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Modelling Two-dimensional Lease Contract with Preventive Maintenance and Servicing Strategy

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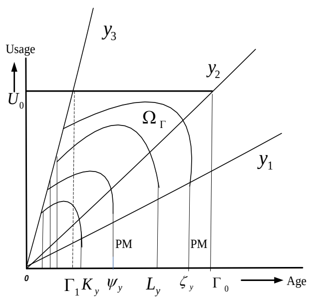
Graphical abstract



Abstract

In this paper, we study a two-dimensional lease contract with preventive maintenance and servicing strategy involving imperfect repair. The lease contract coverage is characterized by two parameters – age and usage. The lessor will carry out preventive maintenance (PM) and a servicing strategy which allows more than one imperfect repairs under the contract. In the lease contract studied, we consider that a penalty cost incurred when the time required to perform an imperfect repair exceeds a target. This servicing strategy aims to reduce equipment failures and hence decreasing the penalty cost and maintenance cost during the leased contract. We find the optimal PM degree and the imperfect repair strategy such that the expected total cost is minimized. Numerical examples are presented to illustrate the optimal PM and servicing strategy for various usage rates (heavy, moderate and low usage rates), and compared results for this servicing strategy with those of minimal repair strategy.

Keywords: Servicing strategy, imperfect repair, expected cost, two dimensional leased contract

1.0 INTRODUCTION

We consider high capital equipment used in mining industry such as dump trucks, excavators, bull dozers, etc. These heavy equipment can cost a large of money (the investment tied to the equipment can reach more than \$1 million). Recently, prices of ore, coal and other mining materials tends to decrease and this results in the decrease of the revenue of a mining company. This situation makes the mining company cut back on capital expenses – including procurement for the heavy equipment. As a result, leasing the equipment to an external agent or Original Equipment Manufacturer (OEM) is a viable option to meet the need of the equipment.

Let us assume that the population be first going through a growth phase in which individuals survive, reproduce and changing from a life stages to other in one location. For each patch *j*, suppose A_j is a $m \times m$ non-negative matrix of representing the dynamics of growth on the location of *j*. Then vector abundance of that population is describe as follow

In general, the agent (or OEM) as a lessor gives a lease contract with a full coverage of the maintenance actions (PM or/and CM). Study of leased products has received much attention. In [1] the authors studied a lease contract in which PM is taken when the failure rate of the leased product reaches a certain threshold value. Further in [2], the failure rate reduction method is also used to obtain the optimal periodical maintenance policy for the leased equipment. The optimal number and degrees of PM have been studied by [3]. Most studies of leased products focus on determining the optimal PM policy in a specified contract period.

Recently, OEM tends to offer a longer lease contract – more than 1 year, in order to attract consumers. For the OEM, offering a leased product with a long period of contract will result in a greater maintenance costs for servicing the lease contract – as more failures are likely to occur. Hence, the lessor is of interest for reducing the maintenance costs. For a repairable product such as heavy equipment, a proper combination of pm and servicing strategy can reduce the maintenance cost significantly, the maintenance costs. A variety of servicing strategies has been studied in the context of warranted product. [4], [5] and [6] have shown that an appropriate servicing strategy gives a significant reduction to the warranty cost. But these servicing strategies only allow at most one imperfect repair over the warranty period. [7] and [8] extend the servicing strategy developed by [4] to allow more than one imperfect repair for products sold with one-dimensional and two-dimensional warranties.

In all of the works on leased contracts, they consider that the coverage of contracts is characterized by time (called a one-dimensional lease contract). In this paper, we study two-dimensional lease contracts where the coverage of the contract is limited by age and usage limits. In addition, due to the lease contract coverage tends to be larger, then it needs PM and a servicing strategy allowing more than one imperfect repair in order to minimize failures under the lease contract.

The paper is organized as follows. In section 2 we give model formulation for the leased contract studied. Sections 3 and 4 deal with model analysis to obtain the optimal improvement level and the numerical example. Finally, we conclude with topics for further research in Section 5.

2.0 MODEL FORMULATION

The following notation will be used in model formulation.

$\Omega_l = [0,\Gamma_0)\times[0,U_0)$	: lease coverage
δ_y	: improvement level ($0<\delta<1$)
ψ_y	: parameter of lease contract type 1 ($0<\psi_y\le\Gamma_y$)
K_y	: threshold time of parameter lease contract type 1 ($0<K_y\le\psi_y/2$)
$Z_y, V_y, \psi_y, \zeta_y$	: the first failure after K_y and the first failure after z_y+K_y , ($0\le z_y\le\psi_y\le v_y\le\zeta_y\le\Gamma_y$)
$F_y(t)$, $f_y(t)$, $h_y(t)$	: failure distribution, density and hazard rate functions for a given y
$A_i(t)$	: virtual age after i^{th} imperfect ($i=1,2$)
Z_1	: random variable of first failure afte τ (S_1) for lease contract 1 (2) with distribution function $F_1(z_1)$
Z_2	: random variable of first failure afte $Z_1+\tau$ (S_2) for lease contract 1 (2) with distribution function $F_2(z_2)$
c_m	: cost of minimal repair
c_p	: cost of perfect repair
$c_{im}(\delta)$	:cost of imperfect repair maintenance as a function of δ
$c_{ip}(\delta)$	: cost of imperfect PM as a function of δ
$J_y^1(\delta, K_y, \psi_y, L_y, \zeta_y)$	: expected total lease contract cost for Case (i)
J_y^2	: expected total lease contract cost for Case (ii)

2.1 Two-dimensional Maintenance Leased Contract

We consider that a mining company operates a number of leased dump trucks and each covered by a two-dimensional lease contract. The lease contract is characterized by a rectangle region $\Omega_\Gamma = [0,\Gamma)\times[0,U)$ where Γ and *U* are the time and the usage limits (e.g. the maximum coverage for Γ (e.g. 3 year) or U (e.g. 150.000 km), and hence the lease contract is characterised by a rectangle region Ω_l (see Figure 1). All failures under lease contract are rectified at no cost to the lessee (consumer).

Let *Y* be the usage rate of a truck. The usage rate varies from truck to truck. For a given truck or usage rate *y*, the lease contract ceases at $\Gamma_y = \Gamma_0$ for $y \le U_0/\Gamma_0$, or $\Gamma_y = U/y$ for $y > U_0/\Gamma_0$. We consider that the lease contract given by the OEM also covers PM action, and hence, during the lease period CM and PM actions are done by the OEM without any charge to the lessee (See Figure 1).

A penalty cost incurs the OEM if the actual down time falls above the target ($\mathfrak{T}$) . If $\mathcal{D}$ is down time (consisting repair time and waiting time) for each failure occuring during the contract, then the OEM should pay a penalty cost ($\mathcal{C}_p$)when $\mathcal{D} > \mathfrak{T}$. The amount of the penalty cost is assumed to be proportional to $\Delta = \mathcal{D} - \mathfrak{T}$. The decision problem for the OEM is to determine the optimal maintenance level such that to minimize the expected maintenance cost.

We model the degradation of a truck influenced by three factors – i.e. age, usage and operating condition where the truck. Here, the accelerated failure time (AFT) model is used as it allows to incorporate the effect of the three major factors on degradation of the truck leading to failure. The usage can be represented by distance travelled (mileages of a truck). Here, we use the one-dimensional approach as in [9] where equipment failures can be viewed as a one-dimensional point process. Then, we use a distribution and a failure rate functions conditional on Y to modelling failures.

If the distribution function for T0 is given by F0(T, a0), where a0 is the scale parameter, then the distribution function for Ty is the same as that for T0 but with a scale parameter given by

$$\alpha_y = (y_0/y)^{\rho}\,\alpha_0 . \tag{1}$$

with $\rho \geq 1$. Hence, we have $F(t,\alpha_y) = F_0((y_0/y)^{\rho} t, \alpha_y)$. The hazard and the cumulative hazard functions associated with $F(t, \alpha_y)$ are given by $r_y(t) = f(t,\alpha_y)/(1-F(t,\alpha_y))$ and $R_y(t) = \int_0^t r_y(x)dx$ respectively where $f(t,\alpha_y)$ is the associated density function. If all failures are fixed by minimal repair and repair times are small relative to the mean time between failures, then failures over time occur according to a non-homogeneous Poisson process (NHPP) with intensity function $r_y(t)$. We will use the accelerated failure time (AFT) model to modelling failures, which allows to incorporate the effect of usage rate on degradation of the product (see [9]).

As mentioned before that Y varies from truck to truck but is a relatively constant for a given truck (or a given equipment). Hence, Y is a random variable with density function $g(y), 0 \leq u < \infty$. Conditional on $Y = y$, the total usage u at age x is given by $u = yx$.

2.2 Servicing Strategy

Here, we consider a larger lease contract coverage – e.g. a dump truck is leased for 3 years or 150.000 km, whichever occurs first. For a larger coverage of a lease contract, the equipment would experience more failures under the contract, and this needs PM and a servicing strategy allowing more than one imperfect repair in order to minimize failures under the lease contract. As PM and a servicing strategy can control the degradation of the equipment, and hence reduce the number of failures. We define the servicing strategy as follows. For a given usage rate y, a lease contract ends at $\Gamma_y = \Gamma_0$ (See Figure 1) and we divide interval $(0, \Gamma_y]$ into five intervals i.e. $(0, K_y]$, $(K_y, \psi_y]$, $(\psi_y, L_y]$, $(L_y, \zeta_y]$ and $(\zeta_y, \Gamma_y]$ (See Figure 1). Each first failure in $(K_y, \psi_y]$ or $(L_y, \zeta_y]$ is imperfectly repaired and all other failures are minimally repaired. When there is no failure in interval $(K_y, \psi_y]$ ($(L_y, \zeta_y]$) then PM is done at ψ_y (ζ_y).

It is assumed that imperfect PM and imperfect repair improve the reliability of the product in the sense that the age of the item after repair is smaller than that before failure. The imperfect PM (imperfect repair) will reduce the age of the product with improvement level $[\delta_2]$. Without losing the generality of improvement levels, it is considered that $\delta_1 = \delta_2 = \delta$ (Note: all imperfect repair for case $y \leq \gamma$ and $y > \gamma$ are done at improvement level δ).

2.3 Modeling Imperfect Repairs

With a wider coverage of a lease contract, it needs a proper PM and several imperfect repairs required to reduce the the number of failures and maintenance costs. For both case, $y \leq \gamma$ and $y > \gamma$ we confine to at most two imperfect repairs over the lease contract coverage. Now, we obtain distribution functions for z_y and v_y defined as $F_{z|y}(z_y)$ and $F_{v|y}(v_y)$] which are important in modelling the expected maintenance cost for the servicing strategy studied. As failures occuring in $(0, K_y]$ and $(z_y, L_y]$ are fixed by minimal repair, then

$$F_{z|y}(z_y) = 1 - \exp[H_y(K_y) - H_y(z_y)] \tag{2}$$

where $H_y(t) = \int_0^t h_y(u)du$. Differentiating (2) with respect to z_y yields

$$f_{z|y}(z_y) = h_y(z_y)\exp[H_y(K_y) - H_y(z_y)] \tag{3}$$

Conditioning on $Z_y = z_y$ and then unconditioning it, we have $F_{v|y}(v_y)$ given by

$$F_{v|y}(v_y) =$$

$$\int_{K_y}^{\psi_y} \left\{1 - \exp\left[H_y(L_y) - H_y(v_y)\right]\right\} h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]dz_y +$$

$$\left\{1 - \exp[H_y(L_y) - H_y(v_y)]\right\}\exp[H_y(K_y) - H_y(\psi_y)]$$

where $H_{z|y}(t) = \int_0^t h_{z|y}(u)du$ and $H'_{z|y}(t) = \int_0^t h'_{z|y}(u)du$ and its density function is given by

$$f_{v|y}(v_y) =$$

$$\int_{K_y}^{\psi_y} h_{z|y}(v_y)\exp\left[H_{z|y}(L_y) - H_{z|y}(v_y)\right]dv_y +$$

$$\int_{K_y}^{\psi_y} h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]dz_y h'_{z|y}(v_y)$$

$$\exp[H'_{z|y}(L_y) - H'_{z|y}(v_y)]\exp[H_y(K_y) - H_y(\psi_y)]$$

Note that distribution function Distribution functions of Z_y and V_y for case $y > \gamma$ can be found in Husniah *et.al* [8].

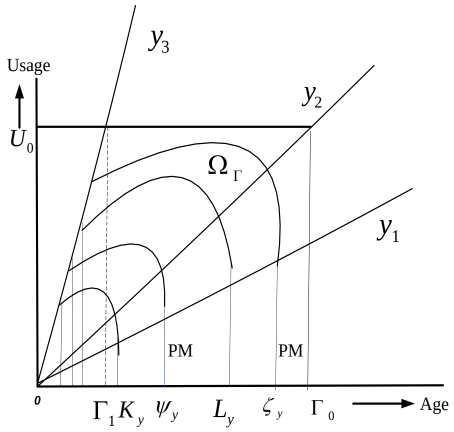


Figure 1 Parameter of with PM and two imperfect repairs

3.0 MODEL ANALYSIS

We consider a wide coverage of leased contract for a dump truck (e.g. for maximum 5 years or 250.000 km). OEM needs to perform PM and more than one imperfect repair for reducing the maintenance cost. a situation where the OEM incurs repair cost for each failure and PM cost. Here, we confine to at most two imperfect repairs over the contract. We first obtain the expected total maintenance cost for the case (i) lease contract with proposed servicing strategy and PM and then the case (ii) lease contract with only minimal repair.

For $y \le \gamma$, the total maintenance cost consists of penalty cost (when down time above the target), repair cost and PM cost. Hence, the expected cost is given by

$$E[\pi_y] = E[\text{ Penalty cost}] + E[\text{Repair and PM cost}]$$

We obtain the expected repair and PM cost and expected penalty cost in (0, Γ0] as follows.

<u>Case(i): PM and imperfect repair,</u> $J_y(\delta, K_y, \psi_y, L_y, \zeta_y)$

The expected maintenace cost with PM and imperfect repair is obtained by a conditional approach. Let t_1 and t_2 be the first and second imperfect maintenance, respectively.

The first (second) imperfect maintenance may occur at either $t_1 = z$ if $Z \le \psi_y$ or $t_1 = \psi_y$ if $Z > \psi_y$ ($t_2 = v$ if $L_y < V \le \zeta_y$ or $t_2 = \zeta_y$ if $V > \zeta_y$). Then, there are four possible conditions for (t_1, t_2) given by (4).

$$t_1 = \begin{cases} z_y, t_2 = \begin{cases} v & \text{if } K_y < z \le \psi_y, L_y < v \le \zeta_y \\ \zeta_y & \text{if } K_y < z \le \psi_y, v > \zeta_y \end{cases} \\ \\ \psi_y, t_2 = \begin{cases} v & \text{if } z > \psi_y, L_y < v \le \zeta_y \\ \zeta_y & \text{if } z > \psi_y, v > \zeta_y \end{cases} \end{cases} \quad (4)$$

Let $J_y^1(\delta, K_y, \psi_y, L_y, \zeta_y | Z = z, V = v)$ be the expected maintenance cost conditional on Z and V. From (4) we have $J_y^1(\delta, K_y, \psi_y, L_y, \zeta_y | Z = z, V = v)$ given by (5),

$$J_y^1(\delta, K_y, \psi_y, L_y, \zeta_y | Z = z, V = v) =$$
$$\begin{cases} 2c_{im}(\delta) + c_m(H_y(K_y) + H_{z|y}(L_y) - H_{z|y}(z) + H_{v|y}(\Gamma_y) - H_{v|y}(v)) & \text{if } K_y < z \le \psi_y, L_y < v \le \zeta_y \\ c_{im}(\delta) + c_{ip}(\delta) + c_m(H_y(K_y) + H_{z|y}(L_y) - H_{z|y}(z) + H'_{v|y}(\Gamma_y) - H'_{v|y}(\zeta_y)) & \text{if } K_y < z \le \psi_y, v > \zeta_y \\ c_{im}(\delta) + c_{ip}(\delta) + c_m(H_y(K_y) + H'_{z|y}(L_y) - H'_{z|y}(\psi_y) + H_{v|y}(\Gamma_y) - H_{v|y}(v)) & \text{if } z > \psi_y, L_y < v \le \zeta_y \\ 2c_{ip}(\delta) + c_m(H_y(K_y) + H'_{z|y}(L_y) - H'_{z|y}(\psi_y) + H'_{v|y}(\Gamma_y) - H'_{v|y}(\zeta_y)) & \text{if } z > \psi_y, v > \zeta_y \end{cases} \quad (5)$$

where $H_{v|y}(t) = \int_0^t h_{v|y}(u)du$ and $H'_{v|y}(t) = \int_0^t h'_{v|y}(u)du$.

Removing the conditional form in (5) yields

$$\begin{aligned}
&J_y(\delta\ ,K_y,\psi_y,L_y,\zeta_y)=c_m H_y(K_y)+\\
&\int_{K_y}^{\psi_y}\int_{L_y}^{\zeta_y}\Big(2c_{im}(\delta)+c_m\left[H_{z|y}(L_y)-H_{z|y}(z)+H_{v|y}(\Gamma_y)-H_{v|y}(v)\right]\Big)h_y(z)\exp[H_y(K_y)-H_y(z)]h_{z|y}(v))\\
&\exp[H_{z|y}(L_y)-H_{z|y}(v)]dvdz+\\
&\int_{S_1}^{\psi_y}\Big(c_{im}(\delta)+c_{ip}(\delta)+c_m H_{z|y}(L_y)-H_{z|y}(z)+H'_{v|y}(\Gamma_y)-H'_{v|y}(\zeta_y)\Big)\exp[H_{z|y}(L_y)-H_{z|y}(\zeta_y)]h_y(z)\\
&\exp[H_y(K_y)-H_y(z)]dz+\\
&\int_{L_y}^{\zeta_y}\Big(c_{im}(\delta)+c_{ip}(\delta)+c_m\left[H'_{z|y}(L_y)-H'_{z|y}(\psi_y)+H_{v|y}(\Gamma_y)-H_{v|y}(v)\right]\Big)\exp[H_y(K_y)-H_y(\psi_y)h'_{z|y}(v)\\
&\exp[H'_{z|y}(L_y)-H'_{z|y}(v)]dv+\\
&\Big(2c_{ip}(\delta)+c_m(H'_{z|y}(L_y)-H'_{z|y}(\psi_y)+H'_{v|y}(\Gamma_y)-H'_{v|y}(\zeta_y))\Big)\exp[H_y(K_y)-H_y(\psi_y)\exp[H'_{z|y}(L_y)-H'_{z|y}(\zeta_y)
\end{aligned}\tag{6}$$

And we get

$$\begin{aligned}
&H_y(K_y,\psi_y,L_y,\zeta_y)=H_y(K_y)+\\
&\int_{K_y}^{\psi_y}\int_{L_y}^{\zeta_y}\Big(\left[H_{z|y}(L_y)-H_{z|y}(z)+H_{v|y}(\Gamma_y)-H_{v|y}(v)\right]\Big)h_y(z)\exp[H_y(K_y)-H_y(z)]h_{z|y}(v))\\
&\exp[H_{z|y}(L_y)-H_{z|y}(v)]dvdz+\\
&\int_{S_1}^{\psi_y}\Big(H_{z|y}(L_y)-H_{z|y}(z)+H'_{v|y}(\Gamma_y)-H'_{v|y}(\zeta_y)\Big)\exp[H_{z|y}(L_y)-H_{z|y}(\zeta_y)]h_y(z)\\
&\exp[H_y(K_y)-H_y(z)]dz+\\
&\int_{L_y}^{\zeta_y}\Big(\left[H'_{z|y}(L_y)-H'_{z|y}(\psi_y)+H_{v|y}(\Gamma_y)-H_{v|y}(v)\right]\Big)\exp[H_y(K_y)-H_y(\psi_y)h'_{z|y}(v)\\
&\exp[H'_{z|y}(L_y)-H'_{z|y}(v)]dv+\\
&\Big((H'_{z|y}(L_y)-H'_{z|y}(\psi_y)+H'_{v|y}(\Gamma_y)-H'_{v|y}(\zeta_y))\Big)\exp[H_y(K_y)-H_y(\psi_y)\exp[H'_{z|y}(L_y)-H'_{z|y}(\zeta_y)
\end{aligned}\tag{7}$$

<u>Expected of Penalty Cost:</u>

Let $\mathscr{D}$ and $\mathfrak{T}$ denote down time (consisting repair time and waiting time) for each failure occuring during the contract and down time allowed. The expected penalty cost is given by $EP(\tau)=\mathscr{C}_{\mathscr{P}}\bar{G}(\mathfrak{T})H_y(K_y,\psi_y,L_y,\zeta_y)$ where $\mathscr{C}_{\mathscr{P}}$ is the penalty cost and $H_y(K_y,\psi_y,L_y,\zeta_y)$ denotes the expected total number of failure during the lease contract.

As a result, the total expected cost of the OEM is

$$E\left[\pi_y\right]= J_y(\delta\ ,K_y,\psi_y,L_y,\zeta_y)+\mathscr{C}_{\mathscr{P}}\bar{G}(\mathfrak{T})H_y(K_y,\psi_y,L_y,\zeta_y)\tag{8}$$

If margin of the profit $\mathscr{M}_{\mathcal{I}}$ is constant then price of the lease contract, $\mathscr{P}_{\mathcal{I}}$ is defined as

$$\mathscr{P}_{\mathcal{I}}=\mathscr{M}_{\mathcal{I}}+E\left[\pi_y\right]$$

For case $y>\gamma$, the expected cost of OEM is given by (8) replacing Γ with Γ_y .

<u>Case (ii): Lease contract with only minimal repair</u>

This case is considered for comparison purposes –showing the advantage of the servicing strategy used in the first case. For case $y\le\gamma$, the expected cost of the OEM is

$$E\left[\varphi_y\right]=c_m H_y(0,\Gamma_0)+\mathscr{C}_{\mathscr{P}}\bar{G}(\mathfrak{T})H_y(0,\Gamma_0)\tag{9}$$

For case $y>\gamma$, the expected cost of OEM is given by (9) replacing Γ with Γ_y .

4.0 NUMERICAL EXAMPLE

Since the complexity of integral equations involved in (8), it is not possible to obtain the optimal values $\delta^*, K_y^*, \psi_y^*, L_y^*, \zeta_y^*$ analytically. Hence, a computational approach will be used to obtain the sub optimal solution. We consider that the time to the first failure for a given usage rate *y* is given by the Weibull distribution with $F_y(t;\alpha_y) = 1 - \exp(-t/\alpha_y)^\beta$, and its hazard function is $r_y(t) = \beta\left(t^{\beta-1}/(\alpha_y)^\beta\right)$ where α_y as in (1). The other parameter values be as follows. β = 2, Γ0 = 5(years), *U*=5(1x10⁴Km) (*y* = *U*/*W* = 1), y_0 = 1, and $c_m = 0.5.c_p$, $\mathfrak{T}$ = 1 (year), $\mathcal{C}_{\mathcal{P}} = 25c_p r$ and $\mathcal{M}_{\mathcal{I}} = 42$. The cost of imperfect repair is a function of δ_y given by $c_i(\delta_y) = c_m + (c_p - c_m)\delta_y{}^4$ as in Yun *et. al* [1]. The down time distribution is given by the Weibull distribution with $\alpha = \beta = 0.5$.

The values of ρ for three different land contours are 1.7, 2.0, and 2.3 coresponding to light incline, high incline and very hilly, respectively

Tables 1 show optimal improvement level for Case (i) for the two usage types –(medium $(1.0 \le y < 1.4)$ and heavy $(y \ge 1.2)$ usages) with ρ = 1.7, 2.0 and 2.3 coresponding to high incline and very hilly, respectively. c_p = 7 and $\alpha_0 = 2$. For a given $y \ge 1$, and α_0 (representing reliability level), the optimal expected cost increases as the usage rate y increases. This is as expected since the increasing in *y* causes the failure rate to increase and this in turn increases the number of failures under Lease Contract, and also it requires a higher δ^*. As a result, the price of the lease contract increases with *y*. Tables 2 show results for Lease Contract Cases (i) and (ii) with C_m =4, c_p =7 and α_0 =2. Case (i) is the appropriate strategy when the equipment is used with moderate to high usage rate but for light usage Lease Contract with Case (ii) is better. As a result, the PM and servicing strategy proposed is appropriate to control the expected maintenance cost for a lease equipment used in moderate to heavy usages.

Table 1 Results for Lease Contract Case (i) with c_p =7, r = 0.7 and $\alpha_0 = 2$

	ρ = 1.7			ρ = 2.0			ρ = 2.5		
$\bar{y}$	δ^*	J_1^*	$\mathcal{P}_{\mathcal{I}}$	δ^*	J_1^*	$\mathcal{P}_{\mathcal{I}}$	δ^*	J_1^*	$\mathcal{P}_{\mathcal{I}}$
1.00	0.43	4.863	46.86	0.43	4.863	46.86	0.43	4.863	46.86
1.20	0.47	6.004	48.00	0.49	6.591	48.59	0.51	7.721	49.72
1.40	0.50	7.014	49.01	0.54	8.284	50.28	0.59	11.657	53.66
1.60	0.53	8.515	50.52	0.58	10.350	52.35	0.66	15.318	57.32
1.80	0.56	9.861	51.86	0.62	12.603	54.60	0.77	20.533	62.53
2.00	0.58	10.616	52.62	0.66	15.044	57.04	0.91	27.259	69.26

Table 2 Results for Lease Contract Case (i) and (ii)with α_0 =2, ρ = 1.7, c_m = 4, c_p =7, r = 0.7, Γ0 = 2(years), *U*=2(1x10⁴Km)

$\bar{y}$	δ^*	K_y^*	ψ_y^*	L_y^*	ζ_y^*	J_1^*	J_2^*
0.50	0.08	0.00	1.25	1.25	2.00	1.429	**0.38**
0.70	0.12	0.00	1.26	1.26	2.00	1.498	**1.19**
1.00	0.21	0.00	1.23	1.23	2.00	**1.791**	4.00
1.20	0.23	0.00	1.01	1.01	1.67	**1.931**	5.16
1.40	0.26	0.00	0.85	0.85	1.43	**2.088**	6.41
1.60	0.28	0.00	0.73	0.73	1.25	**2.263**	7.72
1.80	0.30	0.00	0.64	0.64	1.11	**2.454**	9.11
2.00	0.32	0.00	0.57	0.57	1.00	**2.660**	10.56

5.0 CONCLUSION

In this paper we study a two-dimensional lease contract involving PM and imperfect repair for a repairable product (such as dump truck).Two cases of lease contracts have been studied. Case (i) is always the best strategy compared to Case (ii) – i.e. lease contract with only minimal repair for moderate and heavy usages. The implementation of this Lease Contract of Case (i) is easy as it requires only a simple administrative work (collecting the time elapsed since the last imperfect repair) in deciding whether a failure is minimally repaired or imperfectly repaired. The lease contract of Case (i) considers two imperfect repairs at most under the contract. More general case which allows more than two imperfect repairs under the contract is interesting to research and this is a challenging topic for further research.

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