

# HIGH SPEED COMPUTING OF ICE THICKNESS EQUATION FOR ICE SHEET MODEL

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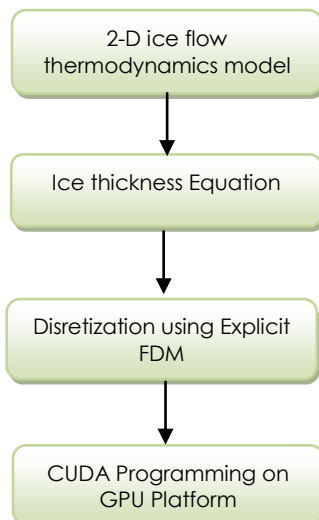
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## Graphical abstract



## Abstract

Two-dimensional (2-D) ice flow thermodynamics coupled model acts as a vital role for visualizing the ice sheet behaviours of the Antarctica region and the climate system. One of the parameters used in this model is ice thickness. Explicit method of finite difference method (FDM) is used to discretize the ice thickness equation. After that, the equation will be performed on Compute Unified Device Architecture (CUDA) programming by using Graphics Processing Unit (GPU) platform. Nowadays, the demand of GPU for solving the computational problem has been increasing due to the low price and high performance computation properties. This paper investigates the performance of GPU hardware supported by the CUDA parallel programming and capable to compute a large sparse complex system of the ice thickness equation of 2D ice flow thermodynamics model using multiple cores simultaneously and efficiently. The parallel performance evaluation (PPE) is evaluated in terms of execution time, speedup, efficiency, effectiveness and temporal performance.

**Keywords:** Ice thickness, 2-D ice flow, Finite Difference Method, Explicit Method, CUDA

## Abstrak

Gandingan model aliran ais termodinamik dua-dimensi (2-D) mempunyai peranan penting dalam menggambarkan tingkah laku lapisan ais di kawasan Antartica dan sistem iklim. Salah satu parameter yang digunakan di dalam model ini adalah ketebalan ais. Kaedah yang jelas dalam kaedah perbezaan terhingga (FDM) telah digunakan dalam pendiskretan persamaan ketebalan ais. Selepas itu, persamaan itu akan dilaksanakan dalam pengaturcaraan Compute Unified Device Architecture (CUDA) dengan menggunakan Graphics Processing Unit (GPU). Kebelakangan ini, permintaan dalam GPU untuk menyelesaikan masalah pengiraan telah meningkat disebabkan oleh harga yang rendah dan ciri-ciri prestasi pengiraan yang tinggi. Artikel ini bertujuan untuk menyiasat prestasi perkakasan GPU yang disokong oleh pengaturcaraan selari CUDA dan mampu untuk mengira persamaan ketebalan ais dengan menggunakan model aliran ais termodinamik 2D dalam sistem yang besar, jarang dan kompleks. Penilaian prsetasi selari (PPE) dinilai dari segi masa pelaksanaan, kecepatan, kecekapan, keberkesanan dan prestasi sementara.

**Kata Kunci:** Ketebalan ais, 2-D pengaliran ais, Kaedah Perbezaan Terhingga, Kaedah Jelas, CUDA

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## 1.0 INTRODUCTION

Ice sheets are regarded as an important interactive element in the global climate system that can contribute to the increasing of sea level. The water stored and released from ice sheets to the ocean made the sea level is varied in part. The two dominant ice sheets on the Earth are Greenland ice sheet and Antarctica ice sheet. If both of the ice sheets completely melt, the global sea level will be rise about 70 m [1]. While the Antarctica ice sheet alone stored an enormous of water which equivalent to 61 m [2]. Intergovernmental Panel on Climate Change (IPCC) stated that the Antarctica ice sheet contributes to the rate of the increasing sea level about 0.41 mm per year [3].

The modeling of ice sheets is studied to predict the reaction of the ice sheets and simulate the dynamic behaviours of ice sheets numerically. The global sea level can be predicted by the change of the ice sheets behaviours. The evolution and changed of the ice sheets will act as an indicator of the climate. In this paper, 2-D ice flow thermodynamics coupled model is used to observe Antarctica ice sheet evolution. This model is simplified from three dimensional (3-D) Glimmer model and become the 2-D ice flow with the coupled conditions of temperature. The evolution and interaction of thermodynamics governed in this model are in terms of ice thickness, ice velocity and ice temperature. But, this paper focuses on one dimensional (1D) ice thickness equation of the model because it is the main variable in the calculation for the depth-averaged of velocity. Ice thickness also can produce the bedrock elevations by subtracting it from surface elevations.

The aim of this paper is to discretize the ice thickness equation of the model using the explicit finite difference method and performed the numerical calculation by using CUDA on GPU platform for parallel computing. In this era, the utilization of GPU is not only for powerful rendering the graphics, but also can be used for general purpose of non-graphical computing. GPU is outstanding in applying for both graphics and computation purpose. CUDA is the parallel programming with the use of GPU. CUDA has been introduced in 2006 as the first general computing that harnessing general purpose computing on graphics processing unit (GPGPU). This programming has been used by many researchers to solve their problem with the large scale and complex computational in a more efficient way. CUDA is flexible to utilize the resources of GPU due to a unified programming model. The high data-parallelism of CUDA enables to speedup the computation [4]. Thus, this study will be implemented and run the ice thickness equation on two computer models which are C programming on the Central Processing Unit (CPU) and CUDA embedded with C language on GPU. GPU computation platform is chosen to reduce the

execution time of the numerical computations. Then, the parallel performance evaluation (PPE) is measured based on speedup, efficiency, effectiveness and temporal performance. The formula of this PPE refers to [5, 6, 7, 8]. But, CUDA with the shared memory architectures depends on the number of threads per block differ from the distributed memory architectures.

This paper is organized as follows: first, in section 2 discussed about the literature review of the paper. Then, the methodology consists of ice thickness equation with specific parameters, the parallel algorithm for the equation and a brief about CUDA presented in section 3. While in section 4 showed and discussed the results that obtained from the study and lastly in section 5 is concluded the paper.

## 2.0 LITERATURE REVIEW

An evolution of mathematical modeling of ice sheets has been developed from many years ago. Many scientists and researchers provided the ice sheet models that approximated to the real system for estimating the future of ice sheets. Some of the equations in ice sheets model are useful to figure out the response of ice sheets towards the climate system. There are many studies of mathematical modeling of ice sheets model including thermodynamics model [9,10], the ice flow dynamics model [11,12] and thermomechanics model [13,14]. Oerlemans was first published the Antarctica ice sheets of dynamic model [15] that used the vertically integrate model to study how the Antarctica ice sheet will respond to the warming of the climate. Then, Oerlemans upgrade the model by added the modelling of ice shelves where the ice thickness will be a function of both distances of grounding line and ice sheets [16]. Besides that, the 3-D thermomechanical model is applied in order to solve for West Antarctic Ice Sheet (WAIS) of the internal flow dynamics [17]. It is used to predict the location of ice streams within WAIS. In this model, it gives the freely evolve for temperature, velocity, ice thickness and temperature. The existence buoyancy effect is considered in this research because the grounded below sea level of the large amount of WAIS. The thermomechanical coupled shallow ice approximation model is presented with exact solutions for solving the Antarctic ice sheet [18]. The exact solutions are used to verify the numerical results of finite difference method. The solutions are good in order to test and compare with numerical solution. The Glimmer model with erosion component in postulated situation is studied by [19]. In this research, they want to review the landscape evolution under the ice sheets. The model used is fully 3-D thermomechanical coupled with a sliding component that can predict the ice thickness, basal velocities and the distribution of areas of pressure melting point.

High performance computing on GPU has gained an interest of many researchers from many fields due to the flexible programming for the high floating point performance. In 1999, GeForce 256 was introduced the first GPU for 3-D graphics processors with the single chip. Then, GPU has been evolved from the graphics processor to the programmable processor for parallel which can execute many parallel threads from the multiple cores. GPU becomes more efficient cost. NVIDIA has invented CUDA programming to be implemented on GPU platform [20]. The implementation of the algorithm was using standard C language with CUDA specific extensions. The computations on GPU can be managed by CUDA due to the computing of data-parallel. Today's, there are many numerical computations that has been computed by using CUDA that utilized GPU to minimize the computational time. In the fluid flow, a compact finite difference method with a higher order is used to obtain the solution by implementing on CUDA [21]. In the study, they compared the results obtained from the CPU in terms of computational time. When the mesh size is bigger, the computational time for GPU is lower than the CPU and it increasing the speedup. They concluded that the fluid flow problem that is solved by using a compact finite difference method can obtain a good speedup when running on GPU. The explicit preconditioned conjugate gradient method is parallelized by using a new proposed parallel technique and performed using CUDA [22]. They discussed the CUDA implementation on the propose technique and stated that the performance of the method can be improved by running on GPU platform with the interface of CUDA programming. The new parallel technique is used to investigate the increasing in the performance of the method. Besides that, CUDA programming is applied in the glaciers and ice sheets modeling [8]. They utilized GPU for the modeling of high order ice dynamics. The model was discretized using finite difference method and the linear system of equations has been solved using Gauss-Seidel method which implemented on CUDA. They showed that GPU has an excellent speedup compared to the CPU.

### 3.0 METHODOLOGY

In this section, the discretization of the ice thickness equation of the 2-D ice flow thermodynamics coupled model is demonstrated. We also presented an overview of the CUDA programming of GPU.

#### 3.1 Ice Thickness Equation

The mathematical modeling for the ice thickness equation of 2D ice flow thermodynamics coupled model and the physical constant are described as below [23, 24].

$$\frac{\partial H}{\partial t} = M + \frac{\partial}{\partial x} \left( D \frac{\partial H}{\partial x} - u_b H \right) \quad (1)$$

where  $H$  is the ice thickness,  $D$  is diffusivity,  $u_b$  is the bottom of horizontal velocity and  $M$  is surface accumulation. It can be assumed that there is zero horizontal velocity,  $u_b = 0$ , so the equation (1) will be written as

$$\frac{\partial H}{\partial t} = M + \frac{\partial}{\partial x} \left( D \frac{\partial H}{\partial x} \right) \quad (2)$$

Then, the initial and boundary condition for  $H$  and  $D$  of equation (2) are carried by

$$\begin{aligned} H(x,0) &= B \left( l^{\frac{4}{3}} - x^{\frac{4}{3}} \right)^{\frac{3}{8}} \\ H(0,t) &= Bl^{\frac{1}{2}}, \quad H(l,t) = 0 \\ D(x,0) &= \frac{\Gamma B^7}{20} x^{\frac{2}{3}} \left( l^{\frac{4}{3}} - x^{\frac{4}{3}} \right)^{\frac{5}{8}} \\ D(0,t) &= D(l,t) = 0 \end{aligned} \quad (3)$$

where  $B = 2^{\frac{3}{8}} \left( \frac{5M}{\Gamma} \right)^{\frac{1}{8}}$ ,  $\Gamma = 2(\rho g)^3 A_0$  and  $l = 750 \text{ km}$ .

**Table 1** The list of the constants used in the model

Symbol	Value	Description
$\rho$	910 $\text{kg}/\text{m}^3$	Ice density
$g$	9.81 $\text{m}/\text{s}^2$	Gravity acceleration
$A_0$	$2.948 \times 10^{-9} / \text{Pa}^3 \text{s}$	Glen law constant
$M$	0.3 $\text{m}/\text{year}$	Surface accumulation

Equation (2) will be discretized using the explicit method of FDM that is inspired by [25, 26]. The approximation of diffusivity is referring to EISMINT [26]. Explicit method is used forward difference in time and central difference for space. The discretization of the ice thickness equation is as follows:

$$\frac{H_i^{j+1} - H_i^j}{\Delta t} = M + \partial_x \left( D_i^j \frac{H_{i+1/2}^j - H_{i-1/2}^j}{\Delta x} \right) \tag{4}$$

$$\frac{H_i^{j+1} - H_i^j}{\Delta t} = M + \frac{D_i^j}{\partial_x} \left( \frac{\partial H_{i+1/2}^j}{\partial x} - \frac{\partial H_{i-1/2}^j}{\partial x} \right) \tag{5}$$

$$\frac{H_i^{j+1} - H_i^j}{\Delta t} = M + \frac{1}{\Delta x} \left( D_{i+1/2}^j \left( \frac{H_{i+1}^j - H_i^j}{\Delta x} \right) - D_{i-1/2}^j \left( \frac{H_i^j - H_{i-1}^j}{\Delta x} \right) \right) \tag{6}$$

$$H_i^{j+1} = M\Delta t + H_i^j + \frac{\Delta t}{(\Delta x)^2} \left\{ D_{i+1/2}^j (H_{i+1}^j - H_i^j) - D_{i-1/2}^j (H_i^j - H_{i-1}^j) \right\} \tag{7}$$

which is  $D_{i\pm 1/2}^j = \frac{1}{2} (\bar{D}_{i\pm 1}^j + \bar{D}_i^j)$  and

$$\bar{D}_i^j = \frac{\Gamma}{5} \left| \frac{H_{i+1}^j - H_{i-1}^j}{2\Delta x} \right|^2 (H_i^j)^5.$$

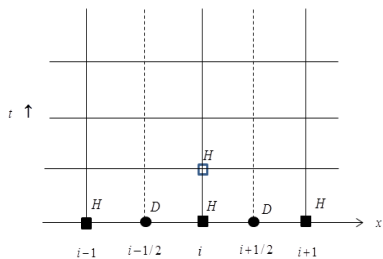


Figure 1 Finite difference grid representation

The parallel algorithm for the ice thickness equation refers as follow:

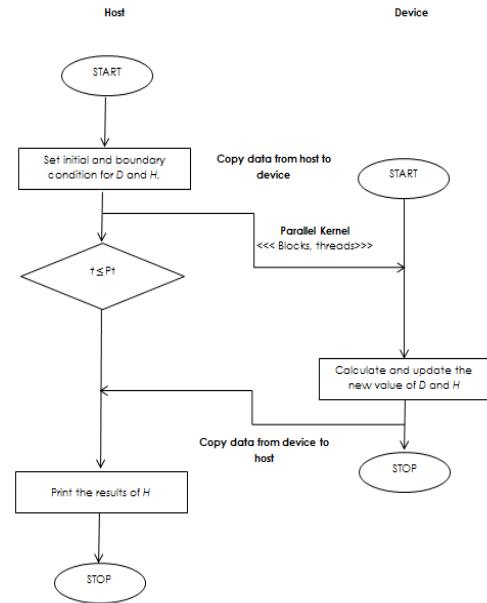


Figure 2 Parallel Algorithm

### 3.2 CUDA Overview

CUDA stands for the Compute Unified Device Architecture. The programming language available in CUDA is like C/C++, Python, Fortran, OpenACC and others. The application programming interface (API) of CUDA is the extensions of C programming language. There are two terminologies used in CUDA which are "host" and "device" that referring to the CPU and GPU respectively. Host will control the device and the tasks will perform freely during the device operation [27]. CUDA programming is dealing with many numbers of threads that is launched from the host which bring the execution to the device by the kernel function. The parallel kernel will be executed by a set of parallel threads that can be organized by the number of threads in block and the number of block in grids. So, the CUDA programming on GPU is different from the programming on the CPU.

CUDA architecture is considered as Single Instruction Multiple Threads (SIMT). It is alike Single Instruction Multiple Data (SIMD) which is one of the classification of computer architecture in Flynn's Taxonomy. SIMT and SIMD performed the parallelism with the multiple data in parallel that using the same instruction [28]. But, SIMT enable the multiple threads to execute independently even though it is in the same warp. The domain decomposition technique was used to partition the domain, so that it is easy to implement in parallel algorithm. The form of the code transfer for parallelization of explicit method of ice thickness equation by using CUDA programming is as follow:

```

//Define CUDA kernel on Device
__global__ void thickness (argument list);
{
int i = blockIdx.x*blockDimx +
threadIdx.x;
}

//CUDA kernel launch on GPU
Thickness<<<grid, thread per block>>>
(argument list);
    
```

### 4.0 RESULTS AND DISCUSSION

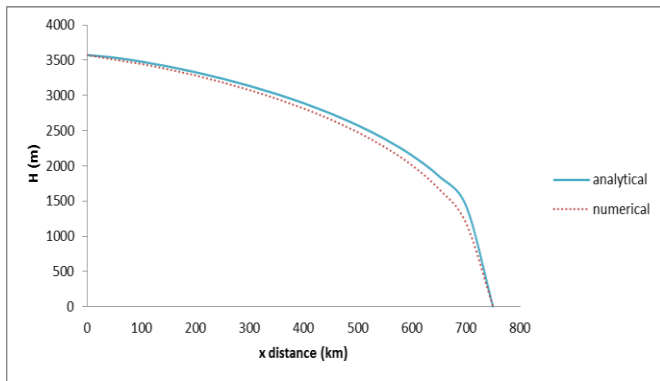


Figure 3 Visualization of the distribution of the ice thickness equation

Table 2 Numerical analysis for ice thickness equation

Numerical Analysis	Ice thickness Equation
Execution time (s)	3.819
Maximum Error	0.171
Root mean square error (RMSE)	0.0428
Number of grid	$N_x=15, P_t=2500$

The ice thickness equation is running on CPU using C programming for sequential algorithm and GPU using CUDA programming for parallel algorithm. Figure 3 shows the results of ice thickness equation computed by numerical and analytical solution of the ice thickness equation in the steady-state [20]. Then, the numerical analysis based on the execution time, maximum error and RMSE are shown in table 2. The graphical graph of numerical solution is approximated to the analytical solution in figure 3 and with the small value of RMSE in table 2 indicate that the numerical method can be used to simulate the ice thickness equation. The execution time of two different computer models and the speedup has been examined based on table 3.

Table 3 Results of Execution time of CPU and GPU and speedup for ice thickness Equation with different number of grid ( $N_x$ )

No. of grid ( $N_x$ )	Execution time of CPU (s)	Execution time of GPU (s)	Speedup
15	3.819	8.088	0.472
50	13.066	18.843	0.693
100	26.320	24.40	1.079
150	46.819	41.105	1.139

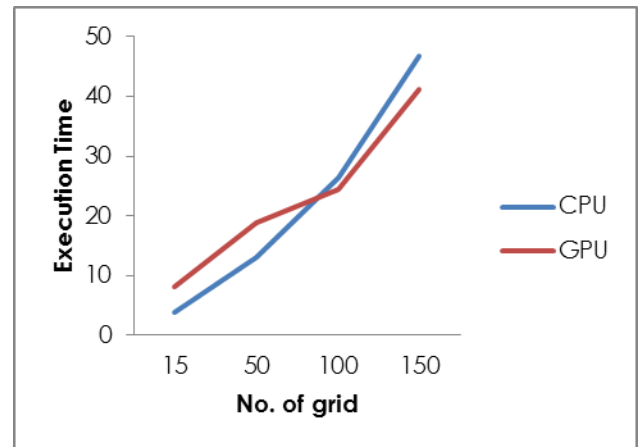


Figure 4 Execution time of ice thickness equation on CPU and GPU

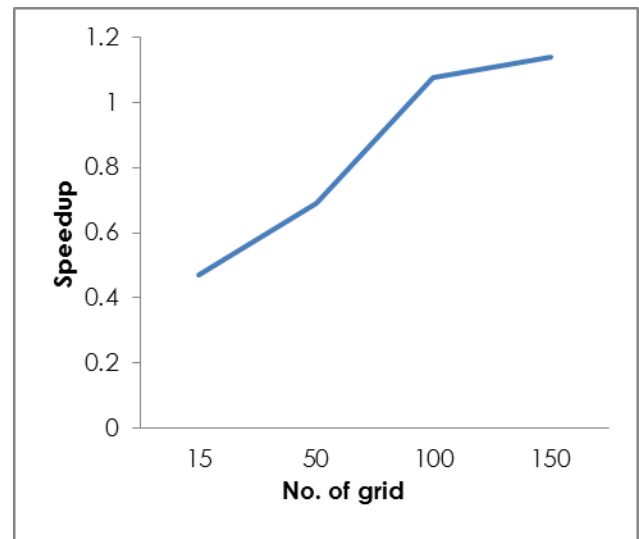


Figure 5 The value of speedup based on the number of grid

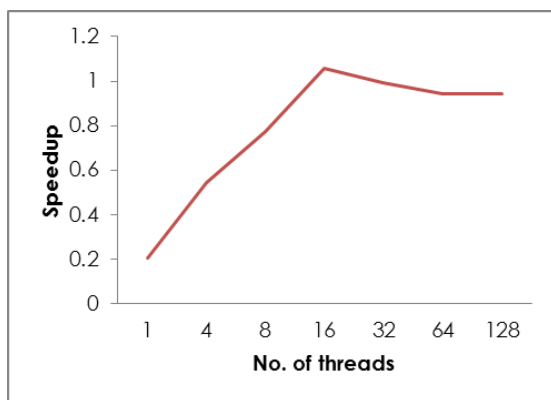
Table 3 and figure 4 shows the execution time of CPU and GPU for ice thickness equation. It is clearly seen that GPU time is much faster than CPU time when the number of grid is increased. Then, figure 5 shows the speedup obtained by comparing GPU time with CPU time and the result shows that the speedup is increasing with the increasing of the number of grid. It can be stated that the

implementation of ice thickness equation on CUDA programming can achieve a good speedup. This is due to the sequential programming on CPU needs to wait the process one by one but for parallel programming on GPU is dividing the computation data by using the number of threads and run simultaneously. Even though the data in this paper is consider small, our goal is to see that the ice thickness equation can be implementing on CUDA programming and reduce the computational time when the data is increased.

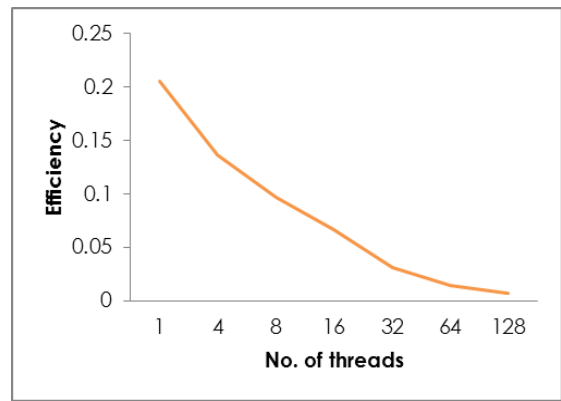
**Table 4** Execution time of GPU with the different number of threads per block for the number of grid of 150

Threads per block	Execution time of GPU (s)
1	227.389
4	85.618
8	60.311
16	44.198
32	47.04
64	49.559
128	49.593

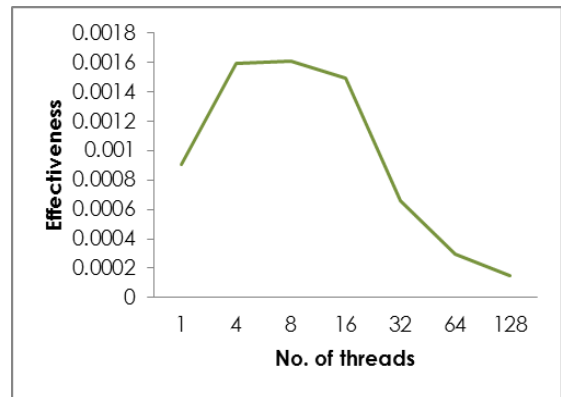
The PPE of ice thickness equation is shown in table 4 and figure 6. In table 4, the results show the execution time of GPU for 150 grids with the different number of threads. Then, in figure 6, we can see the decreasing of the value of effectiveness as the number of the threads per block increasing. It is because of the asynchronous CPU-GPU memory transfers with a bandwidth problem. The temporal performance in CUDA that indicate then, it can be seen that 16 is an optimal number of thread for ice thickness equation from the graph of temporal performance and the execution time of GPU is smaller than CPU.



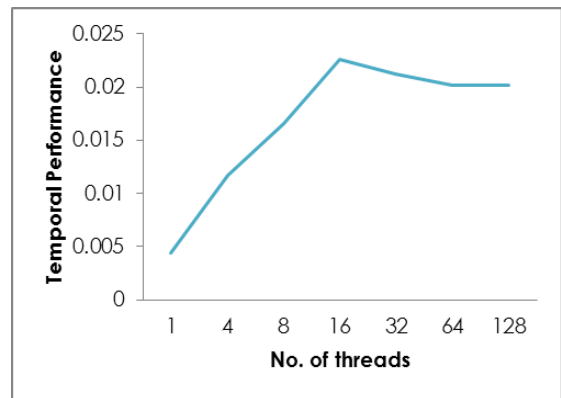
(a)



(b)



(c)



(d)

**Figure 6** Parallel performance evaluation of ice thickness equation based on a) speedup b) efficiency c) effectiveness and d) temporal performance

### 5.0 CONCLUSION

It concluded that ice thickness equation of 2-D ice flow thermodynamics coupled model computing is needed to study the ice sheet behaviour. The utilizing of GPU is presented for solving the problem with the use of parallelizable of explicit method for ice thickness equation by using CUDA programming. Based on the results, it can be seen that GPU can

outperform the CPU in terms of execution time when the size of the data is increasing. This is due to the CUDA programming suitable for the massively parallel programming and can accommodate to execute many or even up to thousands number of threads simultaneously. Also, the good speedup is achieved in this study that can accelerate the computational time by using CUDA on GPU platform.

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