A Comparison Study Between B-Spline Surface Fitting and Radial Basis Function Surface Fitting on Scattered Points

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Graphical abstract
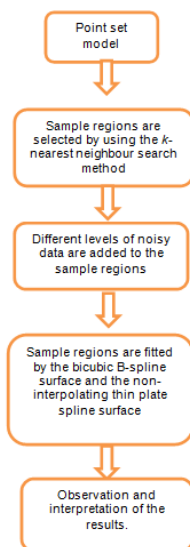
Abstract

This paper looks in the effectiveness of bicubic B-spline surface fitting and radial basis function, specifically the thin plate spline surface fitting in constructing the surface from the set of scattered data three dimensions (3D) points. Modification of the B-spline approximation algorithm is used to determine the unknown B-spline control points, followed by the construction of the bicubic B-spline surface patch, which can be joined together to form the final surface. The non-interpolation scheme of thin plate spline is also used to fit the data points in this study. The sample of scattered data points is chosen from a specific region in the point set model by using k-nearest neighbour search method. Observation is further carried out to observe the effect of noise in the bicubic B-spline surface fitting and the thin plate spline surface fitting. From the visual aspect, non-interpolation scheme of thin plate spline fits the surface better than bicubic B-spline in the presence of noises.

Keywords: 3D scattered data, approximation, bicubic B-spline surface, radial basis function, noisy data

1.0 INTRODUCTION

Surface fitting can be considered as the regression problem, where the model is the surface representation and the data are the sampled points on the surface [1]. The example sources of scattered data points are obtained from the measured values of physical quantities, experimental results, and computational values, which are widely found in scientific and engineering applications [2]. However, the fitting for 3D scattered data points is a tough task due to the amount of data points, as well as its irregularity in distribution.

The most commonly-used approximation methods are fitting methods, which include interpolation by spline, interpolation by radial basis function, and the least square approximation [3]. The tensor product of B-splines surfaces is widely used because of its advantages inherent in working with tensor products [4]. Radial basis function (RBF) is an example of global basis function method, where the concept of a global method is described as the interpolant, which is dependent on all data points [5]. Any addition or deletion of a data point or a change of one of the coordinates of a data point will propagate throughout the domain of definition [6]. RBF is widely used in mesh repair and surface reconstruction such as in range scanning, geographic surveys and medical data, field visualisation in 2D and 3D, image warping, morphing, registration, as well as artificial intelligence [7].

2.0 MATERIALS AND METHODS

In this section, some required mathematical background will be provided as follows to enhance the understanding of this work.

2.1 B-spline Surface

The rectangular B-spline surface patch $f(u,v)$ is constructed by applying tensor product technique to the B-spline curve, which is described as a linear combination of B-spline basis functions in two topological parameter u and v [8]. Furthermore, it is defined by a topological rectangular set of control points $P_{i,j}$ for $0 \le i \le m$, $0 \le j \le n$, and the two knot vectors: $U = (u_0, u_1, u_2, ..., u_{m+k})$ and $V = (v_0, v_1, v_2, ..., v_{m+k})$. B-spline surface patch is given by

$$f(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} P_{i,j} N_i^k(u) N_j^l(v)$$

, where $N_i^k(u)$ and $N_j^l(v)$ are the B-spline basis functions of order k and l respectively. The parameters u and v are the global parameter.

2.2 Radial Basis Function

Radial basis function (RBF) is an example of global basis function method. It is invariant to translations and rotations of the coordinate systems over $\mathbb{R}^n$. The concept of a global method is described as the interpolant, which is dependent on all data points, where any addition or deletion of a data point or a change of one of the coordinates of a data point, will propagate throughout the domain of definition [6]. Besides the interpolating scheme, RBF has a non-interpolating scheme. The general RBF is in the form shown in [9]:

$$s(X) = p(X) + \sum_{j=1}^{N} \lambda_j \phi\left(| X - X_j |\right) \qquad (1)$$

, where p is a polynomial of low degree, N is the total number of distinct data points, λ_i is the weight of center X_i, the Euclidean norm on n-dimensional $\mathbb{R}^n$ is denoted as $r = |\cdot| \ge 0$, basic function ϕ is a real valued function on the interval $[0, \infty)$, which is usually unbounded and of non-compact support.

The good choice of polyharmonic for fitting smooth function of two variables is thin-plate spline $\phi(r) = r^2 \log(r)$, which has C^1. Suppose we want to interpolate the data of two variables and set the polynomial p as linear form, then the interpolant in equation (1) is defined as $s : \mathbb{R}^2 \to \mathbb{R}$, we have

$$s(x_i, y_i) = a_1 + a_2 x_i + a_3 y_i + \sum_{j=1}^{N} \lambda_j \phi\left(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\right) = f_i$$

. Since there is $N + 3$ unknown, three additional solution constraints are added, such that

$$\sum_{j=1}^{N} \lambda_j = \sum_{j=1}^{N} \lambda_j x_j = \sum_{j=1}^{N} \lambda_j y_j = 0$$

, which yield the linear system as written in $(N+3) \times (N+3)$ matrix form as follows:

$$\begin{pmatrix} 0 & P^T \\ P & A \end{pmatrix} \begin{pmatrix} a \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix} \qquad (2)$$

, where P is the matrix with ith row $(1, x_i, y_i)$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_N)^T$, $a = (a_1, a_2, a_3)^T$, and $f = (f_1, f_2, ..., f_N)^T$. By solving the linear system, the value of λ and a can be uniquely determined and hence, the function is obtained. It is appropriate to use the direct method to solve the above matrix when the problem occurs at most few thousand points that is $N < 2000$. If $N \ge 2000$, then the matrix will be conditionally poor and the solution seems unreliable. However, this problem is resolved by the fast method as proposed by [9], therefore fitting and evaluating the large number of data points with a single RBF now is becoming the possible task.

If noise is present in the data points, the parameter controls the quality of approximation or in other words, it controls the trade-off between smoothness and fidelity to the data [10]. The solution for this problem is also a RBF of form in equation (1). A smaller value of ρ will provide a better approximation and will be an exact interpolation if ρ tends to be 0 [11]. By modifying the equation (2), we have the following:

$$\begin{pmatrix} 0 & P^T \\ P & A + \rho I \end{pmatrix} \begin{pmatrix} a \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix} \qquad (3)$$

, where I is an identity matrix. By solving the system of linear equation in equation (3), we will get a and λ, then plug in the equation (1), a RBF in form of approximation is obtained.

2.3 Effect of Noisy Data on Surface Fitting

In this study, we use the proposed algorithm from [12], which is a modification of B-spline approximation from the existing algorithm as can be found in [2]. The existing B-spline approximation algorithm is undergoing a modification in order to minimise the distance between the scattered data points and the approximated bicubic B-spline surface. The distance is the problem in the existing algorithm. The presence of noise will contribute to the bad fitting of the surface. Moreover, the accuracy of the 3D model will be reduced during the surface reconstruction due to the set of data points being contaminated by the noise. Therefore, we will look at the effect of noisy data on the B-spline surface patch in [12] with radial basis function surface, specifically the non-interpolating scheme of thin plate spline surface. We assume that the sets of sample scattered data points, P that are used earlier are noise free. For the experimental purpose, twenty noisy data are added randomly in positive and negative direction to P and then, we construct the surface. The noise levels to be considered are 0.0, 0.3, 0.5, and 0.7.

3.0 RESULTS

We test our model with Stanford bunny as shown in the coming figures. The scattered data points are denoted as green dots, whereas the control points are denoted as red dots. To select the sample of scattered points, *P* from Bunny point set model, we set $k = 100$, that is 100-nearest neighbour.
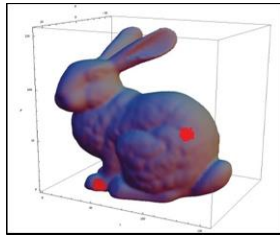

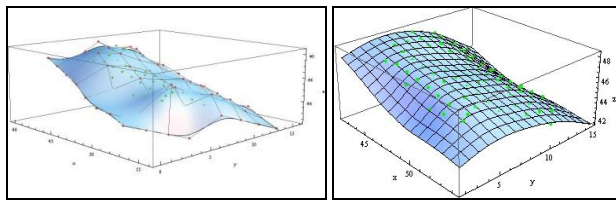**Figure 1** The two sets of selected red sample regions from the Stanford bunny mesh model

Figure 2 (From left to right) Comparison between bicubic B-spline surface fitting and thin plate spline fitting for set 1
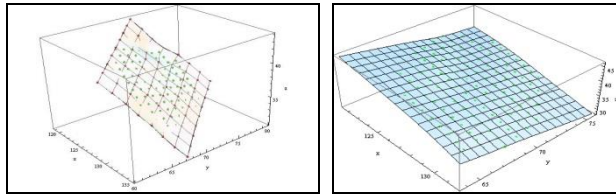

Figure 3 (From left to right) Comparison between bicubic B-spline surface fitting and thin plate spline fitting for set 2
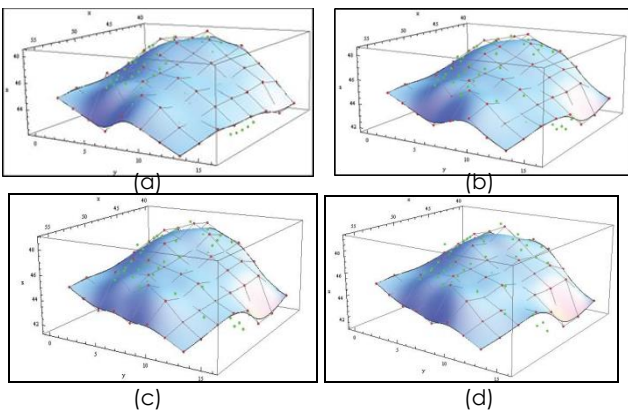

(a)   (b)

(c)   (d)

Figure 4 (From (a) to (d)) Set 1: Reconstruction with noise level at 0.0, 0.3, 0.5, and 0.7 by bicubic B-spline surface patch 1
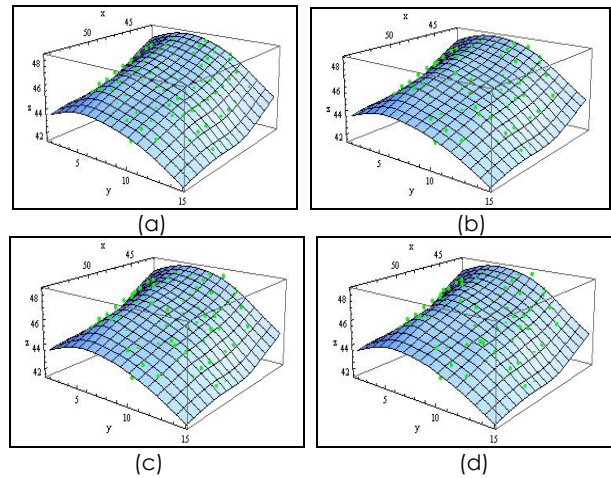

(a)   (b)

(c)   (d)

Figure 5 (From (a) to (d)) Set 1: Reconstruction with noise level at 0.0, 0.3, 0.5, and 0.7 by thin plate spline surface patch 1

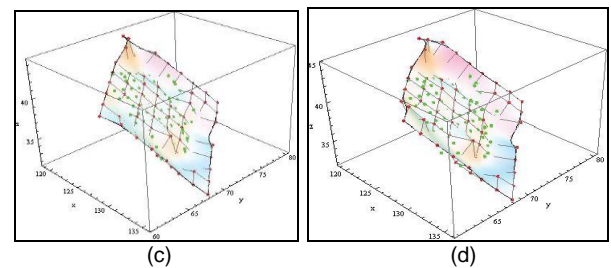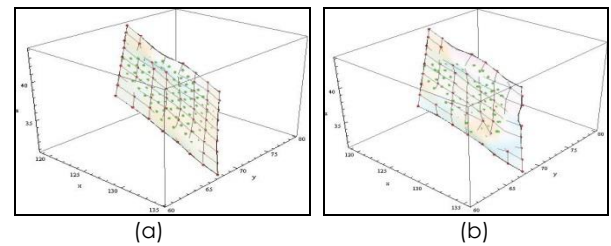(a)   (b)

(c)   (d)

Figure 6 (From (a) to (d)) Set 2: Reconstruction with noise level at 0.0, 0.3, 0.5, and 0.7 by bicubic B-spline surface patch 2
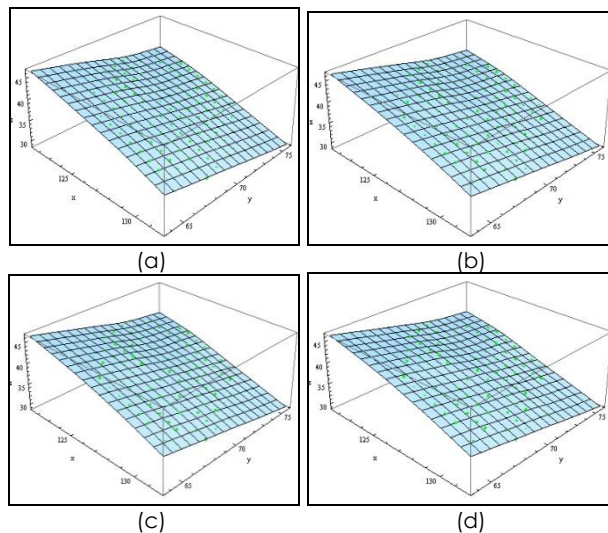
Figure 7 (From (a) to (d)) Set 2: Reconstruction with noise level at 0.0, 0.3, 0.5, and 0.7 by thin plate spline surface patch 2

4.0 DISCUSSION

The two sets of selected sample regions in the Stanford bunny point set model are marked and shown in Figure 1. The two bicubic B-spline surface patches are fitted with the modified B-spline approximation algorithm after resolving the distance issue in the existing algorithm and the thin plate spline surface fitting. They are shown in Figure 2 and Figure 3. The value of ρ , which can be seen from the equation (3) is known as smoothing parameter. In our study, we choose the smaller value for equation (3). The optimum value will be our future work. The value we choose is sufficient to observe the effect of noisy data towards the non-interpolating scheme of the thin plate spline surface fitting.

The effect of noisy data towards the accuracy of surface is inspected visually. Noise levels at 0.0, 0.3, 0.5, and 0.7 are considered in this study. The effect of noise for the bicubic B-spline surface is not obvious as shown in Figure 4, but it is obvious in Figure 6 when the noise level is increasing. Therefore, the accuracy of the bicubic B-spline surfaces is quite sensitive to the presence of noise even at a low level of noise. However, non-interpolating thin plate spline surface is uneasily affected by the presence of the noisy data compared with the bicubic B-spline surface as shown in Figure 5 and Figure 7. In this study, noise level at 0.3 is a good indication of the result as it is neither too low nor too high, whereas the noise level at 0.5 and 0.7 can be considered quite noisy. Therefore, the bad fitting of the surface is expected not due to the fact that b-spline surface fails to estimate, but rather than the case of bad data points given.

5.0 CONCLUSION

This paper shows the bicubic B-spline surface fitting by using the modified B-spline approximation algorithm and the thin plate spline fitting. For the effects of noisy data, the experimental results show that the accuracy and the smoothness of B-spline surfaces can be easily influenced by the presence of noise, which mean that they are sensitive to the noise. However, the non-interpolating scheme of thin plate spline surfaces fits the surface better than bicubic B-spline surface in the presence of the noise. We note that our observation is done visually, which may be prone to a subjective opinion. For future research, we will use an objective inspection, such as statistical methods to search for the optimum smoothing parameter of thin plate spline, as well as the accuracy of the fitting in the presence of noisy data.

References

[1]   Jain, R., Kasturi, R. and Schunck, B. G. 1995. *Curves and Surfaces*. United States: McGraw-Hill, Inc. 360-405.
[2]   Lee, S. Y., Wolberg, G. and Shin, S. Y. 1997. Scattered Data Interpolation with Multilevel B- splines. *IEEE Transactions on Visualization and Computer Graphics*. 3(3): 228-244.
[3]   Feng, R. and Zhang, Y. 2013. Piecewise Bivariate Hermite Interpolations for Large Sets of Scattered Data. *Journal of Applied Mathematics 2013*.
[4]   Lee, B. G., Lee, J. J. and Yoo, J. 2005. An Efficient Scattered Data Approximation Using Multilevel B-splines Based on Quasi-interpolants. *Proceedings of the 5th International Conference on 3-D Digital Imaging and Modeling*. 110-117.
[5]   Duchon, J. 1977. *Constructive Theory of Functions of Several Variables*. Lecture Notes in Mathematics. Berlin: Springer-Verlag. 8-100.
[6]   Franke, R. 1982. *Scattered Data Interpolation*. *Mathematics of Computation* 38: 181-200.
[7]   Qin, H. 2014. *Lecture Notes on Computer Graphics (Theory, algorithms, and applications)*. Department of Computer Sciences, Stony Brook University (unpublished).
[8]   Cohen, F. S., Ibrahim, W. and Pintavirooj, C. 2000. Ordering and Parameterizing Scattered 3D Data for B-spline Surface Approximation. *IEEE Transactions on Pattern Analysis and Machine Intelligenc*. 22(6): 642-648.
[9]   Carr, J. C., Beatson, R, K., Cherrie, J. B., Mitchell, T. J., Fright, W. R., McCallum, B. C. and Evans, T. R. 2001. Reconstruction and Representation of 3D Objects with Radial Basis Functions. *SIGGRAPH '01 Proceedings of the 28th Annual Conference on Computer Graphics and Interactive Techniques*. 67-76.

[10] Wahba, G. 1990. Spline Models for Observational Data, Ch. Estimating the Smoothing Parameter. *Society for Industrial and Applied Mathematics*. SIAM. 314-315.

[11] Bennink, H., Korbeeck, J., Janssen, B. and Romeny, B. M. t. H. 2007. Warping a Neuro-anatomy Atlas on 3D MRI Data with Radial Basis Functions. *Proceedings of the 3rd Kuala Lumpur International Conference on Biomedical Engineering (Biomed) 2006*. New York, USA: Springer-Verlag Berlin Heidelberg. 28-32.

[12] Liew, K. J., Ramli, A. and Abd. Majid, A. 2016. B-spline Surface Fitting on Scattered Points. *Applied Mathematics & Information Sciences*. 10(1): 1-9 (To appear).