Jurnal
Teknologi

Full Paper

EMPIRICAL STRENGTH ENVELOPE FOR SHALE

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Article history
Received
2nd December 2015
Received in revised form
13th March 2016
Accepted
31st March 2016

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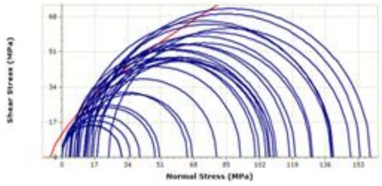
Graphical Abstract



Abstract

Effectively, strength envelope describes behavior of rock when subjected to common stresses in construction, i.e. compressive, triaxial and tensile stresses. This study is aimed at investigating the strength envelope for shale, a sedimentary rock obtained from dam project site in Baram, Sarawak. Series of triaxial compression tests were carried out to obtain the strength envelope for the rock samples. For verification of failure criterion, uniaxial compression and Brazilian tests were also conducted on the rock samples. Results from the relevant tests were analysed using RocData software to obtain the strength envelope. Subsequently, Mohr-Coulomb and Hoek-Brown failure criterion are used to determine failure envelop for the rock samples. Based on the failure envelopes and the related strengths (i.e. compressive and tensile strength), suitability of both approach, in defining strength envelope for shale, is verified. The study shows that for highly laminated sedimentary rock like shale, Hoek-Brown criterion gave a more representative failure behaviour. The failure envelope clearly shown all the strength limits when the rock is subjected to triaxial, uniaxial and tensile stress, which is not clearly shown in the Mohr-Coulomb criterion. Therefore, Hoek-Brown criterion is a more appropriate method for describing strength envelope, as it able to show the limiting stresses when rock samples are subjected to common stresses in construction.

Keywords: *strength envelope; shale; Mohr-Coulomb criterion; Hoek-Brown criterion*

1.0 INTRODUCTION

Civil engineering constructions require a comprehensive approach for characterizing and assessing strength of rock when subjected to various stresses. The present approach includes evaluating the strength of rock samples in laboratory under the effect of stresses like compression and tension. At depth, effect of confinement on the rock is also essential.

Mohr strength envelope is often used to evaluate the failure criterion for rocks. However, this approach requires understanding on the material strengths and mass conditions of the *in situ* rock in order to properly characterize its strength. The reliability of the approach is also affected by anisotropy and inhomogeneity exhibited by the rock samples, as these characteristics affect their failure strengths.

In this study, shale (a sedimentary rock) was used as sample for the related strength tests in the lab. Shale is chosen for its anisotropic behaviour created by its minerals arrangement called lamination. This study is aimed at investigating suitability of existing empirical strength envelopes for describing limiting strengths for shale. Two failure criteria, namely Mohr-Coulomb (MC) and Hoek-Brown (HB), are used in this study. It is important for an empirical failure criteria to be able to describe the strength envelope of rocks consistently and reliably. Such approach is essential for predicting strength of rock when subjected to common stresses in construction.

To verify the strength envelope for shale, 3 types of commonly encountered stresses in rock have used; tension, uniaxial and triaxial compression. Consequently, the related laboratory tests conducted were uniaxial compression, triaxial compression and Brazillian test (an indirect tensile strength test) to obtaining the respective rock strength parameters and strength envelope for the rock samples. Analysis of data and comparison on the suitability of the selected failure criteria were undertaken using RocData software.

2.0 LITERATURE REVIEW

2.1 Shale

In civil engineering, the rock can be defined as a hard, compact and naturally occurring earth material composed of combination of one or more minerals. In addition, it is permanent and durable for engineering applications by Sivakugan, et al.[1]. Rocks types can be divided into three categories; igneous, sedimentary and metamorphic rocks.

In construction, shale is among the most difficult and problematic rock types to be dealt with and it is among the most abundant in sedimentary rocks. Classified as clastic sedimentary rocks, shale is categorised as argillaceous deposits. Shale becomes the most concern rock types in rock engineering construction due to its inhomogeneous and anisotropic behaviour.

Shale composed chiefly more silt than clay grade mineral. Texturally, clay is referring as all material finer than 4 microns while silts range in size from 4 to 63 microns. Pettijohn [2] stated that the averages shale contains about two part silts and one part clays. Due to its fines particles, their permeability is very low where can lead to reduce the effective stress and rock failure. Shale can be characterized based on the fissility and generally parallel to the bedding which most of them are laminated.

Shale exhibits fissile, Bates and Jackson [3] define fissility as the ability for some rock types to split easily into thin layers along closely spaced, rough planar and approximately parallel surface. The term fissile is also used to indicate a class of parting, with parting defined as the tendency of a rock to split along lamination or bedding which the tendency greatly enhanced by weathering, Potter et al. [4].

Bates and Jackson [3] also stated that laminae are the thinnest recognizable unit layers and are usually less than 10mm which commonly 0.05mm to 1.00 mm in thickness. The surface of lamination has their own strength to sustain the compressive stress and shear strength of the rocks itself. It is the orientation of these laminations in shale that affects its strength when loaded in different directions, i.e. anisotropic behaviour.

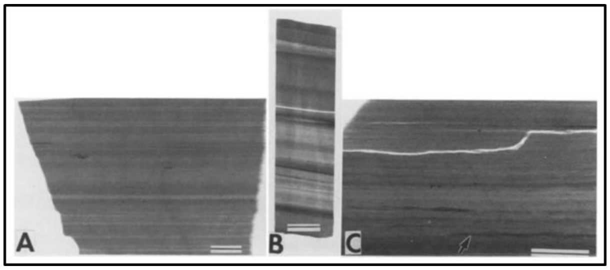


Figure 1 Types of lamination: a) Thin lamination b) Thick lamination c) Wavy lamination (O'Brien, 1990) [5]

2.2 Mohr Coulomb Criterion

MC criterion is a combination of both Mohr and Coulomb theories of rock failure. Mohr Criterion is used for cohesionless material where the failure occurs when applied stress overcomes internal friction resistance or incipient failure surfaces. However, the Coulomb theories represent the cohesive material where the sample failure will occur along any planar orientation. The maximum generated shear stress exceeds the intrinsic bonding strength between sample grains.

The concept of MC Criterion is which rock materials are defined to exhibit strength characteristics that are mobilized both by cohesive (c) and by frictional resistance (φ) effects. According to Labuz and Zang [6], MC failure criterion is a set of linear equations in principle stress describing the conditions for which an isotropic material will fail, with any effect from the intermediate principle stress (σ_2) being neglected.

Among other failure criterion, MC is the most popular criterion that works quite well for geo-materials especially soils, where the failure generally takes place in shear [1]. This failure criterion consists a few advantages which contribute to its popularity. Among others are its mathematical simplicity and clear physical meaning of mineral parameters. The shear strength of rock can be express as Equation 1 below where the failure plane (τ_f) is proportional to normal stress(σ) and the representation of this equation is supported with the Figure 2 by Zhao [7].

$$\tau_f = c + \sigma \, tan\phi \qquad (1)$$

Referring to Equation 1, τ and ϕ represent shear strength and internal friction of rock material, respectively. The failure stresses are outlined by projecting the failure plane with relevant angle on Mohr circle which can be derived from transformation relations, which yielded by Equations 2 and 3 as follows:

$$\sigma n = ½\,(\sigma 1 + \sigma 3) + ½\,(\sigma 1 - \sigma 3)\cos 2\theta \qquad (2)$$

$$\tau = ½\,(\sigma 1 - \sigma 3)\sin 2\theta \qquad (3)$$

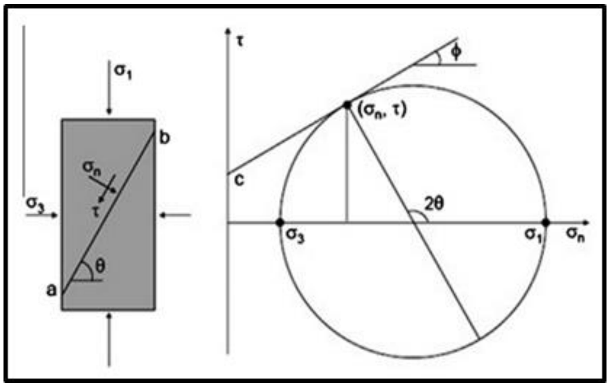


Figure 2 Stress condition on strength envelope a-b and tangent point on Mohr Circle (Zhao, 2005) [7]

2.3 Hoek Brown Criterion

In geotechnical engineering, it is common to present failure criterion in term of shear and normal stress on the failure plane. Accordingly, the HB criterion is empirical with no fundamental relationship between the constants included in the criterion and any physical characteristics of the rock, Hoek [8]. The HB criterion has been widely accepted in rock engineering practice since it was derived based on a wide range of experimental data studied by Lee et al.[9].

The concept for HB criterion is it relates limiting rock failure conditions in terms of principle stress components. The criterion is expressed in terms of major and minor principle effective stresses which acting on an element of the rock mass. The definition of criterion can be define in basic equation as shown in Equation 4 below.

$$\sigma'_1 = \sigma'_3\sqrt{m\,\sigma_c\,\sigma'_3 + s\sigma_c\text{^}2} \qquad (4)$$

Where σ'_1 and σ'_3 are major and minor principal effective stress at failure respectively while σ_c is uniaxial compressive strength of the intact rock. The material constant represent in term of m and s.

This failure criterion only applicable to isotropic rock. Hoek and Brown [10] stated that the Hoek-criterion can be used where the rock contains four or more closely spaced discontinuity sets and none of the discontinuity is weaker than the other. In case there is a contrasting strength, HB Criterion is not applicable.

HB criterion was developed through an extensive evaluation of laboratory test data covering a wide range of intact rock types. In addition, it is a non-linear form which deals with experimental data over a range of confining pressure. Other advantage is that it provides a straight forward empirical means to estimate rock mass properties. Eberhardt [11] pointed out that the higher value of m, gives a higher friction angle. Comparison between MC and HB Criteria [1] are shown in Figure 3 and Figure 4.

3.0 METHODOLOGY

The rock samples were collected from a dam project site in Baram, Sarawak. Samples preparation and laboratory tests were undertaken at Rock Mechanics Laboratory, Faculty of Civil Engineering, UTM Johor. Preparation of samples and test procedures were in accordance to ISRM (2007) [12]. Total of 28 specimens were prepared for the related tests. Figure 5 show the core samples prepared for laboratory test. The laboratory tests undertaken were Brazillian test, Uniaxial and triaxial compression tests. For the triaxial compression, confining pressure (σ3) applied ranging from 2 to 24 MPa.

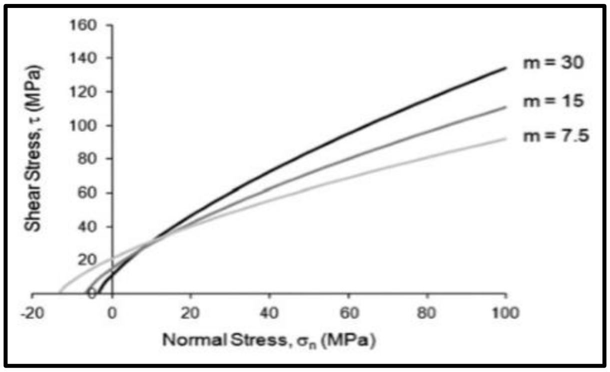


Figure 3 Change in Hoek-Brown failure envelope[11]

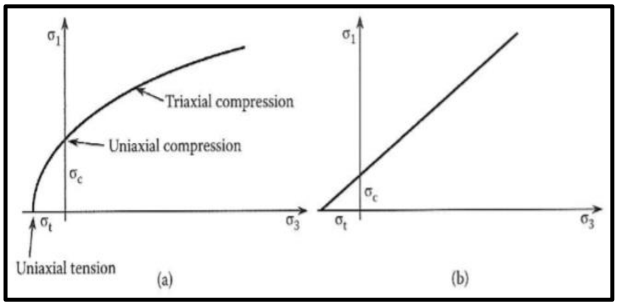


Figure 4 A Comparison of failure criterion a) Hoek-Brown failure criterion b) Mohr-Coulomb failure criterion [1]

Figure 5 The core samples of shale prepared for laboratory test

3.1 Uniaxial Compression Test (UCT)

The uniaxial compressive strength (UCS) of the sample is determined by Uniaxial Compression Test (UCT). The test was carried out on Tinius Olsen (USA) Super-L, closed-circuit servo-controlled Universal Testing Machine (capacity 3000 kN), as shown in Figure 6. The compressive load was applied at constant strain rate (equivalent to platen stroke of 0.5 mm/mm/s). The UCS is calculated by dividing maximum load with cross-sectional area of the sample.

Figure 6 Uniaxial Compression Test

3.2 Triaxial Compression Test

Triaxial compression test is to evaluate compressive strength of rock samples as a function of confining pressures. Essentially it evaluates the strength of rock under confinement (i.e. at depth below ground surface).

Figure 7 shows the test set-up with for the triaxial test. A 54 mm diameter Hoek's cell (see Figure 8) is used to confine the samples, and rubber sealing sleeve is mounted on the specimen in order to seal the specimen from the hydraulic oil (confinement medium). The required confining pressure is applied using constant pressure pump unit.

Figure 7 A Universal servo-controlled testing machine

Figure 8 Hoek's cell and sample sleeve for triaxial test

3.3 Brazillian Test

Brazillian Test or also referred as indirect tensile test. The test is for estimating uniaxial tensile strength of intact rock indirectly, by inducing failure stress along diameter of disc shaped specimen. Based on ISRM (2007), tensile strength of rocks, σ_t, is calculated using the Equation 5 (Diameter, D=2R),

$$\sigma_t = \frac{2P}{\pi Dt} = \frac{P}{\pi Rt} = \frac{0.636\,P}{Dt} \qquad (5)$$

where σ_t is tensile strength MPa), P (kN) is failure load, t is thickness of specimen (mm), D is diameter of specimen (mm) and R is radius of specimen (mm). Figure 9 show the apparatus used to perform the Brazillian test.

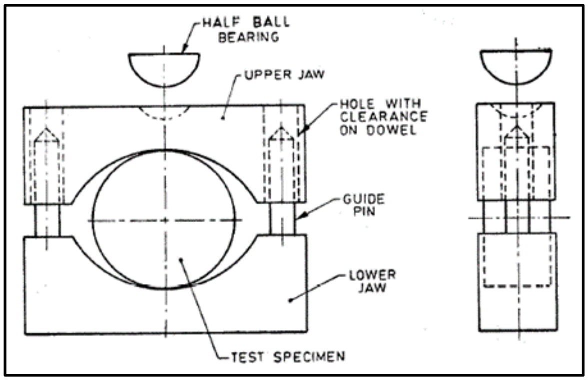


Figure 9 Craddle for Brazillian Test (ISRM, 2007) [12].

Total of 10 samples were prepared for this test. The samples and the equipment used in the Brazillian test is shown in Figure 10.

Figure 10 Brazilian test and disc shaped samples

4.0 RESULTS AND DISCUSSION

The summary of test result for uniaxial, triaxial and Brazillian test are presented in Tables 1 to 3.

Table 1 Uniaxial Compression Test

Groups	Diameter (mm)	Thickness (mm)	UCS (MPa)
Group 1	52	108	35.29
Group 2	51	106	30.82
Group 3	45	95	30.79
Group 4	45	93	50.26
Group 5	45	93	47.72
Group 6	45	93	40.11

Figure 11 Shale sample after UC T

Table 2 Triaxial Test

		Diameter (mm)	Thickness (mm)	Triaxial Compression	
				Applied confining pressure, σ_3 (MPa)	Maximum stress at failure, σ_1 (MPa)
G1	1	52	107	4.50	92.00
	2	52	111	9.00	105.40
	3	52	108	13.50	106.80
	4	52	108	18.00	128.50
G2	1	51	109	3.80	107.60
	2	51	107	7.50	139.40
	3	51	109	11.30	120.30
	4	51	107	15.00	158.90
G3	1	45	92	3.80	99.9
	2	45	97	7.50	101.80
	3	45	93	11.30	140.10
	4	45	93	15.00	153.90
G4	1	45	94	6.30	110.60
	2	45	95	12.50	109.30
G5	1	45	96	6.00	67.20
	2	45	96	12.00	129.40
	3	45	96	16.00	133.90
	4	45	96	24.00	148.90
G6	1	45	94	2.00	64.69
	2	45	94	4.00	79.76
	3	45	94	8.00	117.44
	4	45	94	16.00	109.27

*G=Group

Table 3 Brazillian Test

Diameter (mm)	Thickness (mm)	Load at Failure,F(kN)	Tensile strength, σ_t (MPa)
45	24	7.208	4.31
45	27	8.248	4.38
45	27	11.402	5.90
45	23	3.866	2.35

45	23	5.262	3.23
45	25	8.056	4.55
45	27	5.532	2.86
45	25	7.658	4.39
45	25	6.238	3.58
45	25	5.578	3.22

4.1 Uniaxial Compression Test (UCT)

From Table 1, the highest UCS is from samples in Group 4 (50.26 MPa) while the lowest is from Group 3 (30.79MPa), with average UCS of 39.17 MPa. Based on ISRM (2007), this rock is classified as R3 and R4; medium strong rock and strong rock, respectively.

Based on the image of samples after test, majority failed in a sudden manner, and this is mainly due to the distinctive lamination exhibited by shale. Majority of the failure planes occurred along the laminations (see Figure 11).

4.2 Triaxial Test

From Table 2, sample no. 4 in Group 1 shows the highest maximum stress (128.5MPa) when subjected to highest minor principle stress (18.0MPa). Meanwhile, sample no. 1 from Group 1 shows the lowest maximum stress at failure (92 MPa) and the lowest minor principle stress (4.5 MPa). Similar behaviour is observed for other groups. The triaxial test is to evaluate rock under confinement. Generally rock displays a higher strength with increasing confinement. Figure 12 shows images of sample after the triaxial compression test.

Figure 12 Samples after Triaxial Test

4.3 Brazillian Test

Figure 13 shows the sample after Brazilian test. Table 3 shows the tensile strength for the samples is between 2.35 and 5.90 MPa.

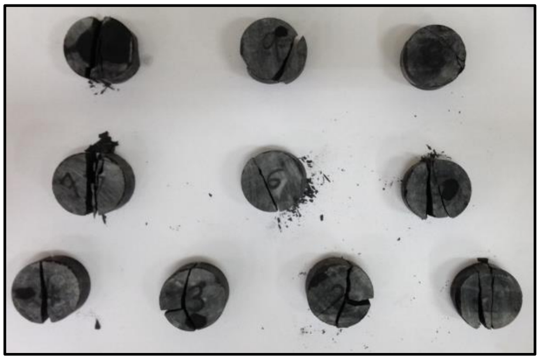


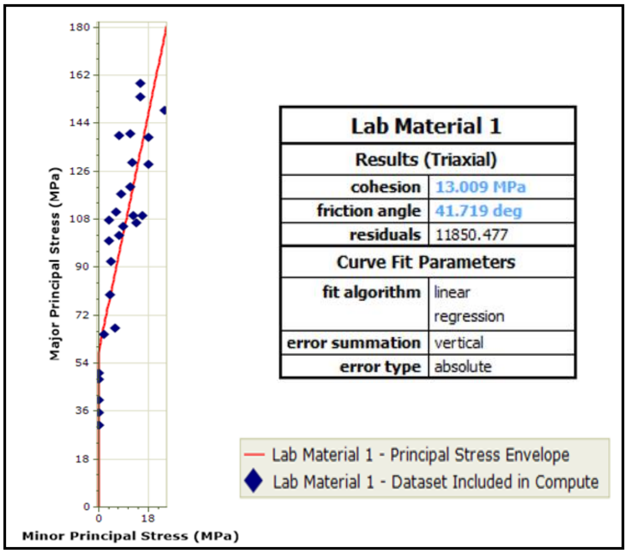
Figure 13 Samples of Shale after Brazilian test

4.4 RocData Analysis Output

RocData program is a versatile software for analysing strength parameters and for determining strength envelope of rock. In addition, it can be used to determine parameters of either a linear or non-linear strength envelopes based on the analysis of triaxial test or direct shear strength test. The failure envelopes were plotted on both shear stress and normal stress space. Furthermore, MC and HB curve fit were based on major principle stress (σ_1) and minor principle stress, (σ_3).

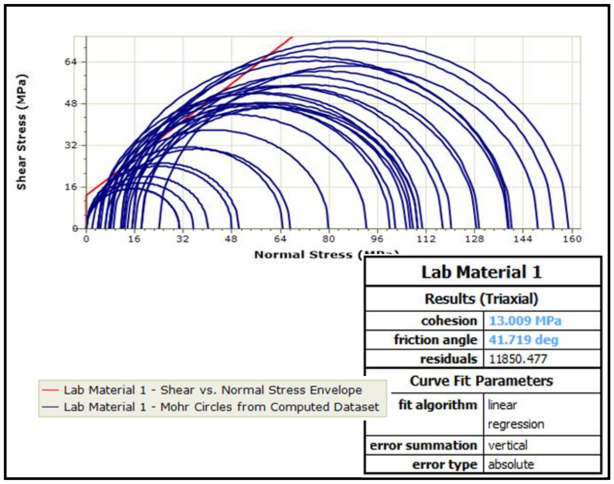
As for MC criterion, there are only two parameters can be obtained. They are friction angle (ϕ) and cohesion (c) of rock which it is paramount for the design purpose. On the other hand, four parameters namely friction angle (ϕ), cohesion (c), uniaxial compression strength (σ_{ci}) and tensile strength (σ_t) represented by using HB criterion.

Figures below are the combination results from all groups. The curve fit and strength envelope for Mohr-Coulomb criterion were presented in Figure 14(a) and 14(b). As for Figure 15(a) and 15(b) show the curve fit and strength envelope for Hoek-Brown criterion.



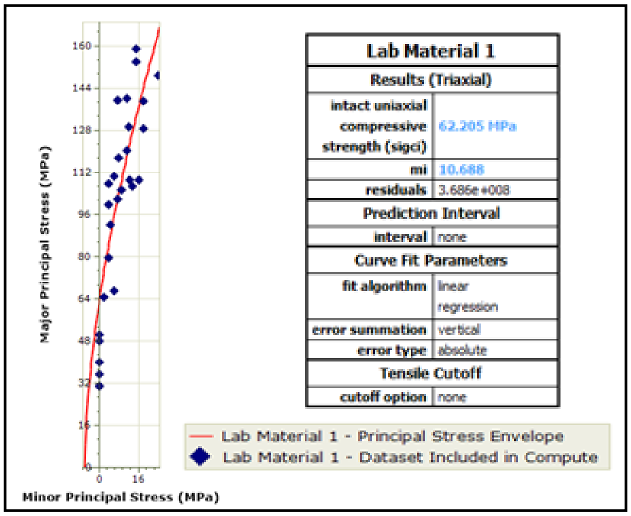
Lab Material 1	
Results (Triaxial)	
cohesion	13.009 MPa
friction angle	41.719 deg
residuals	11850.477
Curve Fit Parameters	
fit algorithm	linear regression
error summation	vertical
error type	absolute

Figure 14(a) Projection of a Mohr-Coulomb curve fit on the data pairs, σ_1, σ_3 for all samples



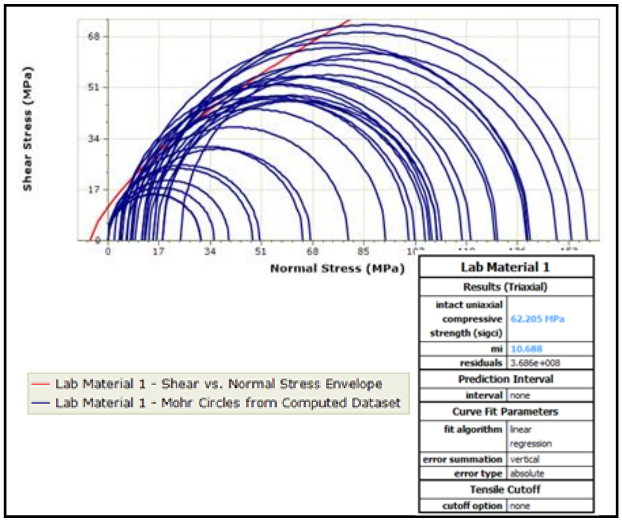
Lab Material 1	
Results (Triaxial)	
cohesion	13.009 MPa
friction angle	41.719 deg
residuals	11850.477
Curve Fit Parameters	
fit algorithm	linear regression
error summation	vertical
error type	absolute

Figure 14(b) The resulting M-C envelope for all samples



Lab Material 1	
Results (Triaxial)	
intact uniaxial compressive strength (sigci)	62.205 MPa
mi	10.688
residuals	3.686e+008
Prediction Interval	
interval	none
Curve Fit Parameters	
fit algorithm	linear regression
error summation	vertical
error type	absolute
Tensile Cutoff	
cutoff option	none

Figure 15(a) Projection of a Hoek-Brown curve fit on the data pairs, σ_1, σ_3 for all samples



Lab Material 1	
Results (Triaxial)	
intact uniaxial compressive strength (sigci)	62.205 MPa
mi	10.688
residuals	3.686e+008
Prediction Interval	
interval	none
Curve Fit Parameters	
fit algorithm	linear regression
error summation	vertical
error type	absolute
Tensile Cutoff	
cutoff option	none

Figure 15(b) The resulting M-C envelope for all samples

Table 4 summarises the results from all the laboratory tests. There are 3 laboratory tests involved to obtain the strength parameters for shale and out of these data, four parameters can be determined.

Firstly, comparison is made between the two failures criteria. From the different failures criteria obtained, the triaxial result, MC give value of ϕ = 41.72° while HB was ϕ = 43.67°. In contrast, MC show slightly higher value, 13.01MPa in term of cohesion parameter compare to HB equal to 12.36MPa. The result from HB was found to be better in describing the rock behavior compared with the MC criterion since HB is a non-linear failure criterion. HB analysis has simplified the work where the rock behaves in a non-linear pattern. Based on the table below, it clearly shown that HB criterion can obtained two extra parameters including UCS and tensile strength of the rock compared with MC failure criterion. Then, it was proven that HB criterion able to show all rock limit strengths over MC criterion.

Further comparison made between the laboratory tests and failure criterion, HB. As tabulated in Table 4 below, the UCS value from UCT was lower compared to the value calculated using HB criterion analyses. The reasons for this statement was HB criterion used Triaxial Test data where the confining pressure was taken into count. When the rock exhibits both confining and axial pressure, the rock become stronger. The concept of UCT was similar with Triaxial Test unless zero confining pressure. Furthermore as for the UCT, the test was conducted on a specific and small scale of samples (6 core samples) while HB failure criterion used the data gathered from Triaxial Test (22 core samples).

The pattern was similar for the tensile strength value; HB shows higher value compared to the indirect strength test by 1.95MPa, if the average tensile strength by Brazillian test was considered. Tensile strength of the shale for this study can be used as guideline for designing purpose since the HB value was within the range of Brazillian Test result. A little differences value between both methods was related to amount of samples used. Only 10 disc-shaped samples used in Brazillian Test compared with 22 core samples for Triaxial Test. In addition, it was reliable because both Brazillian Test and HB failure criterion consider compression stress. Other than that, the range value from 2.35MPa to 5.90MPa were due to the lamination orientation which give high impact on the tensile strength value. The highest tensile strength indicated that the lamination orientation was perpendicular to the load applied. However, the lowest tensile strength was due to the parallel of lamination orientation with axial load. According to the empirical strength envelope of shale produced by both failure criteria, the limit for the material to break can be accessed and depending on the Mohr circle correspond to the line tangent. The plotted graph explain that in case the line does not intersect the Mohr-circle, the shear stress in the medium does not exceed. It informed that the material does not break which it is not critical and the design is safe. In contra, if the stress exceeds the strength envelope, it considers as fail where the rupture is about to occur. The strength envelope which developed from this study only as a guideline for the future design purpose because the rock mass strength is lower compared to the intact rock strength.

Table 4 Summary Laboratory Test Result

Parameters	**Laboratory Test**		**Failure Criteria**	
			Triaxial Test	
	Brazillian Test	**Uniaxial Compression Test (UCT)**	**Mohr Coulomb Criterion**	**Hoek Brown Criterion**
Friction Angle, Ø (°)	-	-	41.72	43.67
Cohesion, c (MPa)	-	-	13.01	12.36
Uniaxial Compression Strength, σ_{ci} (MPa)	-	30.79-50.26	-	62.21
Tensile Strength, σ_t (MPa)	2.35-5.90	-	-	5.82

Based on the analysis, it can be inferred that HB criterion gave a more representative failure behaviour for highly laminated sedimentary rock like shale. The failure envelope clearly shown all the strength limits when the rock subjected to triaxial, uniaxial and tensile stress, which was not observed in MC failure criterion. The result obtained from HB more or less represent the overall strength of the rock mass.

Besides that, it is paramount to bear in mind that this empirical strength envelope only applicable if further analyses using Geological Strength Index value (GSI) was carried out. In a nutshell, it was important to determine the strength envelope of the rock in term of various stress; tensile strength, compression strength and shear strength for the designing purpose.

5.0 CONCLUSION

Based on the findings from this study, the following conclusions can be made:

i. Two types of failure criteria can be used to describe rock failure; Mohr-Coulomb and Hoek Brown. Hoek Brown seems to be more reliable in presenting strength envelope of shale. There are four parameters can be obtained using this criterion; friction angle (ϕ), interlocking cohesion (c), uniaxial compression strength (UCS) and tensile strength (σ_t). However, only two parameters can be obtained using Mohr-Coulomb criteria; i.e. friction angle (ϕ) and interlocking cohesion (c).

ii. Strength envelope for shale has been successfully obtained, which is based on three types of commonly encountered stresses in rock; tensile, uniaxial and triaxial stress.

iii. Comparison made between both failures criteria indicates that Hoek-Brown failure criterion gives a more representative failure behaviour, as it able to show the limit of strength of rock samples under uniaxial, triaxial and tensile stresses.

Acknowledgement

The author wishes to extend appreciation to Universiti Teknologi Malaysia to provide required facilities for this research. Valuable comments from Assoc Prof Mohd For Mohd Amin and Dr Rini Asnida Abdullah are also warmly acknowledged. Sincere gratitude offer to the geotechnical laboratory staffs; especially Mr. Zulkifly Abdul Wahid and Mr. Hidayat Pornam for their kind help in the laboratory work.

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