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EMPIRICAL MODELLING OF ACTIVATED SLUDGE PROCESS VIA SYSTEM IDENTIFICATION

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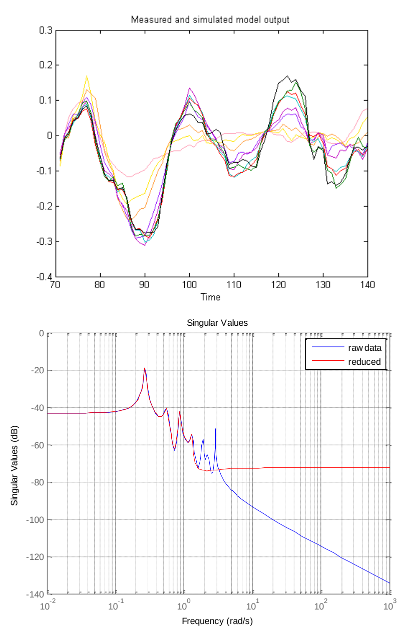
Graphical abstract



Abstract

Activated sludge process is an important stage in Wastewater Treatment Plant (WWTP). In this study, model of the activated sludge process from Bunus Regional Sewage Treatment Plant Kuala Lumpur, Malaysia is developed. This paper focuses on modelling and model reduction of the WWTP system. The model with best fits of higher than 80% and the order of less than 10 is selected. For modelling purposes, data obtained is stimulated and modelled using System Identification technique which employ linear model ARX. For model reduction purposes, the high order model is reduced using model order reduction (MOR) of a combination of Singular Perturbation Approximation (SPA) and Frequency Domain Gramian based Model Reduction (FDIG) method. From the modelling results obtained, the ARX model with best fit of 85.11% is selected. Meanwhile, for the MOR FD-SPA technique, a 9th order model is selected with $2.5\,x\,10^{-2}$ reduction error between frequencies 0.05 rad/s and 1.4 rad/s.

Keywords: Wastewater treatment plant, activated sludge process, system identification, singular perturbation approximation, frequency domain interval gramian

1.0 INTRODUCTION

Wastewater treatment can be defined as the process to remove harmful pollutants in the water so that the water is safe to be reused. The treated water serve many purposes such as for drinking water, industrial process and many more. The concept of wastewater has become an important topic as it deals with health and environment issues. Thus, the improvement of the process is developed from time to time to cope with the consistently change development.

In general, there are several processes involved in the wastewater treatment plant. The process such as biological and chemical treatment are important to ensure that the level of harmful element in the water is successfully removed. Activated sludge process which takes place in the secondary stage is the main focus in this research. The purpose of this research is to remove the level of harmful nitrate through nitrification and denitrification process [1].

The activated sludge process is introduced in the early 20th century. Currently, the process is a widely used technology to get rid of organic pollutants from the wastewater. The process is preferable because of it capability to produce high quality effluent [2]. In addition, the process is very reliable and flexible which can cope with any types of wastewater [3].

In a wastewater treatment plant (WWTP), any unwanted substances product from water sources is removed in order for it to be reused and supplied for other purposes. WWTP is a very important process since the removal of chemical waste products produces clean water to be reused. One of the functions of the system is to treat wastewater from agricultural activity. This includes the removal of residues such as pesticides and animal waste. Besides that, a WWTP system is also designed to process sewage waste water which include human and household wastes. WWTP is also widely used in industrial system whereby waste from various types of industries, such as mining and quarrying are removed.

A WWTP system can be divided into five processes, within three main which are primary treatment, secondary treatment and tertiary treatment. The basic concept of WWTP process is shown in Figure 1 below.

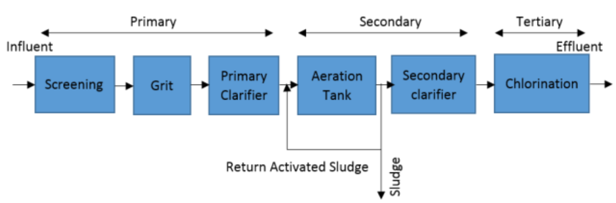


Figure 1 Block diagram of wastewater treatment process

Influent can be defined as the collected raw materials conveyed to the plant to be treated. The raw materials such as water and debris will flow through the collection system. Primary treatment or known as mechanical plant is the first stage of wastewater treatment process. A process called screening is designed to remove the large raw materials such as plastics, metals and papers. The raw materials may cause several damaging issues if it not fully removed, such as causing damage to the sludge removal equipment, block valves, nozzles, channels, pipelines and appurtenances. As a result, it may create serious plant operation and maintenance problem [1].

Small raw materials such as sand are removed by the grit removal system. Then, the remaining lighter materials are suspended in the water and conveyed to the large tanks, called primary classifiers. The purpose of the primary classifiers is to remove the primary sludge which settles in the bottom due to gravity effect. The primary sludge will then pass through the secondary treatment which is the biological stage. The biological process will remove the substances that escape the primary treatment and provide further removal of any suspended solids. The process is achieved by mixing the wastewater with controlled population of microorganisms with the presence of oxygen. The microorganisms consume the impurities as food and thus, removing them from the wastewater [4]. This process is called the activated sludge process, which will be elaborated further in the next section.

Due to the use of microorganisms in the previous stage, a process called disinfection is important to kill harmful organisms in the wastewater that may lead to various waterborne disease. Chlorine is widely used to kill the bacteria due to the fact that it is cheap, effective and available in large quantities, nontoxic in low concentration to higher form of life and builds up a residual [1]. Tertiary treatment or known as chemical stage is the process of removing nutrients remained after the biological treatment stage. These nutrients must be removed to ensure that the water quality is up to the set standard to reuse the wastewater before it can be reutilized to domestic, industrial and other purposes.

WWTP is a very complicated system which can make the analysis of the system time consuming and computationally expensive. In addition to that, complicated system may cause the inclusion of unnecessary information which may interfere with the stability of the original system. Therefore, a lower order system which provides a good approximation of the original system is required. From literature, the process of reducing the high order to lower orders is known as model order reduction. A good reduced order model retained the stability of the original system. A number of model order reduction technique have been proposed in the literature such as hankel norm approximation, balanced truncation and singular perturbation approximation (SPA) [5]. For all these methods, the stability of original system is preserved.

SPA in particular, is a model reduction method where the stability of the original system is preserved with zero steady state error [6-8]. Ideally, a reduction method should minimize the reduction error at the whole frequency range which is sometimes not feasible. However, for several system, the reduction error is more important over certain frequency interval than other frequencies. This motivated the author to utilize frequency weighted in the proposed model order reduction technique.

Enns [9] first implemented this theory with the balanced truncation method to include frequency weightings. However, Enn's methods only yields stability of the reduced order model with single weighting present. With both weighting present, the method may yield unstable reduced order model [10, 11]. To overcome the drawback of instability of the reduced order model, several other technique have been proposed such as Gawronski and Juang's method [10]. In [11], a new model reduction method based on theories of Gramians defined over a desired frequency interval gramians for continuous system is proposed. In this paper, the main focus will be on modeling of the WWTP system as well as model order reduction by utilizing SPA and Gawronski and Juang method.

2.0 ACTIVATED SLUDGE PROCESS

In the activated sludge system, the involvement of air or oxygen is included with the biological floc which composed of bacteria and protozoa. Activated sludge is referred to the mixture of microorganism and air which is clumped together to produce microbial floc [1]. A basic flow process for the activated sludge process is presented in Figure 2.

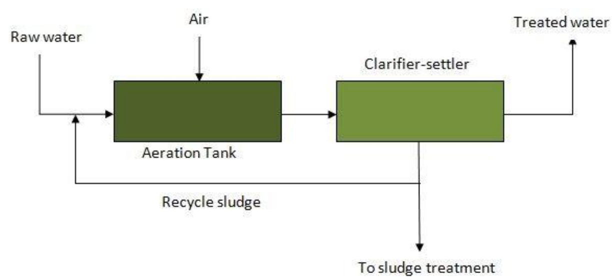


Figure 2 Basic flow process diagram of activated sludge process

In Figure 2, the combination of activated sludge with wastewater is called mixed liquor which flows from the aeration tank to a secondary clarifier (or settler). The clarifier is important to ensure that the activated sludge solids are separated from the wastewater. The sludge is settled at the clarifier for further treatment. A portion of the settled sludge is returned to the aeration basin to maintain a proper food-to-microorganisms (MO) to permit rapid breakdown of the organic matter [1].

The conventional activated sludge process utilizes aeration reactor which contain anoxic tank and aerobic tank. Numbers of required tanks is dependent on size of the distributed area. Several additions of tanks are required if the plant is to distribute wastewater in big cities. In this research, the model and data is obtained from Bunus Regional Sewage Treatment Plant (RSTP), Kuala Lumpur which is Malaysia's sewage company, as depicted in the block diagram in Figure 3.

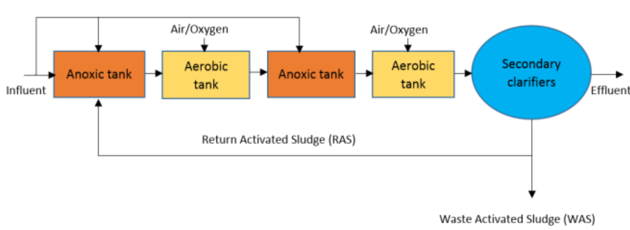


Figure 3 Bunus RSTP's activated sludge process

The block diagram Figure 3 is a real representation of IWK's sludge system. The sludge system is a conventional aerated anoxic system. The first stage of the activated sludge system is anoxic tank. Anoxic refers to the condition where the oxygen is depleted. Raw water entering the tank is combined with sludge that contains large number of microbes. The microorganisms will remove oxygen away from nitrate (NO3) and produce nitrogen gas.

Raw water entering the reactor has high level of ammonia gas (NH3). Oxygen (O2) is introduced to the aerobic tank which will cause the ammonia to convert to nitrite (NO2) and nitrate (NO3). The process is called nitrification. The process is continuous and a portion of the sludge will be returned to anoxic tank to undergo denitrification process [12].

It is important to ensure that the nitrate is fully converted to nitrogen gas before being released to the residential area. This is due to the fact that high level of nitrate will cause disease and causes damaging pollutants [13].

2.1 Modeling of Activated Sludge Process

Modelling of WWTP system provides better understanding of the development, prediction, control and monitoring of complicated system. An accurate model is crucial in process industries, which consists of process design, process control and process optimization [14]. The widely approach used is mathematical modelling [15-17], however a different approach is also available reported in the literature [18].

As previously mentioned, activated sludge process is extensively used in biological method, since the technology available is not costly [3]. Mathematical modelling of the activated sludge process provides a powerful tool for design, predicting future behavior of plants as well as controlling the processes involved [14]. Through computer simulation, the mathematical modelling approach can be conveniently represented as the activated sludge process.

There are several approaches of mathematical modelling found in literature, such as black – box model and white – box model. The black – box model is based on statistical models. From literature [14, 15], this approach has been proposed when analytical equations are unavailable or difficult to develop. In addition, the model is developed based on database approach [14]. For the white – box models, it is a mechanistic model. Generally, mechanistic models define the mechanisms behind the coupling of variables and used for almost any operating condition [15]. Further analysis on mathematical modelling approach in activated sludge process is presented in [14].

3.0 MODEL ORDER REDCUTION TECHNIQUE

Process of reducing high order system to lower order system is known as model order reduction. A good reduced order model retains the stability of the original system. Several literature have proposed a number of model order reduction technique which include hankel norm approximation, balanced truncation, singular perturbation approximation, frequency domain gramian, etc [5, 10]. All these methods preserved the stability of their original system. It has been found that, balanced truncation is simple to implement in contrast to the complex implementation of hankel norm approximation. In addition, both techniques does not approximate the steady state error of the system [5]. In this research, singular perturbation will be applied as a model order reduction, with emphasize on selected frequency range, which is realized by the frequency domain gramian technique.

3.1 Singular Perturbation Approximation

Singular perturbation approximation (SPA) is a non-projection method in MOR [5]. SPA is developed to reduce the complexity of a system. In addition, by utilizing this technique, the stability of the original system is preserved with zero steady state error [6-8]. Ideally, a reduction technique reduces a system in the whole frequency range. However, SPA only provides a good reduction error at low and median frequency [6, 7, 19]. The reduction error goes to zero at lower frequency and tend to increase at high frequency.

In the literature [8, 19], SPA is enhanced by the combination of MOR techniques applied in various higher dimensional systems. In [8] and [19] for example, the purpose of this combination technique is to obtain a stable reduced model with double sided frequency weights at low and high frequency. As mentioned previously, SPA produces good reduction error at lower frequency. Hence, a combination of other reduction method is required to reduce errors at higher frequencies. At present, several combination of SPA with other existing method [20, 21], shows that the system are stable with zero steady state error for low frequency systems.

Consider a linear, time- invariant system

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}u$$

$$y = [C_1 C_2]\begin{bmatrix}x\\ z\end{bmatrix} + Du \tag{1}$$

State variable can be partitioned into two subsystems where the strong subsystem represent slow state variable (autotrophs) and weak subsystem represent fast state variable (dissolved oxygen). Based on, SPA approach, z is a weak subsystem that can be approximated by setting z to zero [22, 23]. To achieve this, equation (1) is replaced by the following

$$\begin{aligned}\dot{x} &= A_{11}x + A_{12}z + B_1u\\ 0 &= A_{21}x + A_{12}z + B_2u\\ y &= C_1x + C_2z + Du\end{aligned} \tag{2}$$

Thus, the singularly perturbed reduced order system is given by

$$G_{red}(s) = \begin{pmatrix}A_{SP} & B_{SP}\\ C_{SP} & D_{SP}\end{pmatrix} \tag{3}$$

where

$$\begin{aligned}A_{SP} &= A_{bal,11} - A_{bal,12}A_{bal,22}^{-1}A_{bal,21}\\ B_{SP} &= B_{bal,1} - A_{bal,12}A_{bal,22}^{-1}B_{bal,2}\\ C_{SP} &= C_{bal,1} - A_{bal,12}A_{bal,22}^{-1}C_{bal,2}\\ D_{SP} &= D - A_{bal,22}^{-1}B_{bal,2}\end{aligned} \tag{4}$$

3.2 Frequency Domain Interval Gramian based Model Reduction

Gawronski and Juang proposed a new model order reduction technique based on Gramian theories defined over a desired frequency interval gramians for continuous system [24]. By utilizing this method, the stability of reduced order models is not guaranteed even though the original system is stable. However, this method has a better approximation error of original system in a desired frequency interval than the existing extended methods [10].

Gawronski and Juang [24] defined the frequency domain interval controllability and observability gramian respectively by the following equations:

$$\begin{aligned}P(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} R(\omega)BB^TR^*(\omega)\,d\omega\\ Q(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} R^*(\omega)C^TR(\omega)\,d\omega\end{aligned} \tag{5}$$

and $\Omega = [\omega_1, \omega_2]$ is the frequency interval, where

$$\begin{aligned}P_\Omega &= P(\omega_2) - P(\omega_1)\\ Q_\Omega &= Q(\omega_2) - Q(\omega_1)\end{aligned} \tag{6}$$

The gramian $P(\omega)$ and $Q(\omega)$ satisfying the following Lyapunov equation;

$$\begin{aligned}AP_\Omega + P_\Omega A^T + W_C(\Omega) &= 0\\ A^TQ_\Omega + Q_\Omega A + W_O(\Omega) &= 0\end{aligned} \tag{7}$$

where

$$W_C(\Omega) = W_C(\omega_2) - W_C(\omega_1) \text{ and}$$
$$W_O(\Omega) = W_O(\omega_2) - W_O(\omega_1)$$

W_C and W_o are positive definite and they act as low pass filter for the signal. This is suitable with WWTP since it is a fast response system. Therefore, the equation yields

$$P(\omega) = PS^*(\omega) + S(\omega)P$$

$$Q(\omega) = S^*(\omega)Q + QS(\omega) \quad (8)$$

and

$$W_C(\omega) = S(\omega)BB^T + BB^TS^*(\omega)$$
$$W_O(\omega) = S^*(\omega)C^TC + C^TCS(\omega) \quad (9)$$

where, $S^*(\omega)$ is the conjugate transpose of $S(\omega)$

$$S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R(\omega)\,d\omega$$

$$= \frac{1}{2\pi}\ln(j\omega I - A)(-jwI + A)^{-1} \quad (10)$$

3.3 Reduction Error

Reduction error is the comparison error between reduced model and estimated models. It is an important element that is utilize as the way to select a reduced models. In this research, the least error reduction is selected. Figure 4 shows the reduction error concept in block diagram.

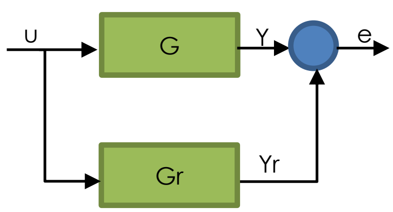


Figure 4 The concept of the reduction error

From Figure 4, the error between the estimated system, G and Gr, reduction system is calculated by utilize $e = y - y_r$ approach.

4.0 RESULTS AND DISCUSSION

4.1 Modelling of Raw Data

The modelling process is important to provide a computational setting within which the behavior of a process can be examined in response to change in the inputs [1]. Identification of nonlinear system is assumed as a complicated task [25]. However, in this research, a linear model is used to estimate the model of the activated sludge process. The data obtained from the real plant will be simulated in MATLAB and estimated using linear model.

The raw data is obtained from Bunus Regional Sewage Treatment Plant (RSTP) Kuala Lumpur, Malaysia. From the plant, four inputs are obtained, which are Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), Suspended Solids (SS) and Ammonium (NH) whereas the output is the water pH. The data obtained is simulated and modelled using linear model ARX (autoregressive exogenous) which is generated heuristically via System Identification toolbox.

The ARX model is used in this research to estimate the model of the activated sludge process. The linear system identification is obtained from the experimental data that consists of input, output and disturbance [26]. The model structure for Single-Input-Single-Output (SISO) ARX model linear difference equation is given by the following equation.

$$y(t) = a_1y(t-1)+..+a_{na}y(t-na) =$$

$$b_1u(t-nk)+..+b_{nb}u(t-nk-nb+1) \quad (11)$$

which relates the current output $y(t)$ to a finite number of past outputs $y(t-k)$ and inputs $u(t-k)$). The equation is defined by three integers na, nb and nk, where na is the number of poles, $nb\text{-}1$ is the number of zeros and nk is the pure time-delay in the system. Typically, nk is equal to 1 if there is no dead time. The equation can be written in a more compact structure which is given in equation (12)

$$A(q)y(t) = B(q)u(t - n_k) + e(t) \quad (12)$$

where $e(t)$ is the value of noise existed in the process. The first part of this paper focuses on modelling of SISO system, where the input is Suspended Solids and output is pH.

From the simulation, the model is selected based on how closely it resembles the original model. The original model is shown in Figure 5.

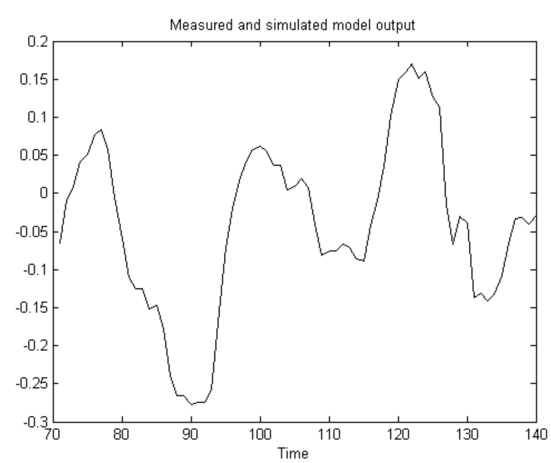


Figure 5 Original ARX model

The best fit of the estimated model should be at least 80% and higher. The data is simulated heuristically by manipulating the order of the estimated model in order to achieve a best fit of 80% and higher. Figure 6 shows several estimated models and the best fit value when compared to the original system.

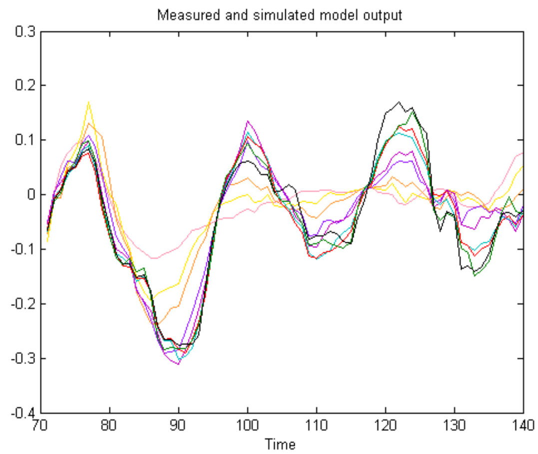


Figure 6 Highest best fits obtained via heuristic approach

From Table 1, it can be observed that the best fit obtained and the respective order of the system

Table 1 Best fit (%) and order of the system

No. of order	Best fit (%)
20th	85.11
12th	73.32
10th	72.97
15th	70.43
4th	14.95

From the table, the model with 20th order is selected because the best fit is more than 80%. However, the order is higher than 10 which cannot be considered as a good model. The order of model should be less than 10 in order for model to be applicable to the real plant. A high order is difficult to be implemented in real plant due to the complexity of the system. Hence, a model order reduction technique is proposed in the next section.

4.2 Model Order Reduction

In this work, a newly proposed model order reduction (MOR) algorithm is presented. The combination of singular perturbation approximation and frequency domain interval gramian (FD-SPA) will be applied to reduce the high order system of a Bunus RSTP system. The algorithm of the proposed technique is given below:

Algorithm 1

1) Given the stable minimal realization A, B, C, D in state-space
2) Compute gramian $P(\omega)$ and $Q(\omega)$ in equation (5) to (10).
3) Transformation matrix is obtained from factorization of gramian P_Ω and Q_Ω as

$$P_\Omega = Q_\Omega = USV^T$$

where U and V are unitary matrices while S is a positive definite diagonal matrix.

4) The reduced order model is $(A_{SP}, B_{SP}, C_{SP}, D_{SP}\}$ as in equation (4).

As mentioned previously, a high order system is complicated and need to be simplified by reducing the order of the system. Hence the proposed MOR technique is applied to Bunus 20th order system to reduce the system order to below 10th.

Applying Algorithm 1 to the 20th order system from 1st to 10th order system yields the following result shown in Table 2 below

Table 2 Error and order of the system

Model order	Error
1	0.1059
2	0.0108
3	0.0160
4	0.0088
5	0.0154
6	0.0027
7	0.0027
8	0.0028
9	0.0025
10	0.0140

Since the reduced model is selected based on the least reduction error, from Table 2, this is given by the 9th order of the Bunus RSTP system. The error describes the comparison between the reduced models with the estimated data. The transfer function for 9th order system is given as

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{0.0002358s^9 + 0.0003343s^8 - 0.001178s^7 + 0.0009753s^6 - 0.001993s^5 + 0.0007665s^4 - 0.0009345s^3 + 8.599x10^5s^2 - 8.703x10^5s - 3.63410^5}{s^9 + 0.3564s^8 + 2.888s^7 + 0.8187s^6 + 2.322s^5 + + 0.522s^4 + 0.5753s^3 + 0.1011s^2 + 0.03026\,s + 0.00524}$$

To show the effectiveness of the proposed technique, a sigma plot is presented in Figure 7. It can be observed that the estimated model is reduced to a 9th order system with 0.0025 reduction error. The frequency interval considered for this technique is 0.05 rad/s and 1.4 rad/s. The reason of this selection is an attempt to eliminate irregularities shown in the original system between desired frequency intervals.

From Figure 7, it is observed that the reduced order system follows the estimated model closely, especially at the low frequency and the frequency interval considered. The proposed technique is capable in reducing a model order while retaining the characteristics of the original system.

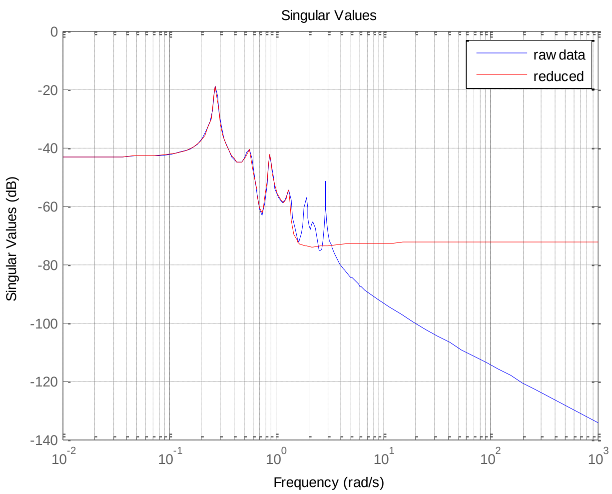


Figure 7 Sigma plot reduced order system

4.0 CONCLUSION

This paper proposed a linear modelling of activated sludge process using ARX model as well as a novel model order reduction technique. The raw data from Bunus RSTP is stimulated via System identification toolbox in MATLAB. A 20th order model is generated via trial and error approach to obtain the best fit of 85 %. This is accomplished by altering the order of the ARX model. To reduce the system's order, a newly proposed model order reduction technique is applied to the 20th order system. From the results obtained, the estimated model is reduced to a 9th order system with 0.0025 reduction error with the desired frequency interval of 0.05 rad/s and 1.4 rad/s. This reduced model order model is readily applicable to the next stage of controller design in order to shows the improvement of pH in a Bunus RSTP system.

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