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OPTIMAL CONTROLLER DESIGN FOR A RAILWAY VEHICLE SUSPENSION SYSTEM USING PARTICLE SWARM OPTIMIZATION

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Abstract. This paper presents the design of an active suspension control of a two-axle railway vehicle using an optimized linear quadratic regulator (LQR). The control objective is to minimize the lateral displacement and yaw angle of the wheelsets when the vehicle travels on straight and curved tracks with lateral irregularities. In choosing the optimum weighting matrices for the LQR, the Particle Swarm Optimization (PSO) method has been applied and the results of the controller performance with weighting matrices chosen using this method is compared with the commonly used, trial and error method. The performance of the passive and active suspension has also been compared. The results show that the active suspension system performs better than the passive suspension system. For the active suspension, the LQR employing the PSO method in choosing the weighting matrices provides a better control performance and a more systematic approach compared to the trial and error method.

Keywords: active suspension control, two-axle railway vehicle, linear quadratic regulator, particle swarm optimization

1.0 INTRODUCTION

Solid-axle wheelset is the most commonly used wheelset on nearly all modern railway vehicles. It consists of two wheels rigidly connected by an axle, so that both wheels rotate at the same angular speed, providing the wheelset with the ability to negotiate curves naturally. This however becomes the primary reason of its instability [1] and has caused a conflict in the design of the vehicle's primary suspension system that deals with its running stability. Longitudinal spring connection between the wheelset and the vehicle body/bogie, which is required to stabilise the wheelset and forms part of the primary suspension system, interferes with the wheelset natural curving action causing poor steering when curves are negotiated. Various strategies have been developed for the primary suspension system to solve the difficult design trade-off

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between maintaining the vehicle's dynamic stability at high speeds and its curving performance using mechanical elements, such as those discussed in [2], [3] and [4]. However, none of these developments have fundamentally overcome the problem.

To deal with the problems described above, active control must come into play. The active control strategy developed in this paper is the linear-quadratic regulator (LQR) that utilizes the Particle Swarm Optimization (PSO) method in selecting the optimum values for the weighting factors for the LQR, which results in an optimised LQR. LQR is chosen as a controller for this system since its design procedure is more systematic and can cater for high order systems such as the railway vehicle system. LQR design involves the selection of some weighting factors for the cost function to be minimised. However, there is no specific technique in choosing these values. The common approach to selecting these weighting matrices is via trial and error but this method could be time consuming, cumbersome and result in a non-optimised performance [5]. Therefore, a more systematic approach in selecting these values is needed. This paper proposes the use of PSO to find the values of the LQR weighting matrices based on a certain criterion that would ensure satisfactory control performance of the two-axle railway vehicle system. The objective of the controller is to minimize the lateral displacement of the wheelset relative to track centerline and its yaw angle, on straight and curved tracks with lateral irregularities.

2.0 TWO-AXLE RAILWAY VEHICLE

Figure 1 shows a plan view diagram of the two-axle vehicle used in the study. This particular vehicle configuration has drawn attention from the railway industry [6]. It mainly consists of a vehicle body and two wheelsets, connected together via the primary and secondary suspensions, resulting in a system of 12th order differential equation.

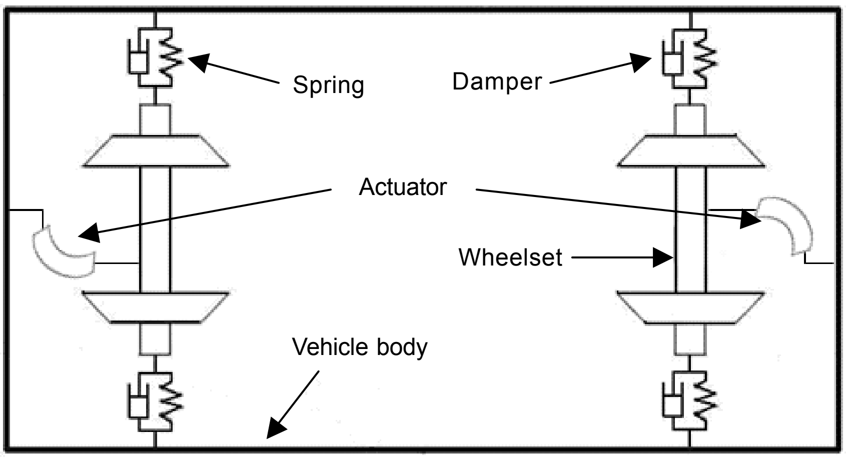

Figure 1 Plan view of a two-axle vehicle

The wheelsets are mounted onto the vehicle body via the springs and dampers, whose values have been chosen appropriately in order to provide good curving performance and stability. The wheelset was controlled using an established technique of active yaw damping in which a rotary actuator replaced the longitudinal springs to provide yaw torque [7].

The mathematical descri ption of the two-axle vehicle can be described by Eq.(1) to Eq.(6) [8]. The symbols and parameters used in the equations are given in the Appendix.

$$m\ddot{y}_F = -\left(\frac{2f_{22}}{v} + C_w\right)\dot{y}_F - K_w y_F + 2f_{22}\psi_F + C_w\dot{y}_B + K_w y_B + C_w l_h\dot{\psi}_B + K_w l_h \psi_B + \frac{mv^2 + 2f_{22}l_h}{R_F} - mg\theta_{cF} \tag{1}$$

$$I_w\ddot{\psi}_F = \frac{-2f_{11}l\lambda}{r}y_F - \frac{2f_{11}l^2}{v}\dot{\psi}_F + \frac{2f_{11}l\lambda}{R_o}y_{iF} + u_F + \frac{2f_{11}l^2}{R_F} \tag{2}$$

$$m\ddot{y}_R = -\left(\frac{2f_{22}}{v} + C_w\right)\dot{y}_R - K_w y_R + 2f_{22}\psi_R + C_w\dot{y}_B + K_w y_B - C_w l_b\dot{\psi}_B - K_w l_b \psi_B + \frac{mv^2 + 2f_{22}l_b}{R_R} - mg\theta_{cR} \tag{3}$$

$$I_w\ddot{\psi}_R = \frac{-2f_{11}l_b\lambda}{r}y_R - \frac{2f_{11}l_b\lambda}{r}y_{tR} + \frac{2f_{11}l_b^2}{v}\dot{\psi}_R + u_R + \frac{2f_{11}l_b^2}{R_R} \tag{4}$$

$$m_v\ddot{y}_B = C_w\dot{y}_F + K_w y_F + C_w\dot{y}_R + K_w y_R - 2C_w\dot{y}_B - 2K_w y_B + \frac{m_v v^2}{2}\left(\frac{1}{R_F} + \frac{1}{R_R}\right) - \frac{m_v g}{2}(\theta_{cF} + \theta_{cR}) \tag{5}$$

$$I_v\ddot{\psi}_B = C_w l_b\dot{y}_F + K_w l_b y_F - C_w l_b\dot{y}_R - K_w l_b y_R - 2l_b^2 C_w\dot{\psi}_B - 2l_b^2 C_w \psi_B - u_F - u_R \tag{6}$$

The mathematical equations can also be written in state-space form as in Eq.(7).

$$\underline{\dot{x}} = A\underline{x} + B\underline{u} + G\underline{w} \tag{7}$$

where;

A = system matrix, with the matrix dimension of 12×12

B = control input matrix, with the matrix dimension of 12×2

G = disturbance matrix, with the matrix dimension of 12×6
x = state vector of dimension 12×1
u = input or control vector od dimension 2×1
w = track disturbance input of dimension 6×1

$$x = \begin{bmatrix} yY_F & y_F & \psi Y_F & \psi_F & yY_{\mathrm{R}} & y_R & \psi Y_R & \psi_R & yY_B & y_B & \psi Y_B & \psi_B \end{bmatrix}^t$$

$$u = \begin{bmatrix} u_F & u_R \end{bmatrix}^T$$

$$w = \begin{bmatrix} y_{tF} & \dfrac{1}{R_F} & \theta_{cF} & y_{tR} & \dfrac{1}{R_R} & \theta_{cR} \end{bmatrix}^T$$

To study the performance of the active suspension system, the vehicle is subjected to two types of track disturbance inputs: the track curvature with a certain degree of cant angle and the lateral irregularities. The track inputs used in this study are as depicted in Figure 2 through Figure 4. The radius of the track curvature is 1500 meter and the cant is 0.12 radian (7 degree) [9]. The cant is the intentionally introduced superelevation on the curved track, where the outer rail is raised above the level of the inner rail by certain amount to reduce the centrifugal force transmitted by the vehicle on the rail when it traverses the curve. The transition period is 1.5 second before and after the curve, during which the track curvature and the cant are gradually increased or decreased to reduce passenger's discomfort when the vehicle entered (at t = 2 second) and left (at t = 7 second) the curve.

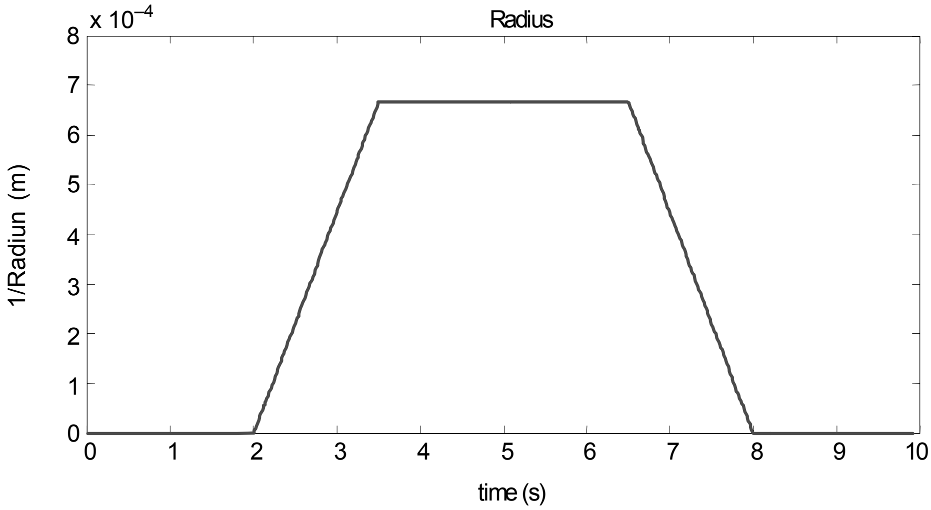

Figure 2 Curve input (1/Radius)

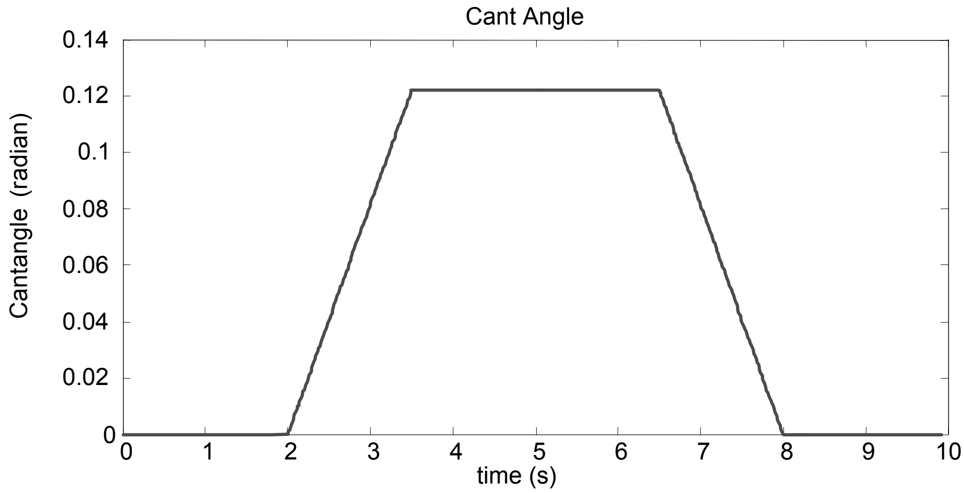

Figure 3 Cant input

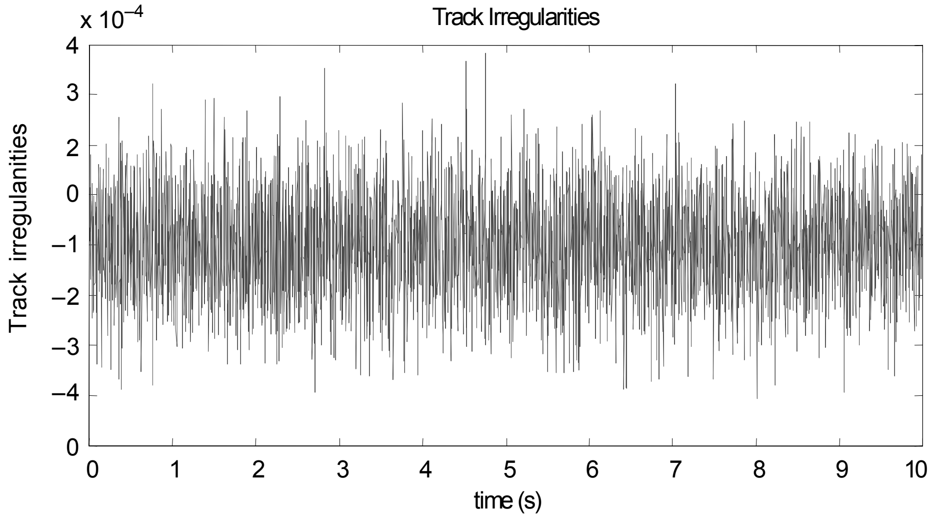

Figure 4 Track irregularities

3.0 LINEAR QUADRATIC REGULATOR (LQR)

The general active control scheme of the railway vehicle is shown in Figure 5 [10]. The inputs of the system are the track inputs, which are the curve radius, the cant angle and lateral track irregularities. Sensors are used to monitor the outputs of the system such as the lateral displacement and yaw angle to provide measurements to the controller. The controller calculates the suitable amount of control action to be applied to the system and the actuators implement this control action. In this work, the wheelset was

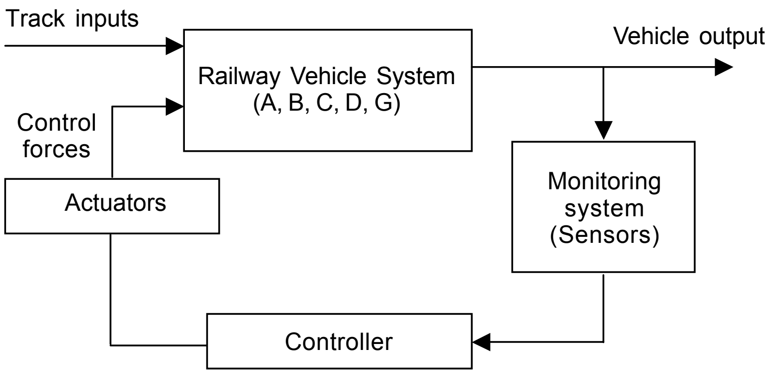

Figure 5 Active Control scheme

controlled using an established technique of active yaw damping in which a rotary actuator replaced the longitudinal springs to provide yaw torque.

When designing the LQR for the railway vehicle system, the system is assumed to be linear and has the state equation in the form below:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{8}$$

$$y = C\underline{x} \tag{9}$$

The controller determines a vector $u(t)$, which will force the behavior of the system under control to minimize the cost function given in Eqn.(10).

$$J = \int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)dt \tag{10}$$

The control law is given by Eqn.(11) and Eqn.(12).

$$u(t) = -K(t)\underline{x}(t) \tag{11}$$

where

$$K(t) = -R^{-1}B^T(t)P_r(t) \tag{12}$$

$P_r(t)$ is obtained by solving the Ricatti equation given in (13).

$$\dot{P}_r(t) = -P_r(t)A(t) - A^T(t)P_r(t) - Q + P_r(t)B(t)R^{-1}B^T(t)P_r(t) \tag{13}$$

The overall aim of the primary suspension control strategy is to avoid flange contact in normal running because this is what causes high wear on the wheel and rail. Therefore, the lateral displacement of the wheelset relative to the track and its yaw angle are the main concern, and the cost function of the LQR needs be modified. For these cases where direct control of certain states of the system state vector, y_{LQ}, is required, a performance index such Eq. (14) is minimized.

$$J_{LQ} = \int_0^\infty \left(y_{LQ}^T(t) \cdot Q \cdot y_{LQ}(t) + u_{LQ}^T(t) \cdot R \cdot u_{LQ}(t) \right) dt \tag{14}$$

where $J_{LQ} = [y_F \ \psi_F \ y_R \ \psi_R]^T$. The weighting matrices, Q and R are important to get a good optimal controller. In this paper, the particle swarm optimization (PSO) is used to assist the selection of the most optimum weighting matrices.

4.0 PARTICLE SWARM OPTIMIZATION (PSO)

The idea on particle swarm optimization (PSO) was first introduced in 1995 [11]. It is a population-based search algorithm inspired by the social behavior of birds, bees or a school of fishes. Each individual within the swarm is represented by a particle in search space. This particle has one assigned vector, which determines the next movement of the particles, called the velocity vector. Each particle searches for global optima by updating the velocity and position of each particle. The PSO algorithm is initialized with iteration = 0, velocity = 0 and position = random position with population optimization technique, in which individuals (particles) "fly" through a multidimensional search space [12].

Each of the particles placed in the problem space has a fitness value evaluated by an objective (fitness) function to be optimized at its current location at every iteration. The particles in a local neighborhood share memories of their best visited positions. Then, it will use those memories to adjust their own velocities and positions based on the Eq. (15) and Eq. (16) [13]. All particles directly fly through the problem space by following the current optimum particles.

$$v_i(t+1) = wv_i(t) + c_1\varphi_1\left(p_i - x_i(t)\right) + c_2\varphi_2\left(p_g - x_i(t)\right) \tag{15}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{16}$$

where;

c_1, c_2 = positive constant

φ_1, φ_2 = random variable with uniform distribution between 0 and 1.

w = inertia weight which shows the effect of previous velocity vector on the new vector.

p_i = The particle position (previous) that resulted in the best fitness so far.

p_g = the neighborhood position that resulted in the best fitness so far.

Each particle remembers its previous best visited position, which is the position obtained when the particle achieves the best fitness function. This position is designated as personal best, p_i, and the fitness value for this position is then stored. The PSO algorithm also tracks the best position obtained by the particle in the swarm. This value is called the global best position, p_g.

In this paper, the PSO technique is used to find the most optimum weighting matrices (Q and R) for the linear quadratic controller to minimize the lateral displacement and yaw angle of the railway wheelset described in section 2. Q and R are the entries that form a particle in the PSO algorithm and are chosen to be diagonal matrices with positive real elements, for simplicity. First, the position and velocity of each particle is initialized. Then, for each particle, the value of $P_r(t)$ is solved from the algebraic Ricatti equation given in Eq.(13) and the feedback gain K is calculated based on Eq.(12). The fitness of each individual in the population is then calculated using the current position value. The fitness value calculation is based on the integrated absolute error (ISE) between the desired and actual values of the states to be controlled,

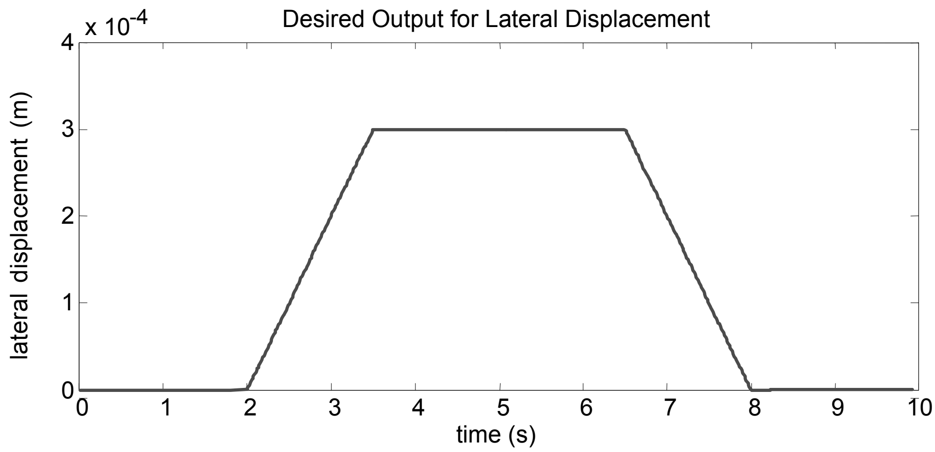

Figure 6 Desired output for wheelset lateral displacement

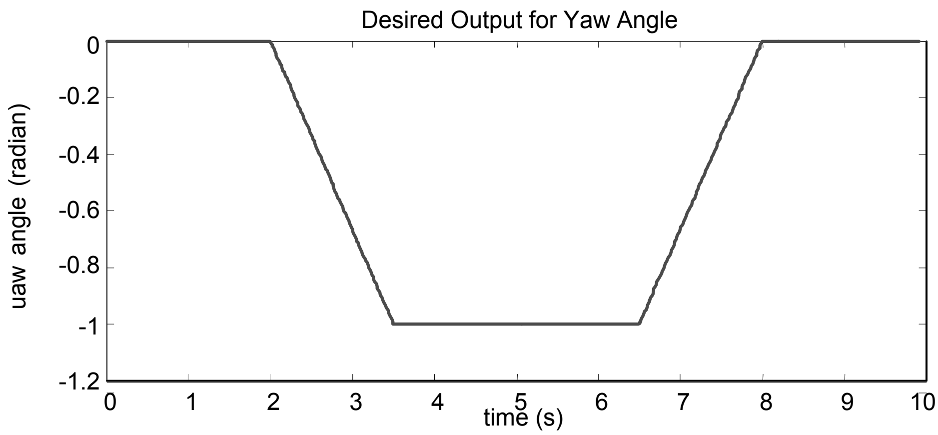

Figure 7 Desired output for yaw angle

$ISE = \sum\left(\underline{y}_{desired}(i) - \underline{y}_{actual}(i)\right)^2$, where $\underline{y}_{desired}$ and $\underline{y}_{actual}$ are the desired and actual trajectory of the states to be controlled respectively. The desired trajectory are shown in Figure 6 for the wheelset lateral displacement, and Figure 7 for the wheelset yaw angle. If the present *ISE* value is less than the previous best *ISE* value, the current position is set as the new p_i. Otherwise, the position with the previous best *ISE* value remains as p_i. The particle with the minimum *ISE* value of all the particles is then chosen as p_g. For each particle, the velocity is calculated using Eq.(15) and the particle position is updated according to Eq.(16). The PSO algorithm also can describe with flow chart. The flow chart is shown in Figure 8. The process is iterated until the stopping criteria, $ISE_{\min} = 6.615 \times 10^{-3}$, is met.

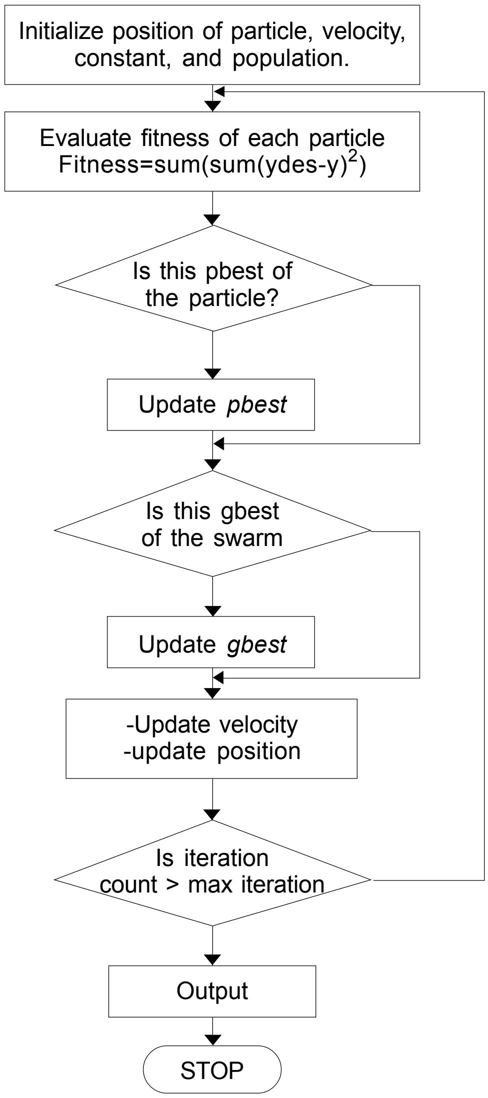

Figure 8 Flow chart for Particle Swarm Optimization

By running the PSO algorithm above, the optimal weighting matrices, Q_{PSO} and R_{PSO}, obtained are:

$$Q_{PSO} = \begin{bmatrix} 5.67 & 0 & 0 & 0 \\ 0 & 0.157 & 0 & 0 \\ 0 & 0 & 5.543 & 0 \\ 0 & 0 & 0 & 0.236 \end{bmatrix}, \quad R_{PSO} = \begin{bmatrix} 10^{-13} & 0 \\ 0 & 10^{-13} \end{bmatrix}$$

The results of the linear quadratic controller performance with weighting factors obtained from both the trial and error method and the PSO method are compared.

5.0 RESULTS AND DISCUSSIONS

The performance of the two-axle railway vehicle lateral suspension system controlled by a linear quadratic regulator that employ the trial and error method and the PSO method in choosing its weighting matrices, Q and R is discussed. A two-axle vehicle travelling along a curved track having a curve radius of 1500m and cant angle of 7 degree with track irregularities, as shown in Figure 2, Figure 3 and Figure 4, is considered. The curving performances of the passive and active suspension systems are compared and the importance of improving this curving performance is discussed. The curving performance is evaluated in terms of the railway wheelset lateral displacement and yaw angle only. Minimization of these values are important in ensuring that wheel flanges do not touch the rail track in any circumstances to reduce noise and wear of wheels and rails, and more importantly, to prevent derailment from occurring.

The lateral displacement of the front wheelset at v =18 ms^{-1} are shown in Figure 9. It can clearly be seen that the passive suspension system is unable to maintain the wheelset/vehicle stability when the vehicle encounters the curve, even at a low speed. Passive suspension system is a traditional suspension that utilizes passive components inside a railway vehicle's suspension system. These conventional passive components consist of coil or leaf springs, viscous dampers and several mechanical parts, such as linkages to support the vehicle mass and the load carried inside the vehicle body. The amplitude of oscillations of the lateral displacement increases with time. This could cause railway derailment unless the suspension parameters are chosen extremely carefully and the vehicle travels at very low speeds. On the other hand, the active suspension system employing the LQR is able to maintain the stability of the vehicle during curving. This shows that the two-axle vehicle configuration requires active primary suspension system in its operation.

The performance of the controller applying the trial and error method and the PSO method in choosing the LQR weighting matrices (Q and R) for the active suspension system are compared. Table 1 shows the Q and R values chosen for each method. For

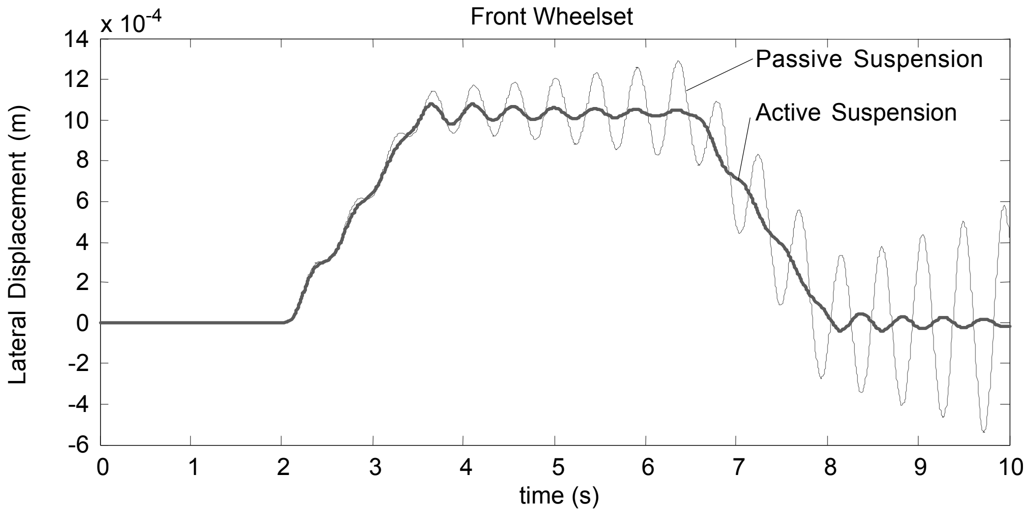

Figure 9 Lateral displacement for the front wheelset at v = 18 ms^{-1}

trial and error method, Q and R the values have been chosen so that satisfactory overall control performance is obtained for the system. The values have been obtained after a cumbersome process of trial and error method. On the other hand, the values for the PSO method are obtained after simply running the PSO algorithm in section 4 of this paper.

Table 1 LQR weighting matrices value

Approach	***Q***	***R***
Trial and error	*diag*[0.1 0.01 0.1 0.01]	*diag*[10^{-12} 10^{-12}]
PSO	*diag*[5.67 0.157 5.543 0.236]	*diag*[10^{-13} 10^{-13}]

Figure 10 and Figure 11 show the lateral displacement and yaw angle of the leading and trailing wheelsets for the two-axle vehicle travelling at 60 ms-1. It can clearly be seen from Figure 10 that the PSO method is able to provide better curving performance in response to the vehicle encountering the track curvature and lateral track irregularities. From Figure 10, the lateral displacement at steady curve is about 33% less than that produced by the trial and error method. In fact, the improvement can be further increased by reducing the stopping criteria of the PSO algorithm. Figure 11 shows that there is no significant difference in terms of the wheelset yaw angle for both methods.

Table 2 compares the curving performance of the railway vehicle for the two LQR weighting matrices selection methods, for different vehicle speeds. It can be seen

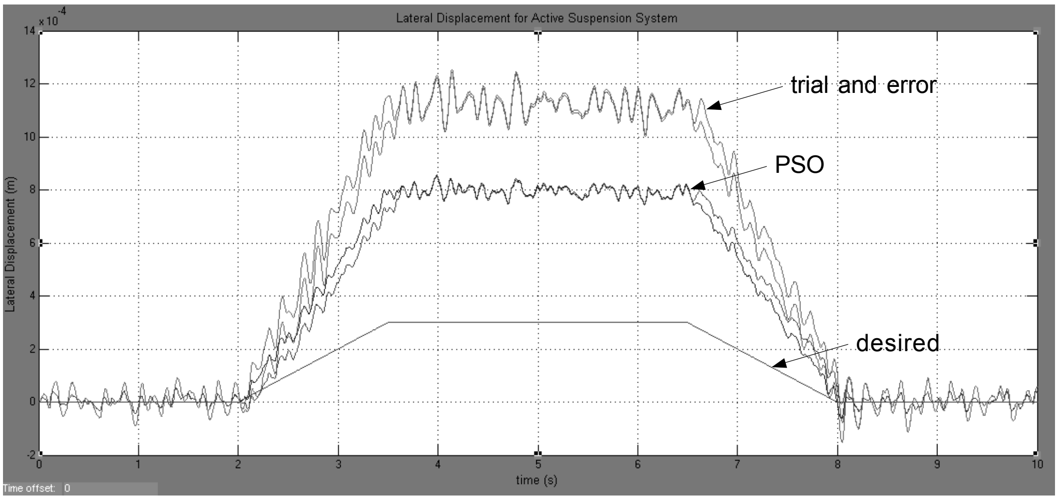

Figure 10 Wheelset lateral displacement at v = 60 ms^{-1}

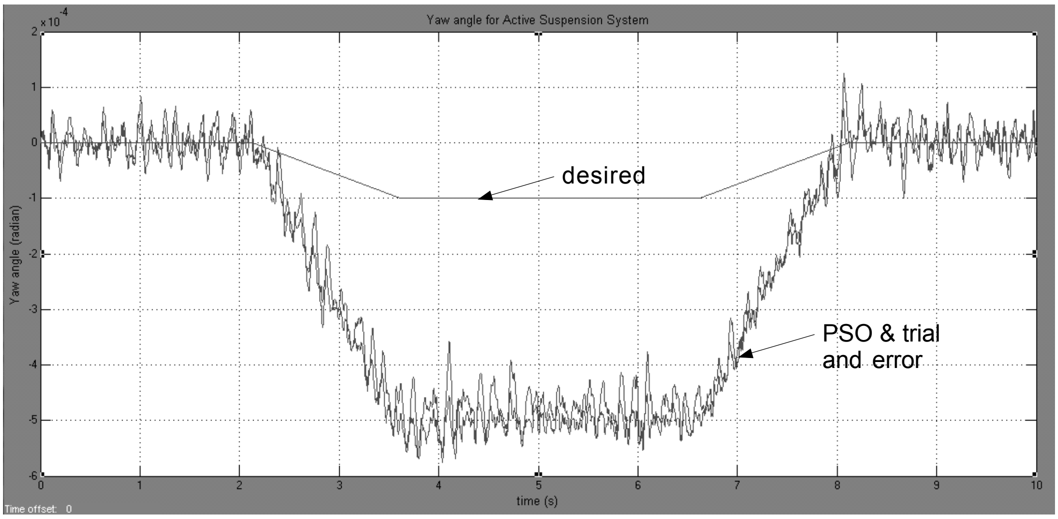

Figure 11 Wheelset yaw angle at v = 60 ms^{-1}

Table 2 Comparison of PSO and trial and error method

Vehicle velocity, v (ms^{-1})	Trial and Error		PSO	
	Maximum lateral displacement (mm)	Maximum yaw angle (mrad)	Maximum lateral displacement (mm)	Maximum yaw angle (mrad)
60	1.1	0.48	0.8	0.48
70	1.2	0.82	0.9	0.82
80	1.3	1.2	1.0	1.2

from Table 2 that for all speed values used in the simulation work, LQR with PSO method provides better curving performance in terms of wheelset lateral displacements. Since the maximum speed for normal railway vehicle is approximately 80 ms^{-1} (or 300 kmh^{-1}) it is chosen as the upper limit for this system.

6.0 CONCLUSION

The results presented in this paper have shown that the two-axle railway vehicle configuration requires active control of its suspension system to maintain its stability when traversing round a curved track, especially when the vehicle is to travel at high speeds.

The results for the active suspension system employing the LQR also show that, by using the Particle Swarm Optimization (PSO) technique to determine the optimum values for the LQR weighting matrices, the performance of active suspension system gives better result compare to the trial and error approach. Although the trial and error method is still generally acceptable, it could be time consuming and cumbersome, as well as not guaranteeing the selection of the most optimum LQR performance. On the other hand, PSO approach is more systematic and the control performance can be further improved by restricting various criteria in the algorithm.

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APPENDIX

Two-axle railway vehicle parameters

Symbol	Description	Value
y_F, y_R, y_B	Lateral displacement of front, rear wheelset and body	
ψ_F, ψ_R, ψ_B	Yaw displacement of front, rear wheelset and body	
v	Vehicle speed	60 m/s
m, m_v	Wheelset and vehicle mass	1250 kg, 13,500 kg
I_w, I_v	Wheelset and vehicle yaw inertia	700 kgm^2, kgm^2
l_B	Half spacing between two wheelset	3.7 m
K_w	Lateral stiffness per wheelset	230 kN/m
C_w	Lateral damping per wheelset	50 kN s/m
r	Wheel radius	0.45 m
λ	Conicity	0.2
l	Half gauge of wheelset	0.7 m
f_{11}	Longitudinal creep coefficient	10 MN
f_{22}	Lateral creep coefficient	10 MN
R_F, R_R	Radius of the curved track at the front and rearwheelsets	1500 m
θ_{CF}, θ_{CR}	Cant angle of the curved track at the front and rear wheelsets	
g	Gravity	
y_{tF}, y_{tR}	Track Lateral displacement irregularities at front (leading) and rear (trailing) wheelsets.	-
u_F, u_R	Controlled torque input for the front (leading) and rear (trailing) wheelsets.	-