phenomenon under consideration. Similarity means certain equivalence between two (2) physical characteristics that are actually different. Dimensional analysis also can be related to a prototype, where sets on independent parameters are chosen to build up the complete characteristics of an actual event. It will reduce the quantity of variables and produce dimensionless parameters. However, experiments or tests are needed to be carried out in order to verify these parameters. Dimensional analysis alone does not give the exact form of an equation, but it can lead to a significant reduction of the number of variables. It is based on two (2) assumptions: i) physical quantities have dimensions (fundamental are mass, M, length, L and time, T). Any physical quantity has a dimension which is a product of powers of the basic dimensions, M, L and T; and ii) physical laws are unaltered when changing the units measuring the dimensions. Units must be taken into consideration when collecting the data.

3.1 Buckingham's Pi Theorem

The Buckingham's Pi theorem is a procedure for determining dimensionless groups from the selected variables. [18] has stated, "If the equation F (q_1, q_2, q_3,...) = 0 is complete, the solution has the form of f(π_1, π_2, π_3, … π_{n-k}) = 0, where the π terms are independent products of the parameters q_1, q_2, q_3, etc., and are dimensionless in the fundamental dimensions." In other word, a complete dimensional homogenous equation, relating n physical quantities which are expressible in terms of k fundamental quantities that be reduced to a functional relationship between n-k dimensionless products. For example, if there are nine (9) physical quantities involved in the relationship of the physical problem with three (3) fundamental physical quantities, six (6) set of dimensionless groups will be formed.

3.2 Dimensional Variables

The first step is identifying selected dimensional variables influencing the streambank erosion rates based on the previous investigators. Based on the physical considerations on river bank erosion processes, the primary factors governing the rate of river bank erosion (ξ) along a channel can be divided into five (5) major categories; bank geometry, hydraulic, soil capacity (resistance to erosion), grain resistance and others.

The parameters for bank geometry consist of channel width, B, water depth, D, bank height, h_b, bank angle, β, and channel slope, S_o. The parameters for hydraulic consist of streambank erosion rate, ξ, streambank depth-averaged velocity, u_b, and boundary shear stress, τ_o. The parameters for soil capacity (resistance to erosion) include critical shear stress, τ_c, porosity, p, and plasticity index, PI. The grain resistance includes mean particle diameter, d_{50}, fall velocity, ϖ, shear velocity, U_*, and concentration of suspended load to equilibrium suspended concentration, C. Other variables include gravity acceleration, g, water density, ρ_w, streambank particle density, ρ_s. The rate of bank erosion (ξ) serves as the dependent variable. Figure 2 describes physical parameters involved. There are eighteen (18) variables selected and three fundamental quantities involved in the relationship.

No.	Variable Category		Variables	Symbol	Units	Fundamental Quantities
i.	Bank geometry	1.	Channel width	B	m	L
		2.	Water depth	D	m	L
		3.	Streambank height	h_b	m	L
		4.	Streambank angle	β	-	-
		5.	Channel slope	S_o	-	-
ii.	Hydraulic	6.	Streambank erosion rate	ξ	m/s	LT^{-1}
		7.	Streambank depth-averaged velocity	u_b	m/s	LT^{-1}
		8.	Boundary shear stress	τ_o	N/ms²	$ML^{-1}T^{-2}$
iii.	Soil capacity (resistance to erosion)	9.	Critical shear stress	τ_c	N/ms²	$ML^{-1}T^{-2}$
		10.	Porosity	p	-	-
		11.	Plasticity index	PI	-	-
iv.	Grain resistance	12.	Mean particle diameter	d_{50}	m	L
		13.	Fall velocity	ω	m/s	LT^{-1}
		14.	Shear velocity	U_*	m/s	LT^{-1}
		15.	Ratio of concentration of suspended load to equilibrium concentration	C	-	-
v.	Others	16.	Gravity acceleration	g	m/s²	LT^{-2}
		17.	Water density	ρ_w	kg/m³	ML^{-3}
		18.	Streambank particle density	ρ_s	kg/m³	ML^{-3}

Figure 2 Selected variables and dimensions influencing streambank erosion rates

3.3 Modelling Procedure

The independent variables consist of the following five (5) categories as mentioned above. The relationship between the independent variables to the dependent variable is formulated as:

$$\xi = f\left(B, D, h_b, \beta, S_o, u_b, \tau_o, \tau_c, p, PI, d_{50}, \omega, U_*, C, g, \rho_w, \rho_s\right) \quad (1)$$

As the number of variables n is eighteen (18) and the number of dimensions m is three (3), it follows that the number of dimensionless groups will be (18 – 3) or 15. Thus there will be three (3) repeating variables chosen to non-dimensionalized the groups that feature as variables of prime interest. The three selected variables are streambank average velocity,

u_b, fluid density, ρ_w, and mean particle diameter, d_{50}. Based on the fieldwork perspective, it is reasonable to choose these repeating variables as they can be directly measured in situ or through laboratory experiments or calculated leaving others as independent variables. The next step is to define the dependent variables Π_1 as $\Pi_1 = \xi\, u_b{}^{a1}\, \rho_w{}^{b1}\, d_{50}{}^{c1}$ and by equating powers of M, L, and T, results in a set of three (3) simultaneous equation that may be solved for the indices of a, b, and c.

Dimensions of Π_1 :

$$
\begin{aligned}
[\Pi_1] &= \xi\, u_b{}^{a1}\, \rho_w{}^{b1}\, d_{50}{}^{c1} \\
&= [\, M^0\, L^0\, T^0\,] \\
&= [\, L^1\, T^{-1}\, (L^1\, T^{-1})^{a1}\, (M^1\, L^{-3})^{b1}\, (L^1)^{c1}\,]
\end{aligned}
$$

By equating the exponents:

$$
\begin{aligned}
a^1 &= -1 \\
b^1 &= 0 \\
c^1 &= 0
\end{aligned}
$$

The first group becomes:

$$\therefore\quad \Pi_1 = \xi\, u_b{}^{-1}\, \rho_w{}^{0}\, d_{50}{}^{0} \tag{2}$$

and can also be rearranged as:

$$\therefore\quad \Pi_1 = \frac{\xi}{u_b} \tag{3}$$

which has the form of streambank erosion rate incorporating the streambank depth-averaged velocity term. The same procedure is repeated for establishing groups of independent variables, Πs (with manipulation to put the Π's into proper established form). The parameters provided a functional relationship in terms of a dimensionless streambank erosion rate using Buckingham Pi theorem that yields the following functional relationship:

$$\frac{\xi}{u_b} = f\left(\frac{B}{d_{50}}, \frac{D}{d_{50}}, \frac{h_b}{d_{50}}, \beta, S_o, \frac{\tau_o}{\rho_w u_b{}^2}, \frac{\tau_c}{\rho_w u_b{}^2}, p, PI, \frac{\omega}{u_b}, \frac{U_*}{u_b}, C, \frac{\rho_s}{\rho_w}, \frac{gd_{50}}{u_b{}^2}\right) \tag{4}$$

However, the dimensionless critical shear stress $\tau_c/\rho_w\, u_b{}^2$ is rewritten in terms of the critical Froude number, Fr_c at which bank erosion is initiated (Park, 2007). To predict the rate of streambank erosion along a channel, it is necessary to collect data from fieldwork investigation with regards to the controlling parameters in streambank erosion. Therefore, the final results of dimensional analysis, after manipulation and simplification, the equation is simplified to:

$$\frac{\xi}{u_b} = f\left(\frac{B}{d_{50}}, \frac{D}{d_{50}}, \frac{h_b}{d_{50}}, \beta, S_o, \frac{\tau_o}{\rho_w u_b{}^2}, Fr_c, p, PI, \frac{\omega}{u_b}, \frac{U_*}{u_b}, C, \frac{\rho_s}{\rho_w}, \frac{gd_{50}}{u_b{}^2}\right) \tag{5}$$

3.4 Streambank Erosion Rate and Parameter Classes

Based on the established functional relationship with regards to the streambank erosion, the dimensionless parameters are clustered into their dimensionless group following the five (5) categories mentioned previously; hydraulic (resistance to flow), soil capacity (resistance to erosion), bank geometry, grain resistance and sediment. Figure 3 shows the dimensionless group of controlling parameters in streambank erosion based on their parameter class. The importance of these parameters and their functions describing the rate of streambank erosion rationalizes the use of these parameters in the development of the new equation.

4.0 DEVELOPMENT OF STREAMBANK EROSION RATES EMPIRICAL MODEL

The development of the proposed empirical model is made by conducting a statistical nonlinear regression analysis on a total of 318 number of observed streambank erosion monitoring data. The primary objective is to identify a relationship for prediction of streambank erosion rates. This will lead to the acquisition of the most accurate model that yields a reliable and most accurate streambank erosion rates predictor. Equations regressed will be validated using data from the present study.

No.	Parameter Class	Dimensionless Group
1.	Hydraulic (flow resistance)	$\left(\dfrac{\tau_o}{\rho_w u_b{}^2}\right), \left(\dfrac{g\, d_{50}}{u_b{}^2}\right), \left(\dfrac{U_*}{u_b}\right), \left(\dfrac{\omega}{u_b}\right), \left(\dfrac{\tau_c}{\rho_w u_b{}^2}\right)$
2.	Resistance capacity of soil	$\left(\dfrac{\rho_s}{\rho_w}\right), (p), (PI)$
3.	Bank Geometry	$(S_o), (\beta)$
4.	Grain Resistance	$\left(\dfrac{B}{d_{50}}\right), \left(\dfrac{D}{d_{50}}\right), \left(\dfrac{h_b}{d_{50}}\right)$
5.	Sediment	(C)

Figure 3 Controlling parameters in streambank erosion

3.5 Selection of Reach

Two locations has been selected for the streambank erosion monitoring sites. The first location is along 50 m reach of Sg. Lui at Kampung Sg. Lui, Hulu Langat and the second location is at Sg. Bernam, Tanjung Malim, Hulu Selangor. From the streambank erosion monitoring, a total of 390 data has been collected for both rivers. However, only 318 data was used for development of streambank erosion rates empirical model. The balance of 72 data was used for model verification. Figure 4 and 5 show the distribution of streambank monitoring data and distribution of data for model development and model verification.

Location	Monitoring period	No. of data
1. Sg. Lui, Kampung Sg. Lui, Hulu Langat	July 2014 – August 2014	78
2. Sg. Bernam, Tanjung Malim, Hulu Selangor	October 2014 – September 2015	312
Total number of data up to September 2015		**390**

Figure 4 Total number of streambank erosion monitoring data

Location	No of data for model development	No of data for model verification
1. Sg. Lui, Kampung Sg. Lui, Hulu Langat	260	52
2. Sg. Bernam, Tanjung Malim, Hulu Selangor	58	20
Total data	**318**	**72**

Figure 5 Distribution of data for empirical model development and model verification

4.0 NON-LINEAR MULTIPLE REGRESSION

Multiple regression analysis provides a useful tool for identification of significant relationships. The model predictors described above are used in a non-linear multiple regression analysis. From all 318 data for model the development, total of 20 data were removed due to outliers. The remaining 298 data were used in the model development. First, each independent variable individually is regressed against the dependent variable. Multiple non-linear regression analysis is then applied to different combinations of the independent variables. A total of 10 regression equations have been derived and these are shown in Figure 6 with their respective coefficient of determination and coefficient of correlation, R^2 values. These equations are evaluated on their performances graphically.

No.	Model	R^2	Equation
1.	$\frac{\xi}{u_b} = 3.464 \times 10^{-6} \left(\frac{\tau_o}{\rho_w u_b^2}\right)^{0.794} \left(\frac{\tau_c}{\rho_w u_b^2}\right)^{-0.171} \left(\frac{B}{d_{50}}\right)^{-0.604} \left(\frac{D}{d_{50}}\right)^{0.401} \left(\frac{h_b}{d_{50}}\right)^{-0.057} (\beta)^{-0.472}(S_o)^{-0.775}(C)^{0.070}$	0.479	6.1
2.	$\frac{\xi}{u_b} = 4.023 \times 10^{-6} \left(\frac{\tau_o}{\rho_w u_b^2}\right)^{0.653} \left(\frac{B}{d_{50}}\right)^{-0.524} \left(\frac{D}{d_{50}}\right)^{0.431} (\beta)^{-0.539}(S_o)^{-0.679}(C)^{0.074}$	0.473	6.2
3.	$\frac{\xi}{u_b} = 4.460 \times 10^{-7} \left(\frac{\tau_o}{\rho_w u_b^2}\right)^{0.643} \left(\frac{D}{d_{50}}\right)^{-0.524} \left(\frac{h_b}{d_{50}}\right)^{-0.392} (\beta)^{-0.399}(S_o)^{-0.668}(C)^{0.073}$	0.460	6.3
4.	$\frac{\xi}{u_b} = 2.700 \times 10^{-5} \left(\frac{\tau_o}{\rho_w u_b^2}\right)^{0.469} \left(\frac{\tau_c}{\rho_w u_b^2}\right)^{0.019} \left(\frac{B}{d_{50}}\right)^{-0.541} \left(\frac{h_b}{d_{50}}\right)^{0.469} (\beta)^{-0.630}$	0.365	6.4
5.	$\frac{\xi}{u_b} = 2.421 \times 10^{-6} \left(\frac{\tau_o}{\rho_w u_b^2}\right)^{0.414} \left(\frac{\tau_c}{\rho_w u_b^2}\right)^{0.126} \left(\frac{D}{d_{50}}\right)^{0.478} \left(\frac{h_b}{d_{50}}\right)^{-0.262} (\beta)^{-0.458}$	0.388	6.5
6.	$\frac{\xi}{u_b} = 6.304 \times 10^{-8} \left(\frac{U_*}{u_b}\right)^{1.198} (\beta)^{-0.413}(S_o)^{-0.719}$	0.422	6.6
7.	$\frac{\xi}{u_b} = 6.142 \times 10^{-8} \left(\frac{U_*}{u_b}\right)^{1.198} \left(\frac{h_b}{d_{50}}\right)^{0.005} (\beta)^{-0.416}(S_o)^{-0.719}$	0.422	6.7
8.	$\frac{\xi}{u_b} = 5.019 \times 10^{-7} \left(\frac{U_*}{u_b}\right)^{1.206} \left(\frac{B}{d_{50}}\right)^{-0.481} \left(\frac{h_b}{d_{50}}\right)^{0.383} (\beta)^{-0.559}(S_o)^{-0.721}$	0.433	6.8
9.	$\frac{\xi}{u_b} = 2.097 \times 10^{-7} \left(\frac{U_*}{u_b}\right)^{1.232} (\beta)^{-0.467}(S_o)^{-0.728}(C)^{0.070}$	0.432	6.9
10.	$\frac{\xi}{u_b} = 5.019 \times 10^{-7} \left(\frac{U_*}{u_b}\right)^{1.206} \left(\frac{B}{d_{50}}\right)^{-0.481} \left(\frac{h_b}{d_{50}}\right)^{0.383} (\beta)^{-0.559}(S_o)^{-0.721}$	0.433	6.10

Figure 6 Results from the non-linear multiple regression analysis for Sg. Lui and Sg. Bernam

4.1 Model Performance and Verification

Non-linear multiple regression analysis was conducted in order to obtain a significant relationship between combinations of independent variables to the dependent variable. A total number of 72 data were used for model verification. This data were obtained from the streambank erosion monitoring at Sg. Lui and Sg. Bernam. There should be a good agreement between the predicted and measured values. This can be obtained from the distribution pattern of the data in which the scatter should be homoscedastic and possess a high positive correlation within the acceptable limit. In addition to the above, the accuracy of the equations are measured based on the discrepancy ratio distribution of 0.5 - 2.0 limit.

Discrepancy ratio is the ratio of the predicted values to the measured values and these values are deemed accurate if the data lie within this limit. Based on these criteria, Equation 6.6 and 6.7 performs better than other equations having a correlation coefficient, R^2 of 0.422.

Figure 7 shows the percentage of discrepancy ratio for all respective models. The prediction accuracies for both Equation 6.6 and 6.7 are 60% respectively. Figure 8 and 9 shows the graphical comparison for Equation 6.6 and 6.7. The plot of both equations demonstrates a good correlation between the predicted data and observed data. Almost all of the data fall within the acceptable range of discrepancy ratio (0.5 – 2.0).

No.	Model	Discrepancy ratio (0.5 – 2.0) %	Equation
1.	$\dfrac{\xi}{u_b}=3.464\times10^{-6}\left(\dfrac{\tau_o}{\rho_w u_b^2}\right)^{0.794}\left(\dfrac{\tau_c}{\rho_w u_b^2}\right)^{-0.171}\left(\dfrac{B}{d_{50}}\right)^{-0.604}\left(\dfrac{D}{d_{50}}\right)^{0.401}\left(\dfrac{h_b}{d_{50}}\right)^{-0.057}$ $\left(\beta\right)^{-0.472}\left(S_o\right)^{0.775}(C)^{0.070}$	50.0	6.1
2.	$\dfrac{\xi}{u_b}=4.023\times10^{-6}\left(\dfrac{\tau_o}{\rho_w u_b^2}\right)^{0.653}\left(\dfrac{B}{d_{50}}\right)^{-0.524}\left(\dfrac{D}{d_{50}}\right)^{0.431}\left(\beta\right)^{-0.539}\left(S_o\right)^{-0.679}(C)^{0.074}$	48.6	6.2
3.	$\dfrac{\xi}{u_b}=4.460\times10^{-7}\left(\dfrac{\tau_o}{\rho_w u_b^2}\right)^{0.643}\left(\dfrac{D}{d_{50}}\right)^{-0.524}\left(\dfrac{h_b}{d_{50}}\right)^{-0.392}\left(\beta\right)^{-0.399}\left(S_o\right)^{-0.668}(C)^{0.073}$	52.8	6.3
4.	$\dfrac{\xi}{u_b}=2.700\times10^{-5}\left(\dfrac{\tau_o}{\rho_w u_b^2}\right)^{0.469}\left(\dfrac{\tau_c}{\rho_w u_b^2}\right)^{0.019}\left(\dfrac{B}{d_{50}}\right)^{-0.541}\left(\dfrac{h_b}{d_{50}}\right)^{0.469}\left(\beta\right)^{-0.630}$	58.3	6.4
5.	$\dfrac{\xi}{u_b}=2.421\times10^{-6}\left(\dfrac{\tau_o}{\rho_w u_b^2}\right)^{0.414}\left(\dfrac{\tau_c}{\rho_w u_b^2}\right)^{0.126}\left(\dfrac{D}{d_{50}}\right)^{0.478}\left(\dfrac{h_b}{d_{50}}\right)^{-0.262}\left(\beta\right)^{-0.458}$	56.9	6.5
6.	$\dfrac{\xi}{u_b}=6.304\times10^{-8}\left(\dfrac{U_*}{u_b}\right)^{1.198}\left(\beta\right)^{-0.413}\left(S_o\right)^{-0.719}$	60.0	6.6
7.	$\dfrac{\xi}{u_b}=6.142\times10^{-8}\left(\dfrac{U_*}{u_b}\right)^{1.198}\left(\dfrac{h_b}{d_{50}}\right)^{0.005}\left(\beta\right)^{-0.416}\left(S_o\right)^{-0.719}$	60.0	6.7
8.	$\dfrac{\xi}{u_b}=5.019\times10^{-7}\left(\dfrac{U_*}{u_b}\right)^{1.206}\left(\dfrac{B}{d_{50}}\right)^{-0.481}\left(\dfrac{h_b}{d_{50}}\right)^{0.383}\left(\beta\right)^{-0.559}\left(S_o\right)^{-0.721}$	58.3	6.8
9.	$\dfrac{\xi}{u_b}=2.097\times10^{-7}\left(\dfrac{U_*}{u_b}\right)^{1.232}\left(\beta\right)^{-0.467}\left(S_o\right)^{-0.728}(C)^{0.070}$	58.3	6.9
10.	$\dfrac{\xi}{u_b}=5.019\times10^{-7}\left(\dfrac{U_*}{u_b}\right)^{1.206}\left(\dfrac{B}{d_{50}}\right)^{-0.481}\left(\dfrac{h_b}{d_{50}}\right)^{0.383}\left(\beta\right)^{-0.559}\left(S_o\right)^{-0.721}$	55.6	6.10

Table 7 Performances of all 10 regression equations on 72 data for Sg. Lui and Sg. Bernam

Figure 8 Graphical comparison of Equation 6.6.
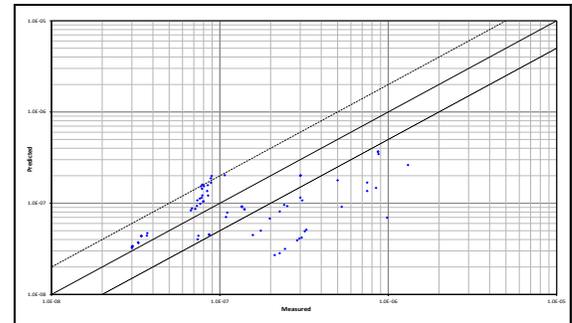
Figure 9 Graphical comparison of Equation 6.7

5.0 CONCLUSION

This paper established factors governing the rate of river bank erosion. The functional relationship between the independent variables to the dependent variable has been established using dimensional analysis. Non-linear multiple regression analysis was performed using 298 data of streambank erosion monitoring for Sg. Lui and Sg. Bernam. From the analysis, a total of 10 regression equations were derived using non-linear multiple regression technique. All 10 regression equations were tested for their individual performances based on graphical plot distributions and discrepancy ratio using 72 data. It can be concluded that Equation 6.6 and 6.7 show the highest prediction accuracies of 60% respectively. The final parameters for prediction of streambank erosion rates using regression for Equation 6.6 and 6.7 are as follows:

$$\frac{\xi}{u_b}=6.304\times10^{-8}\left(\frac{U_*}{u_b}\right)^{1.198}\left(\beta\right)^{-0.413}\left(S_o\right)^{-0.719} \qquad (7)$$

$$\frac{\xi}{u_b}=6.142\times10^{-8}\left(\frac{U_*}{u_b}\right)^{1.198}\left(\frac{h_b}{d_{50}}\right)^{0.005}\left(\beta\right)^{-0.416}\left(S_o\right)^{-0.719} \qquad (8)$$
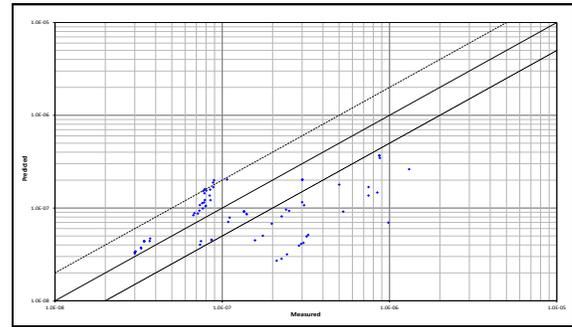
Figure 9 shows the graphical comparison of Equation 6.7. This prediction of streambank erosion is important and can be greatly associated with the rate of meander migration and the evolution of the river meandering processes and thus a very important aspect in river engineering study.

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