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# **Full Paper**

#### LOAD CAPACITY COEFFICIENT **EVALUATION** OF THREE-LAYERED JOURNAL BEARING WITH SLIP

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# Graphical abstract



### Abstract

Analysis of three-layered journal bearing with slip on bearing surface is presented. A modified classical Reynolds equation is derived for slip on bearing surface taking into consideration of bearing surface, core and journal surface layers. The modified Reynolds equation is derived taking into consideration of lubricant layer's film thickness, viscosities and slip on the bearing surface. Navier slip boundary conditions are used to analyze slip. Results of load capacity coefficient are presented for three-layered and two-layered journal bearing with slip. The load capacity coefficient decreases with bearing surface with slip. For a three-layered journal bearing with slip, high viscosity bearing surface layer results in higher load capacity coefficient.

Keywords: Journal bearing; surface layer; core layer; load capacity coefficient

#### Abstrak

Analisis tiga lapisan galas jurnal dengan slip pada permukaan galas dibentangkan. Satu persamaan Reynolds klasik diubahsuai untuk slip pada permukaan galas dengan mengambil kira permukaan galas, teras dan permukaan jurnal. Persamaan Reynolds yang diubah suai mengambil kira ketebalan filem lapisan pelincir, kelikatan dan slip pada permukaan galas. Syarat sempadan slip Navier digunakan untuk menganalisis slip. Keputusan pekali kapasiti beban dibentangkan untuk tiga lapisan dan dua lapisan galas jurnal dengan slip. Pekali kapasiti beban berkurangan pada permukaan galas dengan slip. Untuk tiga lapisan galas jurnal dengan slip, lapisan permukaan galas berkelikatan tinggi menghasilkan pekali kapasiti beban yang lebih tinggi.

Kata kunci: Galas jurnal; lapisan permukaan; lapisan teras ; pekali kapasiti beban

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# **1.0 INTRODUCTION**

Bearing with slip surface influences thin film lubrication analysis of journal bearing. The effective viscosity of adsorbed layer in thin film lubrication is greater than the bulk Newtonian viscosity. Tichy [1] studied the effect of lubricant microstructure in thin film lubrication using surface layer of higher viscosity that adheres to solid surfaces and derived modified Reynolds equation for convergent wedge contact. Meurisse

and Espejel [2] also presented Reynolds equation for a three-layered film model in thin film lubrication. Analysis of load capacity and coefficient of friction in layered journal bearing is influenced by dynamic viscosity and thickness of fluid film layers. Szeri [3] investigated composite film journal bearing configuration with high viscosity lubricant for load support and low viscosity lubricant for friction reduction. The composite film journal bearing consists of immiscible high and low viscosity fluid layers adjacent to bearing and journal respectively. Nabhan et al. [4] analyzed composite film slider bearing and derived modified Reynolds equation for convergent wedge contact.

Spikes [5] investigated half-wetted bearing and analyzed the influence of bearing slip on hydrodynamic lubrication. Fortier and Salant [6], analyzed the extent of slip/no-slip on bearing surface that results in low friction and high load support in journal bearings. Tauviqirrahman et al. [7] investigated the influence of slip in texture region of slider bearing for higher load capacity and lower friction. Li et al. [8] derived the extended Reynolds equation using power-law fluid lubrication with Navier-slip boundary conditions. Rao et al. [9] investigated the effects of partial slip bearing configuration on load capacity and friction coefficient for two-layered journal bearing.

This study investigates the influence of load capacity coefficient for three-layered journal bearing with slip surface based on one-dimensional analysis. The parameters analyzed are: dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid) ( $\beta_s$ ), dynamic viscosity ratio of bearing core layer to journal surface layer (base fluid) ( $\beta_c$ ), journal surface layer (base fluid) ( $\beta_c$ ), journal surface layer thickness ratio ( $\gamma_c$ ) and nondimensional slip coefficient ( $\lambda$ ).

### 2.0 ANALYSIS

A three-layered lubricant film with slip on bearing surface is presented in Figure 1. The three-layered film consists of bearing surface layer, core layer and journal surface layer (base fluid) occupying the clearance space. The dynamic viscosities of bearing adsorbent surface and core layers are considered to be higher than the journal adsorbent surface layer (base fluid). A modified classical Reynolds equation for a three-layered journal bearing is derived considering slip on bearing surface. The load capacity coefficient ( $C_w$ ) is determined based on composite film journal bearing analysis [3].





Based on variation of pressure along the circumferential (sliding) direction, and neglecting

variation of pressure along the radial (film thickness) direction, the momentum equations for velocity components in each layer are simplified as

$$\begin{aligned} \frac{dp}{dx} &= \mu \frac{d^2 u_1}{dy^2} \text{ in } 0 \le y \le (h - h_c - h_s), \\ \frac{dp}{dx} &= \mu_c \frac{d^2 u_2}{dy^2} \text{ in } (h - h_c - h_s) \le y \le (h - h_s), \\ \text{ and } \quad \frac{dp}{dx} &= \mu_s \frac{d^2 u_3}{dy^2} \text{ in } (h - h_s) \le y \le h \end{aligned}$$
(1)

The velocity boundary conditions the journal surface, at the journal surface (base fluid) and core layer interface, at the core and surface layer interface, and at the bearing surface in a threelayered a journal bearing are

$$y = 0, u_1 = u_j,$$
  
at  $y = (h - h_c - h_s), u_1 = u_2 = u_{12}, \mu \frac{du_1}{dy} = \mu_c \frac{du_2}{dy},$   
at  $y = (h - h_s), u_2 = u_3 = u_{23}, \mu_c \frac{du_2}{dy} = \mu_s \frac{du_3}{dy}$   
and at  $y = h, u_3 = -b \frac{du_3}{dy}$  (2)

Integrating Eq. (1) using the boundary conditions in Eq. (2), the velocity components are derived as

$$U_{1} = \frac{1}{2} \frac{dP}{d\theta} H^{2} Y(Y - Y_{1}) + (U_{12} - 1) \frac{Y}{Y_{1}} + 1$$
(3)  

$$U_{2} = \frac{1}{2\beta_{c}} \frac{dP}{d\theta} H^{2} (Y - Y_{1}) (Y - Y_{2}) - U_{12} \left(\frac{Y - Y_{2}}{Y_{2} - Y_{1}}\right) + U_{23} \left(\frac{Y - Y_{1}}{Y_{2} - Y_{1}}\right)$$
(4)  

$$U_{3} = \frac{1}{2\beta_{s}} \frac{dP}{d\theta} H^{2} \left( (Y^{2} - 1 - 2\lambda) - \frac{(Y - 1 - \lambda)(1 - Y_{2}^{2} + 2\lambda)}{(1 - Y_{2} + \lambda)} \right) + U_{23} \left(\frac{1 - Y + \lambda}{1 - Y_{2} + \lambda}\right)$$
(5)

where

$$Y_1 = 1 - \gamma_c - \gamma_s, \quad Y_2 = 1 - \gamma_s \tag{6}$$
$$U_{12} = F_1 - H^2 \frac{dP}{F_2}, \quad U_{22} = F_2 - H^2 \frac{dP}{F_4} F_4 \tag{7}$$

$$F_{1} = \frac{a\theta}{E_{12}E_{132}}, \qquad F_{2} = \frac{E_{22}E_{131} - E_{12}E_{231}}{E_{11}E_{22} - E_{12}E_{21}}, \qquad F_{3} = \frac{-E_{21}E_{132}}{E_{11}E_{22} - E_{12}E_{21}}, \qquad F_{4} = \frac{-E_{21}E_{131} + E_{11}E_{231}}{E_{11}E_{22} - E_{12}E_{21}} \qquad (8)$$

$$E_{11} = \frac{1}{Y_{1}} + \frac{\beta_{c}}{Y_{2} - Y_{1}}, \qquad E_{12} = E_{21} = -\frac{\beta_{c}}{Y_{2} - Y_{1}}, \qquad E_{22} = \frac{\beta_{c}}{Y_{2} - Y_{1}} + \frac{\beta_{s}}{1 - Y_{2} + \lambda} \qquad (9)$$

$$E_{131} = \frac{Y_2}{2}, \ E_{132} = \frac{1}{Y_1}, \quad E_{231} = \frac{1}{2} \left( 1 - Y_1 + \frac{(1 - Y_2)\lambda}{(1 - Y_2 + \lambda)} \right)$$
(10)

Using Eqs. (3) – (5), in equation of continuity for steady flow, yields the modified one-dimensional classical Reynolds equation as

$$\frac{d}{d\theta} \left( \frac{\Delta_s H}{2} - \frac{\Delta_p H^3}{12} \frac{dP}{d\theta} \right) = 0 \tag{11}$$

where

$$\Delta_{s} = (F_{1}+1)Y_{1} + (F_{1}+F_{3})(Y_{2}-Y_{1}) + F_{3}(1-Y_{2})\frac{(1-Y_{2}+\lambda)}{(1-Y_{2}+\lambda)}$$
(12)

$$\Delta_p = Y_1^3 + \frac{(Y_2 - Y_1)^3}{\beta_c} + \frac{(1 - Y_2)^3}{\beta_s} \frac{(1 - Y_2 + \lambda)}{(1 - Y_2 + \lambda)} + 6F_2Y_1 + 6(F_2 + F_4)(Y_2 - Y_1) + 6F_4(1 - Y_2)\frac{(1 - Y_2 + 2\lambda)}{(1 - Y_2 + \lambda)}$$
(13)

For 
$$Y_2 = Y_1$$
, Eqs. (12)-(13) reduce to

$$\Delta_s = (G_1 + 1)Y_1 + G_1(1 - Y_1)\frac{(1 - Y_1 + 2\lambda)}{(1 - Y_1 + \lambda)}$$
(14)

$$\Delta_p = Y_1^3 + \frac{(1-Y_1)^3}{\beta_s} \frac{(1-Y_1+4\lambda)}{(1-Y_1+\lambda)} + 6G_2Y_1 + 6G_2(1-Y_1)\frac{(1-Y_1+2\lambda)}{(1-Y_1+\lambda)}$$
(15)

where

$$G_1 = \frac{(1-Y_1+\lambda)}{(1-Y_1+\lambda+\beta_s Y_1)}, \ G_2 = \frac{Y_1}{2} \left( \frac{Y_1(1-Y_1+\lambda)+(1-Y_1)(1-Y_1+2\lambda)}{(1-Y_1+\lambda+\beta_s Y_1)} \right)$$
(16)

For  $\lambda = 0$ , Eqs. (14)-(15) reduce to  $\Delta_s = \frac{1-Y_1^2 + \beta_s Y_1^2}{1-Y_1 + \beta_s Y_1}$  and  $\Delta_p = Y_1^3 + \frac{(1-Y_1)^3}{\beta_s} + \frac{3Y_1(1-Y_1)}{1-Y_1 + \beta_s Y_1}$  and for  $Y_1 = 0$ , Eqs. (14)-(15) reduce to  $\Delta_s = \frac{1+2\lambda}{1+\lambda}$  and  $\Delta_p = \frac{1+4\lambda}{1+\lambda}$ .

The non-dimensional pressure gradient and pressure are expressed as

$$\frac{dP}{d\theta}(0 \le \theta \le \theta_r) = \frac{\Delta_s}{\Delta_p} \left[\frac{6}{H^2} - \frac{12c}{H^3}\right]$$
(17)

$$P(0 \le \theta \le \theta_r) = \frac{\Delta_s}{\Delta_p} \left[ \int_0^\theta \frac{6}{H^2} d\theta - \int_0^\theta \frac{12c}{H^3} d\theta \right]$$
(18)

The radial and tangential nondimensional load capacity are expressed as

 $W_{\varepsilon} = -\int_{0}^{\theta_{r}} P \cos\theta \ d\theta$  and  $W_{\phi} = \int_{0}^{\theta_{r}} P \sin\theta \ d\theta$ The nondimensional load capacity is (19)

$$W = \sqrt{W_{\varepsilon}^2 + W_{\phi}^2} \tag{20}$$

The load capacity coefficient  $(C_w = \Delta_s / \Delta_p)$  is expressed as the ratio of the nondimensional load capacity of three-layered journal bearing with slip on bearing surface to the nondimensional load capacity of journal bearing lubricated with base fluid of uniform viscosity film.

# 3.0 RESULTS AND DISCUSSION

The parameters considered in the analysis of threelayered lubricant film with slip on bearing surface are: dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid) ( $\beta_s=1$ , 10); dynamic viscosity ratio of core layer to journal surface layer (base fluid) ( $\beta_c$ =1, 10); journal surface layer thickness ratio ( $Y_1$ =0.001); core layer thickness ratio ( $\gamma_c$ =0.001-0.998) and nondimensional slip coefficient ( $\lambda$ =0.001-1.0). Results of load capacity coefficient  $(C_w)$  are presented for three-layered and two-layered film configurations. The load capacity coefficient is significantly influenced by dynamic viscosity ratio of bearing surface layer and core layer to journal surface layer (base fluid) ( $\beta_s$  and  $\beta_c$ ), and thickness ratio of core and journal surface layer ( $\gamma_c$  and  $Y_1$ ).

Figures 2a-2b depict load capacity coefficient  $(C_w)$  of three-layered journal bearing with slip. Figure 2a shows the variation of load capacity coefficient  $(C_w)$  with core layer thickness ratio  $(\gamma_c)$  of 0.001-0.998 for the journal surface layer thickness ratio  $(Y_1)$  of 0.001 and nondimensional slip coefficient ( $\lambda$ ) of 0.001. As shown in Fig. 2a, the load capacity coefficient  $(C_w)$  is significantly influenced by higher dynamic viscosity ratio of (i) bearing surface layer to journal surface layer (base fluid) ( $\beta_s$ =10) of core layer thickness ratio ( $\gamma_c$ ) of 0.001 and (ii) core surface layer to journal surface layer (base fluid) ( $\beta_c$ =10) of core layer thickness ratio ( $\gamma_c$ ) of 0.998. The load capacity coefficient  $(C_w)$  significantly increases with increase in dynamic viscosity of fluid film.

The influence of variation in nondimensional slip coefficient ( $\lambda$ ) for higher dynamic viscosity ratio of bearing surface and core layers on the load capacity

coefficient is shown in Fig. 2 (b). The load capacity coefficient  $(C_w)$  decreases with increase in nondimensional slip coefficient ( $\lambda$ ) from 0.001-1. In the range of nondimensional slip coefficient ( $\lambda$ ) from 0.001-1, higher dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid)  $(\beta_s=10)$  results in higher load capacity coefficient  $(C_w)$ . Higher dynamic viscosity of fluid film results in greater resistance to fluid flow on a bearing with slip surface.



Figure 2 Load capacity coefficient ( $C_w$ ) of three-layered journal bearing with slip

The influence of two-layered journal bearing with slip on load capacity coefficient  $(C_w)$  is presented in Figs. 3a-3b. Figure 3a shows the variation of load capacity coefficient ( $C_w$ ) with journal surface layer thickness ratio  $(Y_1)$  of 0.001-0.99 and nondimensional slip coefficient ( $\lambda$ ) of 0.001. The load capacity coefficient  $(C_w)$  for higher dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid) ( $\beta_s$ =10) significantly decreases with decrease in journal surface layer thickness ratio  $(Y_1)$  from 0.001 to 0.1.

The influence of variation in nondimensional slip coefficient ( $\lambda$ ) for higher dynamic viscosity ratio of bearing surface layer on the load capacity coefficient is shown in Fig. 3 (b). The load capacity coefficient ( $C_w$ ) decreases with increase in nondimensional slip coefficient ( $\lambda$ ) from 0.001-1. The decrease in load capacity coefficient ( $C_w$ ) for higher dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid) ( $\beta_s$ =10) in the range of nondimensional slip coefficient ( $\lambda$ ) from 0.1-1 is higher than 0.001-0.1.



(b)  $Y_1 = 0.001$ 

Figure 3 Load capacity coefficient ( $C_w$ ) of two-layered journal bearing with slip

#### 4.0 CONCLUSION

The present study examines the load capacity coefficient for a one-dimensional three-layered journal bearing with slip on bearing surface. The load capacity coefficient  $(C_w)$  significantly (i) decreases with core layer thickness ratio  $(\gamma_c)$  in the range of 0.001-0.1 for higher dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid)  $(\beta_s=10)$  and (ii) increases with core layer thickness ratio  $(\gamma_c)$  in the range of 0.9-0.998 for higher dynamic viscosity ratio of core layer (base fluid) ( $\beta_c=10$ ) for nondimensional slip coefficient  $(\lambda)$  of 0.001. The load capacity coefficient  $(C_w)$ 

decreases with increase in nondimensional slip coefficient ( $\lambda$ ) from 0.001-1.

Higher dynamic viscosity ratio of bearing surface layer to journal surface layer (base fluid) ( $Y_1$ =0.001,  $\gamma_c$ =0.001,  $\beta_c$ =1,  $\beta_s$ =10) results in higher load capacity coefficient ( $C_w$ ) compared to higher dynamic viscosity ratio of core layer to journal surface layer (base fluid) ( $Y_1$ =0.001,  $\gamma_c$ =0.998,  $\beta_c$ =10,  $\beta_s$ =1).

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#### Nomenclauture

- b Slip length, m
- C Radial clearance, m
- C<sub>w</sub> Ratio of load capacity of three-layered lubricant film with slip on bearing surface to homogeneous film without slip
- $h, h_c, h_s$  Film thickness, thickness of core and bearing surface layer, m; H = h/C
- *p* Pressure distribution, N/m<sup>2</sup>;  $P = pC^2/\mu UR$

R Journal radius, m

- *u*<sub>1</sub>, *u*<sub>2</sub>, *u*<sub>3</sub> Velocity component along circumferential direction in journal surface, core and bearing surface layers respectively, m/s
- $u_j$  Shaft speed, m/s;  $u_j = \omega R$
- $u_j$  Shaft speed, m/s;  $u_j = \omega R$
- $W_{\varepsilon}, W_{\phi}$  Radial and tangential nonondimensional load capacity, N
- W Nondimensional load capacity, N
- x Coordinate along circumferential direction, m;  $\theta = x/R$
- y Coordinate along radial direction, m; Y = y/h
- *Y*<sub>1</sub>, *Y*<sub>2</sub> Journal surface layer thickness ratio, core and surface layer interface thickness ratio
- $\beta_s$ ,  $\beta_c$  Dynamic viscosity ratios of bearing surface and core layer to journal surface layer (base fluid);  $\beta_s = \mu_s/\mu$ ,  $\beta_c = \mu_c/\mu$
- $\gamma_s, \gamma_c$  Bearing surface and core layer thickness ratio;  $\gamma_s = h_s/h, \gamma_c = h_c/h$
- $\lambda$  Nondimensional slip coefficient;  $\lambda = b/h$
- ε Journal bearing eccentricity ratio
- $\mu_s, \mu_c, \mu$  Viscosity of bearing surface layer, core layer and journal surface layer, Ns/m<sup>2</sup>
- θ Angular coordinate measured from the direction of maximum film thickness in journal bearing
- $\theta_r$  Angular extent of fluid film
- $\omega$  Angular velocity of journal bearing, rad/s

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