FLOW PROFILE AROUND BLUFF BODY

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Synopsis

This paper presents the results of an analytical study on local scour around bluff bodies. An equation has been developed which allowed the velocity distribution of the vertical component of velocity induced by the presence of a body to be predicted. The equation, equation (16) was obtained by re-organising known facts and incorporating them in a new fashions e.g. the logarithmic velocity profile was modified to include turbulence effect. Much of this has been done before but it is believed that this is the first time that the particular combination of turbalence effects, roughness effects, etc has been presented. Equation (16) is a modification of equation (16) in reference (7).

The predicted vertical velocity distributions have been compared with analytically obtained data from previous investigations. The results of the comparison are encouraging but unfortunately not conclusive due to the lack of sufficient reliable experimental data.

Introduction

In a open channel flow system the flow pattern and velocity profiles changes when a pier or similar bluff obstruction is introduced into the flow. Firstly, a stagnation plane develops on the upstream face of the body and secondly, within the region affected by the body, a vertical pressure gradient forms as a result of variations in velocity over the flow depth. These pressure gradients give rise to vertical secondary flows. In the case of bluff bodies, the pressure field is strong enough to produce a marked secondary flow in downwards direction immediately upstream of the body. The secondary flow is usually confined to the lower layer of the flow. The flow in these lower layers is further modified by the distortion of the streamlines caused by the presence of the body. Turbulence levels in the flow increase and hence give rise to increased velocities. The vertical velocity distribution thus produced will differ considerably from that of the undistrubed flow.

In general, computations of flow patterns have been based most on the logarithmic velocity distribution of the underdisturbed flow even though it is well established that the disturbed flow velocity profile may deviate substantially from the logarithmic profile (1,3, 8).

It has long been recognised that turbulence and velocity fluctuations play a major role in sediment transport and some method must be found in future studies to take their effect into consideration. In order to do this a greater understanding of the properties of the flow induced by the presence of an obstruction will have to be achieved. This form the basis of the present work i.e. to establish, analytically, the velocity distribution of the vertical component of velocity induced by the presence of a body. This starts from the Euler equations of fluid motion into which terms making allowance for properties such as the approach velocity distribution, the bed roughness, and eddy viscosity are substituted. This ultimately leads to an equation (16) which is derived basically from an extension of Bata's work (1). The major difference between Bata's and the present work involves the use of two distinctively different velocity distributions.

(i) Bata used the logarithmic law of velocity distribution for the change in velocity in the vertical direction. This is modified by means of a series of assumptions and approximations to produce an equation for the vertical velocity produced upstream of a body. In the process the possibility of reversal of flow in the vertical is eliminated. This clearly leads to errors since the vortex action in front of the body is known to produce flow reversal (5,7).

(ii) In the present works, the logarithmic velocity profile has been modified to include turbulent effects in the form of the eddy viscosity coefficient, E. The immediate result was the introduction of a non-zero velocity at bed level which must be the case if scour is to occur.

Analysis

The velocity profile for uniform channel flow is sketched in Figure 1.0 which clearly shows that two basic regions I and II can be identified. Region I is roughly identified with the "content stress layer" which follows the logarithmic distribution law. Region II has much more of the characteristics of free turbulence, the structure of which is measured by the eddyviscosity, E.

Assuming region II is uniform, E can be regarded as constant (6). This enable the velocity profile in region II to be calculated to a first approximation from equation (1).

$$\frac{du}{dz} = pE \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where

is the turbulent shear stress, U is the mainstream velocity and z is the depth of flow under consideration. The thickness of region I is very small compared with the depth of flow h and a good approximation to the real velocity profile may be obtained by extending the velocity distribution of region II down to the bed level. This introduces the concept of a non-zero velocity U_b at the bed. U_b in the case of a rough bed could be obtained from the expression (2)

$$\frac{U_b}{U_f} = 8.3 + \frac{1}{x} \ \text{In.} \frac{E}{K_s \ U_f} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

where U_f is the bed material shear velocity, x is von Karman constant and K_s is the effective roughness of the bed. Considering the shear stress distribution with respect to depth, in equation (1) is given as

$$= p.U_f^{\,2} \ 1 - (z/h) \quad \ldots\ldots\ldots\ldots \quad (3)$$

Substitution for in equation (1) and integrating from z = 0 to h yields

$$U = U_b + U_f^{\,2} \ \frac{h}{E} \quad \frac{z}{h} - \frac{1}{2} \left(\frac{z}{h}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (4)$$

For the flow condition of negligible viscosity effect for x = 0.4, E can be expressed in the form of equation (5)[5]

$$E = (0.1).\ U_f.h \quad \ldots\ldots\ldots\ldots \quad (5)$$

Where 0.1 is the value of eddy-viscosity constant, α. Substituting for E in equations (2) and (4) and putting x = 0.4 in equation (2) yields on simplification

$$\frac{U}{U_f} = K + 10 \quad \frac{z}{h} - \frac{1}{2} \left(\frac{z}{h}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (6)$$

with

$$K = 2.54 + 2.5.\ \text{In.} \frac{h}{K_s}$$

The shear velocity, U_f, may be related to the mean flow velocity by means of a friction factor, C. In general C depends on the factors influencing the resistance to flow, such as the granular roughness of the bed surface, the size and shape of the sand waves and the amount of material carried by the flow.

For the case of clear-water scour in a steady uniform flow (at least in the initial stage) the factor is given as

$$C = \frac{U_m}{U_f} = K + \frac{1}{3\alpha}$$

i.e. $\quad U_f = C.U_m \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$

with C = K + 3.45 for α = 0.1, where U_m is the mean velocity of the flow. Combination of equations (6) and (7) gives the desired expression for the velocity profile,

$$U = U_{m.} \; f_1 \qquad \ldots\ldots \qquad (8)$$

$$\text{with } f_1 = 1 - \frac{1}{C} \; 3.45 - \left(\frac{z}{h} - \frac{1}{2} \; (\frac{z}{-})^2 \right) \qquad \ldots\ldots\ldots \qquad (8a)$$

Thus local scour is open channel is basically a turbulent transport process and as such is dependent on bed roughness, fluid turbulent levels and vorticity, the latter two being expressed in terms of E and x values. For any channel bed made up of granular material of uniform size, effective roughness is given by $K_s = n.D_m$ where D_m is the mean size of bed material and $n = 2^{(4)}$.

Consideration must now be given to the derivation of a relationship for the prediction of velocity when the flow encounters an obstruction. The analysis is confined for flow past a cylindrical body with a uniform approach flow. Choosing a cylindrical coordinates (Fig. 2.0) and denoting by U_r, U_0 and U_z the components of velocity in the radial, tangential and axial directions, Euler's equation of motion in the z-direction may be written as

$$U_r. \frac{2U_r}{\alpha r} + \frac{U_{\theta.}}{r} \frac{2U_z}{\alpha 0} + U_z. \frac{2U_z}{\alpha z} = \frac{1}{\rho} \frac{\alpha \rho}{\alpha z} \qquad (9)$$

where

U_r and U_θ are given by a well-known relationship

$$U_r = -U(1 - \frac{a^2}{r^2}) \cos\theta \qquad \ldots\ldots\ldots \qquad (10)$$

$$\text{and } U_\theta = U\,(1 + \frac{a^2}{r^2}) \sin\theta$$

where U is the velocity at level z above bed and α is the radius of the cylinder. The 1.H.S. terms of equation (9) are balanced mainly by the pressure gradient $\frac{\alpha \rho}{\alpha z}$ and in these circumstances potential flow solution are required.

Considering 2-D flow in the z-plane and denoting the pressure at (r, θ) by p and the pressure far upstream by p_1. By Bernoulli's Theorem and equation (10) the rate of change of pressure at a point in the flow at any radius, r, from the body and at any angle θ relative to the direction of the flow is given as

$$p - p_1 = p. \frac{U^2}{2} \; (2.\frac{a}{r^2} - \cos 2\theta - \frac{a^4}{r^4}) \qquad \ldots\ldots\ldots \qquad (11)$$

The maximum downnards velocity is located along the stagnation plane [1], that is along the centreline of and on the upstream side of the cylinder (i.e. $\theta = 0$). After differentation, equation (11) simplifies to

$$\frac{\alpha \rho}{\alpha z} = p.U\,(2.\frac{a^2}{r^2} - \frac{a^4}{r^4}) \; \frac{\alpha U}{\alpha z} \qquad \ldots\ldots\ldots \qquad (12)$$

Substitution of equation (10) and (12) in equation (9) results in equation (13) after dividing throughout by U, so

$$(1 - (a/r)^2 \cos\theta. \frac{\alpha U_z}{\alpha r} - \frac{(1 + (a/r)^2)}{r} \; \sin\theta \cdot \frac{\alpha U_z}{\alpha \theta} \; +$$

$$\frac{U_{z\cdot}}{U} \frac{\alpha U_z}{\alpha z} = -\,(\frac{2a^2}{r_2}. \cos 20 - \frac{a^4}{r^4}) \; \frac{\alpha U}{\alpha z} \qquad \ldots\ldots \qquad (13)$$

Assuming that at any point in the stagnation plane very close to the upstream face of the body the instantaneous change in $\frac{\alpha U_z}{\alpha r}$ is negligible, the simplification and integration of equation (13) yields

$$U_z^{\,2} = -U\,(\frac{2a^2}{r^2} - \frac{a^4}{r^4}) + C \qquad \ldots\ldots\ldots \qquad (14)$$

Applying a possible set of boundary condition at water surface, so z = h, $U_z = 0$ and $U = U_s$, equation (14) simplifies to

$$U_z^{\,2} = -U^2\left(\frac{2a^2}{r^2} - \frac{a^4}{r^4}\right) + U_s\left(\frac{2a^2}{r^2} - \frac{a^4}{r^4}\right) \quad \ldots \quad (15)$$

As the flow approaches the zone of influence of the cylinder, the net result would be the retardation of the flow. Accompanying this will be a rapid increase in the thickness of the boundary layer and redistribution of flow along the vertical, followed by change in the shape of the velocity profile. If it is assumed that eventually the mean and surface velocities are approximately equal then the term U_m in equation (8) may be replaced by U_s. Thus, substituting this into equation (15), the non-dimensional vertical velocity at any height z above bed level is given as

$$\frac{U_z}{U_s} = -f_1^{\,2}\left(\frac{2a^2}{r^2} - \frac{a^4}{r^4}\right) + \frac{2a^2}{r^2} - \frac{a^4}{r^4} \quad \ldots \quad (16)$$

in which

$$f_1 = 1 + \frac{1}{c}\; 3.45 - 10\left(\frac{z}{h} - \frac{1}{2}\left(\frac{z}{h}\right)^2\right)$$

Equation (16) provides the basis for obtaining vertical velocity profiles within the zone of influence of the cylinder.

Comparison of Mathematical Model Results With Those From Previous Theory

The data presented for verification are limited by the following restrictions. The channel bed is considered to be made up of non-cohesive, almost uniformly graded mterial (D_{60} = 0.63 mm).

The flow condition is for the clear-water scour case with minimum suspension of bed material. The results for the flow with continous supply of sediment case might be entirely different from those predicted by equation (16). The effect of suspended sediment within a flow is to dampen the turbulent fluctuations and this in turn may well influence the size of the main vortex formed in front of a cylinder and the point in the vertical where flow changes direction.

Equation (16) may be used in two ways either:-

(i) to predict a series of longitunal vertical distributions at various distances upstream of the cylinder, as in Figure 3.0 or

(ii) to provide a series of longitunal vertical velocity distributions at different levels with the flow depth, as in Figure 4.0.

Figure 3.0[7, 8] shows how the effect of the cylinder decreases as distance upstream of the cylincer increases. From this it is considered that the effective limit of the cylinder's influence is at some four (4) times its diameter, upstream, thus confirming the result of Bata [1].

Support for flow reversal at approximately mid-depth as predicted by equation (16) comes from the work of Song and Yang [5] which indicates that the vortex layer is about half the flow depth.

Figure 4.0, on the other hand, clearly indicate the diminishing influence of the downwards current as the depth of the flow increases.

The results predicted by use of equation (16) are compared with the results predicted by use of Bata's equation,[1] and Qadar's experimental results (by personal communication).

When the profile given by equation (16) is plotted along with that given by Bata's equation as in Figure 5.0, there is little overall resemblance in shape but better agreement in the lower portion of the flow, particularly at bed level.

Thus it may be seen that the present mathematical model as given by equation (16) is an improvement on that produced by Bata but no doubt required further refining and checking against more accurately obtained laboratory data.

The vertical velocity distribution as given by equation (16) and as measured by Qadar is shown in Figure 6.0. There is a fair degree of agreement between the distributions in the upper part of the flow from z/h = 0.7 to z/h = 1.0.

However, there is considerable difference in the lower part of the flow with equation (16) predicting a much larger portion of downwards velocity and most significantly from the point of view of scour a larger velocity at bed level. Some of the differences may be accounted for by the fact that Qadar's data was obtained for a rigid bed situation and consequently a flow with no sediment in suspension. The effect of suspended sediment within a flow is to dampen the turbulent fluctuatuions and this in turn may well influence the size of the main vortex formed in front of a cylinder and the point in the vertical where flow changes direction.

Thus it was considered sufficient to make use of the model in its present form to provide background information for the choice of scour protection devices such as ring plates[8].

Conclusions

1. By applying conditions of a constant eddy viscosity together with finite value of bottom velocity an equation, equation (16), has been derived which will allow the vertical velocity upstream of an obstruction to be predicted. The equation presented does yield better agreement and more realistic profiles than the comparable equation presented by Bata. On this basis, it is concluded that the model presented herein can be used as a guide in the prediction of the vertical velocity distribution produced upstream of an obstruction in a clear-water flow.
2. The influence of the obstruction, or bluff body, extends over a distance of some four times its diameter in the upstream direction. Within this zone the magnitude of the downwards velocity at bed level is considerable, amounting to some 60% of the corresponding surface velocity. It is this bed level velocity which leads to the development of scour in front of the body (e.g. bridge pier).
3. An alternative way of presenting the information which may be obtained from equation (16) is in the form of longitudinal vertical velocity distribution each for a specific level within the flow depth. Such profiles may be used in the planning of the experiments to assess various types of scour protection devices, such as riprap, and ring plate concentric with pier.

Notation

U — velocity
U_b velocity at particle level
U_f — shear velocity
U_m — meanstream velocity
U_r U_θ, U_z — Components of velocity in r, θ and z cylindrical coordinate directions
U_s — surface velocity
h — upstream depth of flow
a — half width of cylinder
r — distance from centreline of cylinder to point in the flow considered
p — pressure
z — depth of flow under consideration
— Boundar shear stress
E — Eddy viscosity
x — Von Karman Constant
α — Eddy-viscosity constant (equation (5))
f_1 — As defined by equation (8a)
ρ — Density of water
θ — Angular displacement

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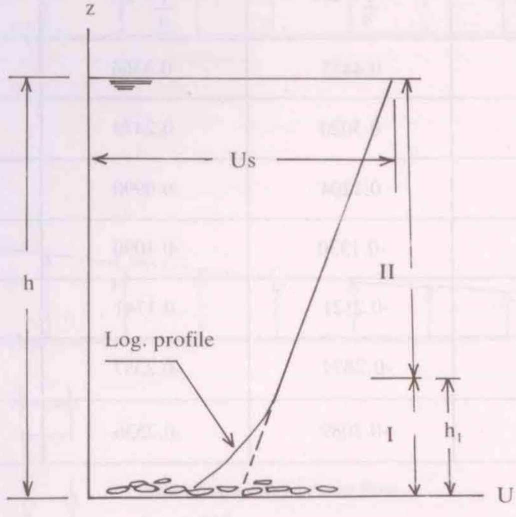

Figure 1.0: Modified Velocity Profile for Uniform Channel Flow

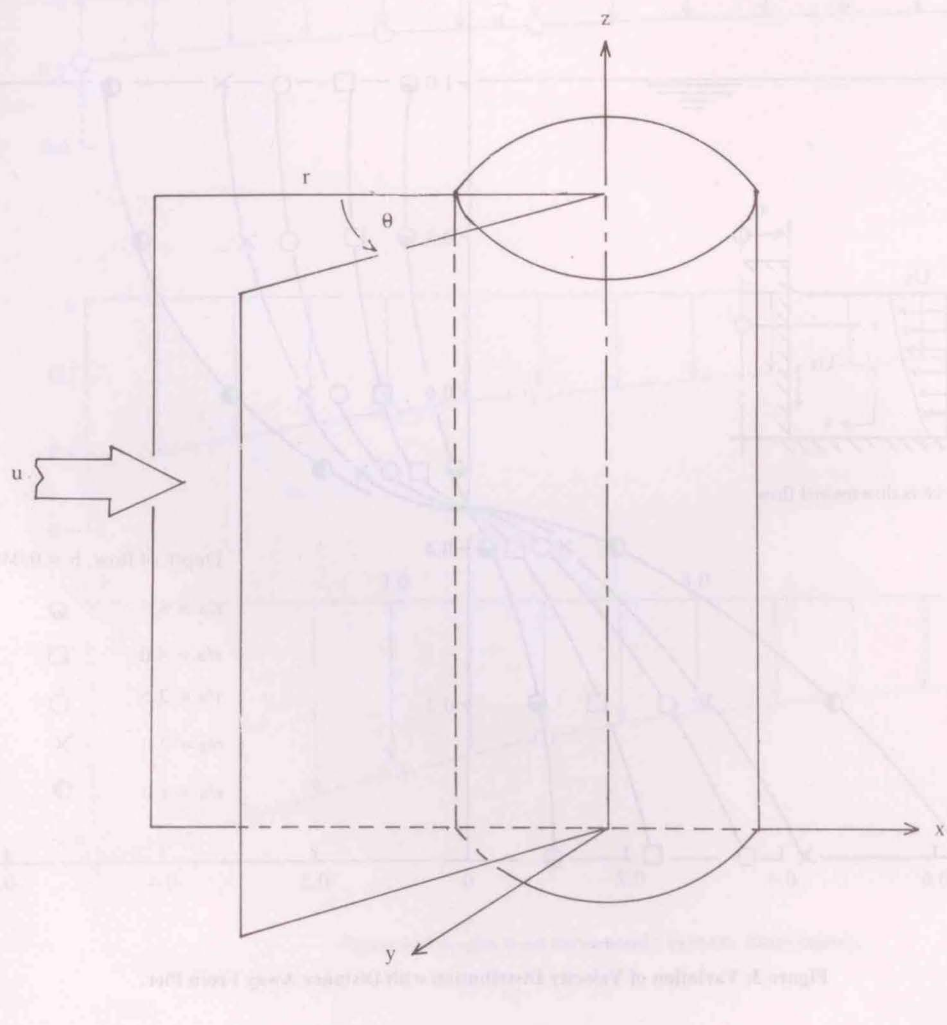

Figure 2.0: Cylinder Coordinate System

$\frac{z}{h}$	Ratio U_z/U_s				
	$\frac{r}{a} = 1.0$	$\frac{r}{a} = 2.0$	$\frac{r}{a} = 2.5$	$\frac{r}{a} = 4.0$	$\frac{r}{a} = 8.0$
0.0	0.6391	0.4425	0.3466	0.2223	0.1127
0.2	0.4567	0.3020	0.2479	0.1588	0.0806
0.4	0.1822	0.1204	0.0990	0.0633	0.0321
0.5	-0.2010	-0.1330	-0.1090	-0.0700	-0.0350
0.6	-0.3207	-0.2121	-0.1741	-0.1116	-0.0565
0.8	-0.4347	-0.2874	-0.2357	-0.1502	-0.0765
1.0	-0.4673	-0.3089	-0.2536	-0.1512	-0.0823

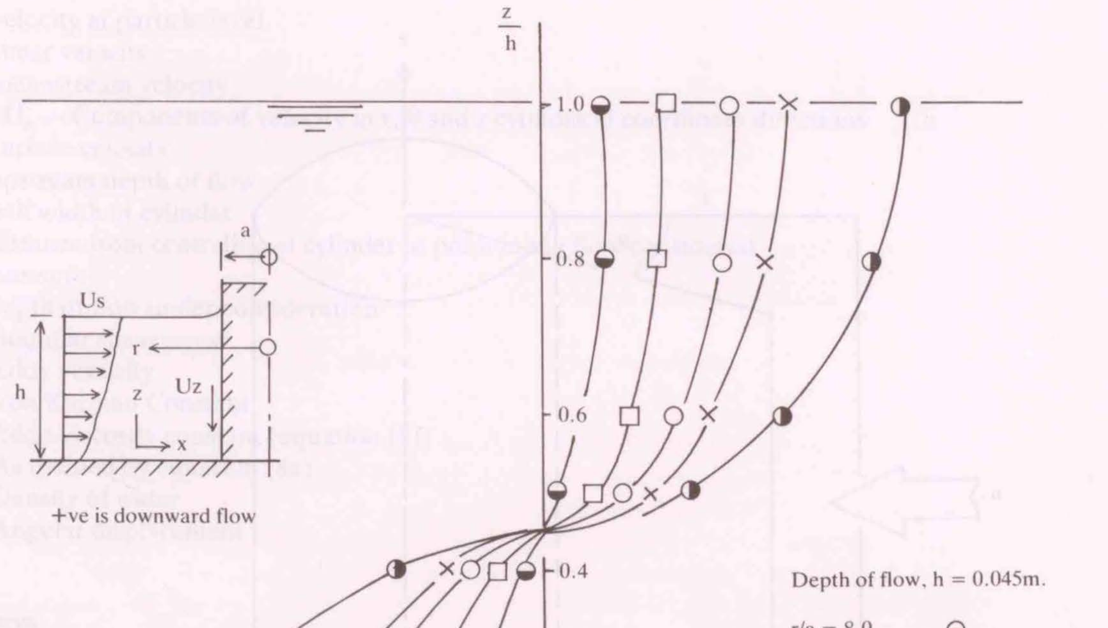

Figure 3: Variation of Velocity Distribution with Distance Away From Pier.

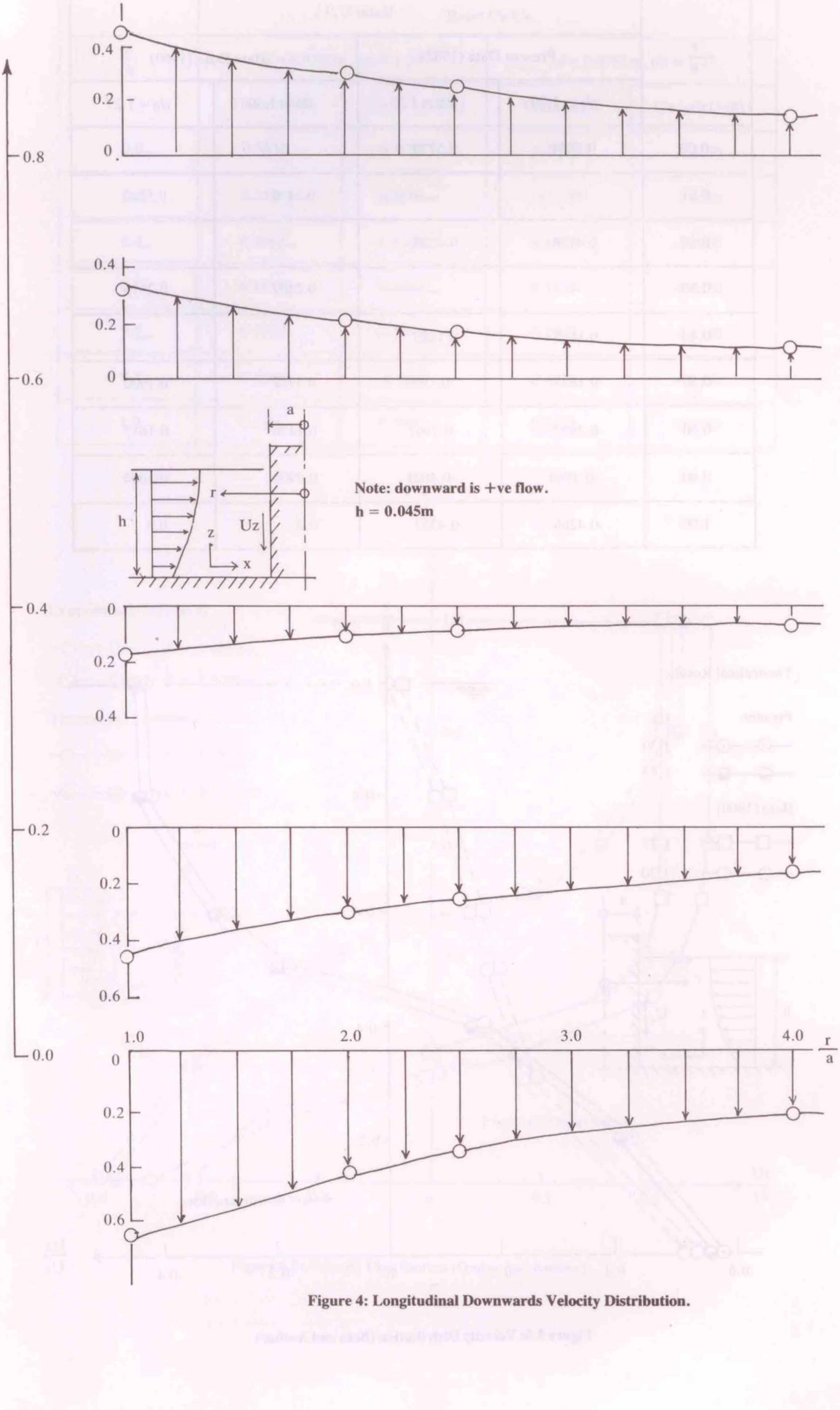

Figure 4: Longitudinal Downwards Velocity Distribution.

$\frac{z}{h}$	Ratio U_z/U_s			
	Present Data (1982)		Bata Data (1960)	
	r/a = 1.30	r/a = 1.27	r/a = 1.30	r/a = 1.27
0.00	0.5834	0.5720	—	—
0.01	—	—	0.5479	0.5642
0.20	0.4170	0.4226	—	—
0.30	—	—	0.2407	0.2615
0.40	0.1663	0.1685	—	—
0.50	-0.1835	-0.1859	0.1782	0.1920
0.60	-0.2927	-0.2967	0.1476	0.1617
0.80	-0.3969	-0.4021	0.0900	0.1004
1.00	-0.4266	-0.4323	0.0	0.0

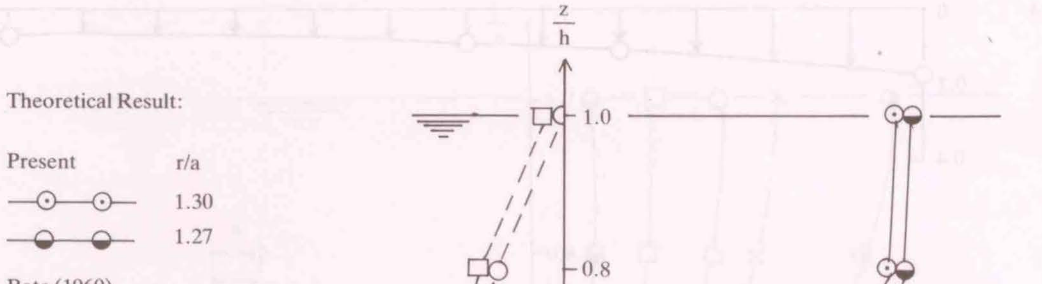

Figure 5.0: Velocity Distribution (Bata and Author)

z/h	Ratio Uz/Us			
	d = 0.0507m , r/a = 1.30		d = 0.0762 m, r/a = 1.27	
	Theoretical	Qadar's (1981)	Theoretical	Qadar's (1981)
0.0	0.5834	0.4950	0.5720	0.4150
0.2	0.4170	-0.0850	0.4226	0.0580
0.4	0.1662	-0.4500	0.1685	-0.3900
0.5	-0.1835	-0.4850	-0.1859	-0.4150
0.6	-0.2927	-0.5000	-0.2967	-0.4150
0.8	-0.3969	-0.5000	-0.4021	-0.4150
1.0	-0.4266	0.5000	-0.4323	-0.4150

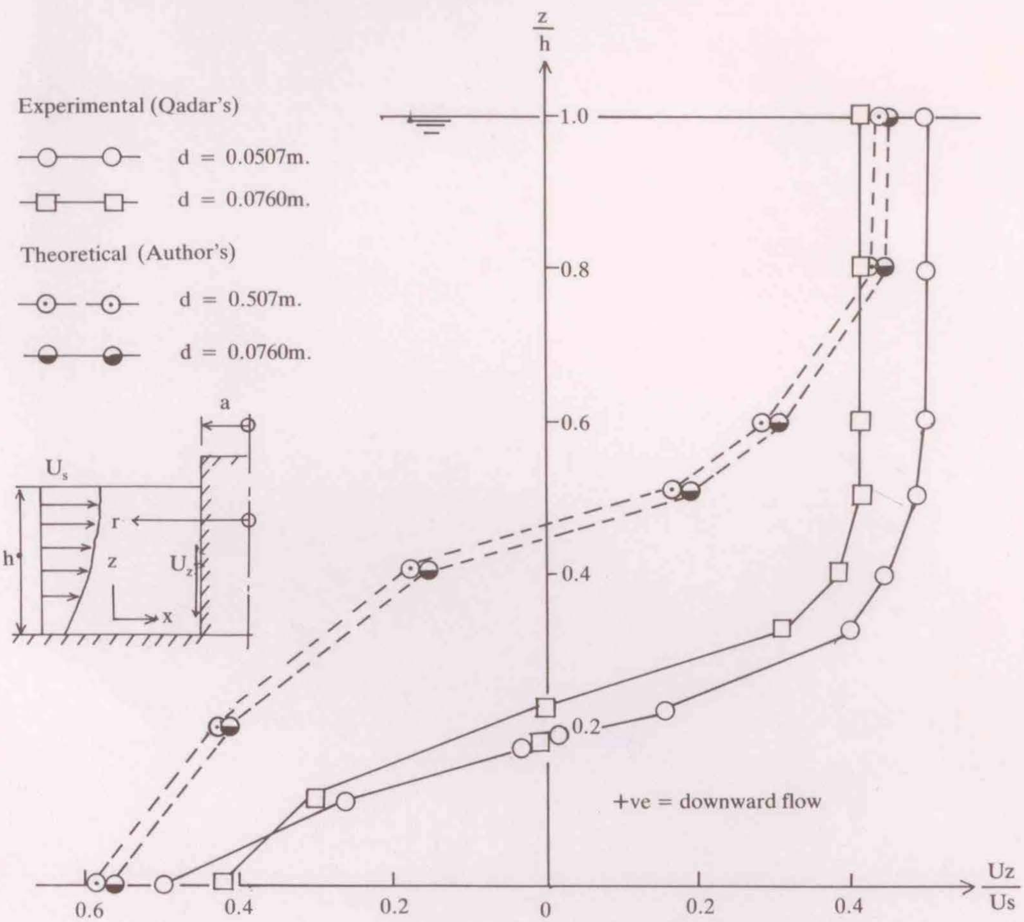

Figure 6.0: Velocity Distribution (Qadar and Author)