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GAUGING PERFORMANCE OF A QUEUING SYSTEM USING MULTI-SERVER WAITING LINE MODEL (M/M/S): AN INVESTIGATION ON A LOCAL HOSPITAL'S OUTPATIENT DEPARTMENT

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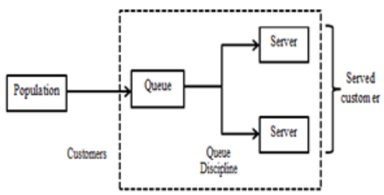

Abstract

Healthcare is a fast growing industry demanding perfection, top notch services and optimal utilization of available resources. Despite shortages in doctors and nurses as well as frequent overcrowding, the number of incoming patients to the government hospitals is increasing every year. The outpatient department in Malaysian Government hospitals is consistently experiencing frequent congestion particularly in the morning. The challenge for hospital administrators is to rightly and timely allocate resources in relevant departments. Henceforth, this study aims to further gauge the performance of an outpatient department in a local hospital in Kedah. In particular, it aims to investigate the current efficiency level of a queuing system of the said department and to propose potential solutions. The study uses Multi-server Queuing Models (M/M/s) to measure the performance of the waiting line. A computational work is then presented to value the operating characteristics of the model. The findings exhibit the performance of existing queuing system and the need to re-examine the number of doctors on duty. In all, the study may be beneficial as a decision making aid to the administrator of the outpatient department particularly to re-look and improve existing queuing system in tandem with the center's commitment for better customer satisfaction and minimal patients' turnover.

Keywords: Healthcare, multiple-server waiting line model, queuing system

1.0 INTRODUCTION

Over the years, hospitals face arduous challenges to provide efficient and satisfactory services with limited resources [1]. The situation occurred mainly in the government hospitals in which the number of patients increases every year whilst number of key personnel such as doctors and nurses are considerably limited. The worrying development applies to many different departments including the outpatient for which queues for treatment can be up for hours. Producing faster services with very minimal waiting time for the outpatient department has also been the goal of many service providers in recent years [2], [3]. The adverse complications between high quality services, limited resources and the increment in the number of patients have been clearly evident in the outpatient department [4]. Predictably, numerous studies had been conducted to examine efficiency of waiting lines in services industry [5], [6], [7], [8] and [9]. In particular, the importance of the outpatient department as a 'front-line battlefield' has also been highlighted in many previous studies [10], [11], [12], [13], [14], [15] and [16]. A research by [17] proposed two main principles for scheduling patients in outpatient departments. First, the scheduled time slot between two patients depends on the average consultation time of each physician. The best ratio of average consultation time to scheduled time slot between two patients is from 0.85 to 0.95 seconds, it is better for the time point to be in multiples of five minutes. Because many patients are unsure about the time of their appointment, they tend to arrive earlier than they should; hence, their waiting times increase even more. A study by [18] used a computer simulation model of an outpatient clinic built within the specified time frame. Patient waiting time results from the interaction of multiple variables present in an outpatient clinic (e.g., appointment schedules, clinic staffing, number of Physicians, room, etc). Determining how changing the variables could affect patient waiting time requires "what if" analysis which simulation effectively performs. The results of the "what-if: analysis provided an abundance of useful information that would have been virtually impossible to obtain through any other analytical method using the same time frame, resources and effort. The simulation gave the outpatient clinic management exactly what they were looking for: the variables influencing patients waiting time. This information was then used to formulate improvement strategies to keep patient waiting time at the desired minimum level in each of the clinics. In Malaysia, perceived quality services in government hospitals remains inconclusive. Several studies highlighted issues related to the outpatient department in Malaysian Government hospitals [19] and [20]. In general, the outpatient department in Malaysian Government hospitals is consistently experiencing frequent overcrowding particularly in the morning. A research by [19] found that on average the waiting time for patient to secure prescriptions slip from the outpatient department is more than two hours. Contact with the doctors (treatment) is only 15 minutes. Unsurprisingly, satisfaction level was low and complaints are mounting. Despite shortages in doctors and nurses as well as frequent overcrowding, the number of incoming patients to the government hospitals is increasing every year. In 2012, a total of 2.6 million admissions were recorded in the government hospitals and about 19 million were seen as outpatients, as compared to about 2.1 million admissions recorded in 2010 and about 17.6 million were seen as outpatients [21]. Apparently, the challenge for hospital administrators is to rightly and timely allocate resources in relevant departments. For outpatient department, patients are normally expecting speedier departure from the hospital (upon completion of treatment). Unfortunately, that has not been the case due to long waiting time and limited number of available doctors. Henceforth, this study aims to further gauge the performance of an outpatient department in a local hospital in Kedah. In particular, it aims to investigate the current efficiency level of a queuing system of the said department and to propose potential solutions. The remaining parts of this paper are organized as follows. *Section 2* revisits the waiting line model that will be used to analyse the queuing performance. *Section* 3 focuses on the problem definition. *Section 4* exhibits the computation works and findings from the analysis. Finally, *Section 5* discusses the implications of the research and concludes the paper.

2.0 THE WAITING LINE MODEL

The waiting line model is originated from the work of Erlang in 1917 when he published a paper on telephone traffic theory [22]. His work has inspired many subsequent studies on the queuing theory and relevant application problems in many different kind of industry such as healthcare and engineering-related fields [23]. There are two basic main approaches to the queuing problems; (1) analytical methods using either single-server waiting line model (M/M/1) or the multi-server waiting line model (M/M/s) and (2) the simulation method [24] and [25]. For the purpose of the study, the model adopted is the multi-server queuing model with Poisson arrivals and exponential service times (M/M/s). The model assumes that arrival rate is following a Poisson probability distribution from an infinite calling population. Service rate is exponential and the queue follows a single phase, first-in-first-out (FIFO) discipline as depicted in Figure 1.

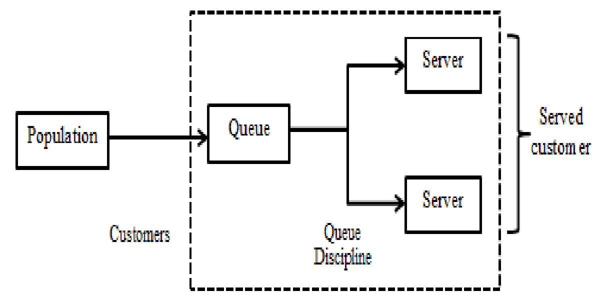

Figure 1 The elements of a multi-server waiting line (M/M/s)
Source: [22]

Figure 1 above shows the elements of a waiting line system. Basically elements of waiting line are the calling population, arrivals, servers and the waiting line [22]:

Calling population – The source of customers to the waiting line system, and it can be either infinite or finite.

Arrival – The rate at which customers arrive at the service facility during a specified period. The variability of arrivals at a service facility often conforms to a probability distribution.

Servers – A queuing process may also be any one a large number of different probability distributions.

Queue Discipline and Length – The order in which waiting customers are served. There are several possibilities in terms of the sequence of customers to be served such as FIFO (first in First Out), LIFO (Last in, First Out), and random order.

The multi-server queuing system refers to two or more servers or channels that are available to handle arriving customers. The system includes a single waiting line and a service facility with several independent servers in parallel. The formulas for determining the operating characteristics (performance measures) for the multiple server models are based on the same assumptions as the single server model with Poisson arrival rate, exponential service times, infinite calling population and queue length, and FIFO queue discipline [22]. Tabvle 1 shows the queuing performance measures - M/M/s). The basic formula for each operating characteristics of this multi-server model (M/M/s) is as follows:

λ : The mean customers arrival rate
μ : The mean service rate
s : Number of servers

Table 1 Queuing performance measures - M/M/s) *Source*: [22]

No	Operating characteristics	Formula
1	The probability that there are no customers in the system	$P_0 = \dfrac{1}{\left[\sum_{n=0}^{n=s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu-\lambda}\right)}$
2	The probability of n customers in the queuing system	$P_n = \begin{cases} \frac{1}{s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, & \text{for } n > s \\ \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, & \text{for } n \le s \end{cases}$
3	The probability that a customer arriving in the system must wait for service	$P_W = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu-\lambda} P_0$
4	Average number in system	$L = \frac{\lambda\mu(\lambda/\mu)^s}{(s-1)!(s\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu}$
5	Average time is system	$W = \frac{L}{\lambda}$
6	Average number in queue	$L_q = L - \frac{\lambda}{\mu}$
7	Average time in queue	$W_q = W - \frac{1}{\mu}$

3.0 PROBLEM DEFINITION

The outpatient department of a local hospital in Kedah has been selected as the focal point of the study (later known as the medical center). The department is divided into three sections namely registration counter, treatment and pharmacy. The opening hour for the department is from 7.00am until 5.00pm every day except Friday. However, treatment hours are only started at 8.00 am, and will stop for an hour break between 1.00pm to 2.00pm. The outpatient department is also open for half a day (until 1pm) on Thursday. As it is one of the biggest medical centers in the district, the number of incoming patients is considerably huge. According to the administrator, on average 140 to 200 patients were visiting the center on a daily basis all year round. However, the number of doctors on duty is limited to only two to three. On Sunday, number of doctors on duty is three from 9am until 12.30pm. For the rest of the week, two doctors are on duty for similar time period. The following Table 2 shows detail of the schedule:

Table 2 Number of doctors and the duty schedule

Time	Number of doctor					
	Sun	Mon	Tues	Wed	Thu	Sat
0800-0830	0	2	2	3	3	2
0830-0900	1	2	2	3	3	2
0900-0930	3	2	2	3	2	2
0930-1000	3	2	1	3	2	2
1000-1030	3	2	2	2	2	2
1030-1100	3	2	1	2	2	2
1100-1130	3	1	1	2	1	2
1130-1200	3	1	2	3	2	2
1200-1230	3	2	2	2	2	1
1230-1300	1	1	1	1	2	2
1300-1330	0	0	0	0		0
1330-1400	0	0	0	0		0
1400-1430	1	2	2	3		1
1430-1500	3	2	2	3		2
1500-1530	1	1	2	2		2
1530-1600	2	1	1	2		2
1600-1630	1	1	1	0		2

The average process time in registration and pharmacy were identified as 1.5 and 2.0 minutes respectively. The process times were determined by interviewed the staffs that responsible for the departments. The medical treatment is more difficult. Processing time in this department is not constant, and it's varying according to the treatment that needed by the patients. To overcome the problem, treatment processing time were discussed with the doctors and the administrators. According to the doctors, process' time to treat one patient is usually between 4 to 5 minutes. However most of the time, it is almost 5 minutes. By using this information, and by comparing each result, it is assumed that the most likely processing time is 5 minutes. The comparing results were based on patients' average waiting time. It's an average waiting time that every patient that arrives at certain time must experience before they get medical treatment. The process flow of in-and-out patients is illustrated in the following Figure 2:

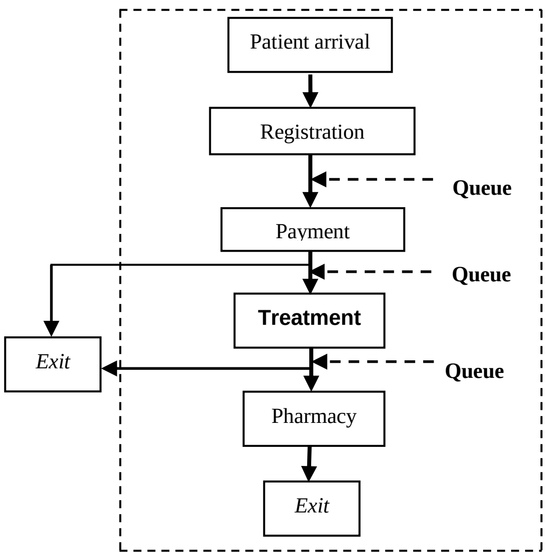

Figure 2 Patient flow in the Outpatient Department

A total of 1089 patients were visiting the center during the study period. Nonetheless, only 942 patients managed to secure treatment from doctors. The number showed that 15.6% or 147 patients were leaving the center early before seeking the treatment. It is believed that one of the main reasons for this is the long waiting time. Table 3 depicted arrival of patients to the medical center in six days period. The table also shows number of patients leaving the center without obtaining doctor's treatment.

Table 3 Patients' total arrival and turnover rates

Day	Total Arrival of patients	No of departure (completed the treatment process)	No of patients leaving without treatment	% of turnover
Sunday	242	203	39	19.21
Monday	181	152	29	19.08
Tuesday	169	147	22	14.97
Wednesday	198	183	15	8.20
Thursday	116	98	18	18.37
Saturday	183	159	24	15.09
Total	1089	942	147	

It is also observed that around 70-77% patients arrived at the center in the morning before 12pm every day (except on Thursday). Table 4 depicted patients' arrival on a daily basis during the one week study period. According to the center's administrator, the arrival rates and times follow similar pattern all year round. However, limited number of available doctors and the long waiting time indicates the need to re-examine the allocation of doctors on duty in order to improve current performance of the queuing system. Assignment of doctors based on peak and non-peak hours is also crucial due to patients' imbalance arrival pattern and limited resources.

Table 4 Patients' arrival to the Outpatient Department

Time	*Sun*	*Mon*	*Tue*	*Wed*	*Thu*	*Sat*
700	55	50	35	49	38	30
800	44	41	37	44	20	33
900	17	19	21	18	15	22
1000	18	1	12	12	12	15
1100	13	6	8	12	9	13
1200	4	0	3	4	4	10
1300	11	8	5	10		12
1400	31	18	17	21		20
1500	9	8	9	13		4
1600	1	1	0	0		0

**On Thursday, the center's opening hours is only until 1pm*

4.0 COMPUTATION WORKS AND THE FINDINGS

Collection of data has been carried out for six days (1 week) in which arrival and departure of patients were recorded based on an hourly basis. The data that were collected are times and number of patients in-and-out of the outpatient department. For the purpose of the study, average values for both arrival and service rates are then calculated. Table 5 illustrated hourly average values of patients' arrival rate. The values are then rounded up to the nearest absolute values:

Table 5 Average patients' arrival on hourly basis

	Average hourly arrivals	
Days	**7am-12pm**	**12pm-4pm**
Sunday	29	11.2
Monday	23	7.0
Tuesday	23	6.8
Wed	27.0	9.6
Thursday	20	
Saturday	23	9.2

The above table showed average arrival of patients per hour. For instance, on Sunday there were 147 patients arrived at the department before 12pm. This indicated an average hourly arrival of 29.4 (147/5 hours). The purpose of having two separate average values is to highlight the significant differences between the morning and afternoon arrivals. The imbalance arrival patterns highlighted the need to emphasize on the morning session. Hence, the following value of mean customer arrival rate will only represents morning data (before 12pm). The mean arrival values (λ) are taken from the morning data (7am – 12pm). Table 5 depicted the average values of patients' arrival each day. It is worth noting that the average values of Monday, Tuesday and Saturday are not significantly differ. Average for Sunday and Wednesday are higher while Thursday represented lowest mean values of patients' arrivals. Therefore, arrival patterns can be categorized into three: (1) high arrival in Sunday and Wednesday, (2) medium arrivals for Monday, Tuesday and Saturday, and (3) low arrival for Thursday. The aim of the categorization is to determine potential impact of each category to resource utilization and queuing performance. In the meantime, the mean service rate for a single server is 12 patients per hour (60 minutes/5 minutes per patients per treatment). The number of servers (doctor on duty) is varied as shown earlier in Table 2. However, for the purpose of the research, number of servers will be based on a single value for which highest frequent number of available doctors is taken into account. Therefore, number of servers is as follows: (1) Sunday – 3 servers, (2) Monday – 2 servers, (3) Tuesday – 2 servers, (4) Wednesday – 3 servers, (5) Thursday – 2 servers, and (6) Saturday – 2 servers. Values for the service rate and number of servers are assumed to be constant. The analysis has been carried out using the QM for Windows software. The study uses multiple server queuing models (M/M/s) for which are based on the same assumptions as the single server model, Poisson arrival rate, exponential service times, infinite calling population and queue length, and first-in-first-out (FIFO) queue discipline. The results in Table 6 shows that higher server utilization of 96% were recorded for Monday, Tuesday and Saturday while the lowest one was on Wednesday (75%). Correspondingly, highest numbers of patients waiting for treatment were documented on those three days. Analysis on the average waiting time for those three days also indicated that it took slightly more than an hour for a patient to secure treatment (waiting time + treatment time). On Sunday, Wednesday and Thursday, average waiting times are considerably very minimal. The results also highlighted better queuing performance when three doctors are available for treatment as compared to only two.

Table 6 Queuing performance of the Outpatient Department

	Operating Characteristics				
Day	**(i) Average server utilization**	**(ii) Average number in the system**	**(iii) average number in the queue**	**(iv) Average time in the system (*minute*)**	**(v) Average time in the queue (*minute*)**
Sun	0.81	5.14	2.72	10.63	5.63
Mon	0.96	23.49	21.57	61.28	56.28
Tue	0.96	23.49	21.57	61.28	56.28
Wed	0.75	3.95	1.7	8.79	3.79
Thu	0.83	5.45	3.79	16.36	11.36
Sat	0.96	23.49	21.57	61.28	56.28

However, the results were only valid based on the assumption that arrival of patients follows a Poisson distribution. In actual fact, arrivals of patients were imbalance for which majority came early in the morning (between 7am-9am). Hence, actual average waiting time for a patient was greater than what the results were showing. If each patient that leaves the center early due to long waiting time can be translated into a loss of sales of RM100, then for that week alone the center had lost 147 x RM100 = RM14,700. Let say the cost of hiring a doctor is RM2000 a week, and the cost of waiting time for each customer is assumed to be RM100. The following analysis compares the operating characteristics and the total costs (labor + waiting cost) for Monday, Tuesday and Saturday:. Table 7 shows the queuing performance, number of servers and the total waiting costs

Table 7 The queuing performance, number of servers and the total waiting costs

	No of Doctors/servers				
	2	3	4	5	6
U	0.96	0.64	0.48	0.38	0.32
L	23.49	2.64	2.06	1.95	1.92
L_q	21.57	0.72	0.14	0.03	0.01
W	61.28	6.87	5.37	5.08	5.02
W_q	56.28	1.87	0.37	0.08	0.02
Total Costs based on waiting	**RM6157.27**	**6071.84**	**8014.18**	**10003.18**	**12000.7**

It is worth noting that based on the above table, the optimal number of doctors on duty should be three. By having three doctors, average waiting time and number of patients in the queue can be reduced significantly with minimal cost. The amount of possible losses that could be saved are 54.41 minutes (reduction in average waiting time: 56.28 – 1.87 = 54.41 minutes) and RM5441 (Reduction in loss of sales: 54.41 minutes x RM100 per customer = RM5441). The reduction benefit is quite significant, potentially representing 37% savings from the total loss of sales when 147 patients walk away from the outpatient department before seeking the treatment. From the hospital point of view, total waiting cost is also the lowest compared to having less or more than three doctors on duty. Although more doctor on-duty reduces waiting time, the total costs that need to be borne by the hospital will be greater as well.

5.0 MANAGERIAL IMPLICATIONS AND CONCLUSION

Healthcare is a fast growing industry demanding perfection, and optimal utilization of available resources. Notwithstanding, the trade-off between providing top medical services and keeping the corresponding costs low is evident. This include the outpatient department in which the interplay between the cost of customers waiting time and the cost of providing faster services has been depicted by [26]. The study investigated queuing efficiency of a local hospital's outpatient department for six days. The aims are to examine the efficiency level of the existing queuing system and to determine the optimal number of doctors on duty. As stated earlier, 15.6% or 147 patients return home without treatment. The findings exhibit utilization rate of the server, average waiting time and number of customer in the queue as well as total waiting cost. It indicates three working days (Monday, Tuesday and Saturday) in which the servers (doctors on duty) were almost 100% fully utilized, and that the average waiting times were significantly higher compared to the other three working days. It is worth noting that the analysis uses the multiple servers queuing models (M/M/s) for which are based on the assumptions that arrival rate follows Poisson distribution while the servicing times were exponential in nature and that the infinite calling population is following the first-in-first-out (FIFO) queue discipline. However, actual waiting times were apparently greater than what the results highlighted. This is due to the fact that majority of patients arrived early in the morning between 7am – 9am and doctors' treatment hours were only begins at 8am. Hence, imbalance arrival patterns were affecting queuing efficiency of the center in which number of available doctors was indifferent between morning and afternoon sessions. Although results from this study may not be a true reflection of a much longer actual waiting time, management could still leverage the findings as a yardstick for further relevant investigations. After all, the findings may still indicate similar outcomes had it been tested using actual arrival data. Nonetheless, to improve the queuing analysis the management may consider setting average arrival data based on the first two-hours of the center operations to closely reflect actual higher number of patients during these hours. For future study, analysis using simulation technique would be valuable to enhance the practicality of the analysis. In all, the study is still beneficial as a decision making aid to the administrator of the outpatient department particularly to re-look and improve existing queuing system for better customer satisfaction and minimal patients' turnover.

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