

OPTIMIZATION OF THE STACK IN A STANDING WAVE THERMOACOUSTIC REFRIGERATOR AT DIFFERENT DESIGN TEMPERATURES

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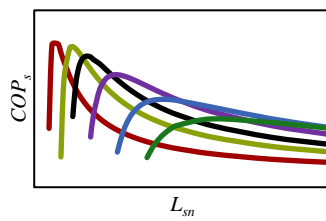
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Graphical abstract



Abstract

Although numerous successful thermoacoustic refrigerators have been reported to date, the performance of these systems is still lower than their vapor compression counter parts. Optimization is imperative to identify the upper limit of the performance in order to be competitive and accepted by the general public. However, optimization methods adopted so far, experimentally and numerically, involved discrete variations of the selected parameters of interest. This paper presents the results of an optimization using the Lagrange Multiplier method, a mathematical approach never used before. The simultaneous optimization of the stack length and center position at various design temperatures is performed for a standard thermoacoustic refrigerator design. Results show similar pattern and trend with previous results with a 24.7% higher stack coefficient of performance achievable. This is promising considering that only two of the design parameters have been optimized.

Keywords: Thermoacoustic refrigerators; optimization; Lagrange Multiplier method; stack length; stack center position

Abstrak

Walaupun banyak kejayaan penyejuk termoakustik telah dilaporkan setakat ini, perlakuan sistem ini masih lebih rendah daripada sistem penyejukan mampatan wap. Pengoptimuman adalah penting untuk mengenal pasti tahap prestasi tertinggi yang boleh dicapai untuk menjadikan sistem kompetitif dan diterima oleh orang ramai. Walau bagaimanapun, kaedah pengoptimuman diterima pakai setakat ini, secara eksperimen dan engan kaedah berangka, melibatkan perubahan berkalake atas parameter berkepentingan yang dipilih. Kertas kerja ini membentangkan keputusan pengoptimuman menggunakan kaedah Lagrange Multiplier, pendekatan matematik yang belum pernah digunakan setakat ini. Pengoptimuman serentak ke atas panjang stack dan kedudukan tengahnya pada pelbagai suhu reka bentuk dilakukan untuk reka bentuk peti sejuk termoakustik yang biasa. Keputusan menunjukkan corak dan perlakuan yang sama dengan keputusan sebelum ini dengan pekali stack 24.7% lebih tinggi boleh diperolehi. Keputusan ini amat memberangsangkan memandangkan hanya dua daripada parameter reka bentuk telah dioptimumkan.

Kata kunci: Penyejuk termoakustik; pengoptimuman; kaedah Lagrange Multiplier; panjang stack; kedudukan tengah stack

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1.0 INTRODUCTION

Pressure and displacement oscillations caused by acoustic waves are accompanied by temperature displacements. Interactions of these oscillating fluid particles with a solid boundary generate a temperature difference across that boundary. As is what happened when one speaks, air particles that move over any solid boundary will generate a temperature difference. At room temperature and pressure the temperature may be negligible, but enclosed within a chamber at high

completed in 2004 [2]. This was possible at a very high cost and with participation by many committed researchers. However, with global concern over the finite resources and deterioration of our environment due to hazardous by-products, thermoacoustic technology remains as one of the alternatives that should be investigated.

Despite considerable amount of cooling attained that has been reported before [3-6], the practical application of the thermoacoustic refrigerator is hindered primarily by its performance and cost, the former of which is being addressed here. The thermoacoustic stack, the core of the thermoacoustic refrigerator, is where the desired cooling effects occur. The coefficient of performance of the stack (COP_s) sets the upper limit of the entire refrigerator performance. Thus, its optimized design has been the focus of attention of past researchers [7-16]. Of these and many more researches completed, the stack geometry, length, center position, and the separation gap between the stack walls have been much studied. However, optimization techniques implemented involved discrete variations of the identified parameter(s) of interest over a selected range. The so-called optimized parameters are then selected from the results after careful graphical representation or/and tabulated outcomes are available. The authors have yet to come across a mathematical optimization tool being used. Thus, this paper presents the outcomes of the optimization of the COP_s of the standing wave thermoacoustic refrigerator using the mathematical technique, Lagrange Multiplier method. An advantage of this method is that the solution is searched for without the need to explicitly solve for the constraint functions [17]. The variables to be optimized simultaneously are the stack length, L_s , and center position, x_s , both of which are dependent on each other. Hence, a simultaneous optimization in this case is advantageous.

2.0 DESIGN PARAMETERS

A schematic of the simplest standing wave thermoacoustic refrigerator is shown in Figure 1.

pressure the cooling attained at one end of the solid wall could be very significant, particularly when the oscillations is generated at the resonance frequency of the enclosure. The technology associated with this phenomenon is the thermoacoustic technology. Without the need for any refrigerant and a compressor the thermoacoustic refrigerator is environmentally friendly and perhaps easier to maintain due to the lack of moving parts. Unfortunately, although the first successful thermoacoustic cooler has been demonstrated about thirty years ago by Hofler [1], the first commercial thermoacoustic chiller was only

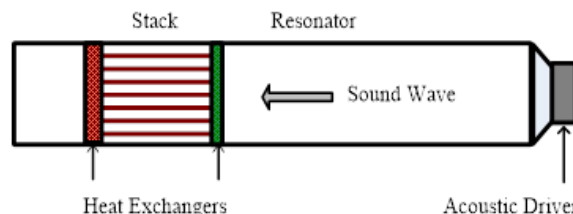


Figure 1 A schematic of the simplest thermoacoustic refrigerator.

It consists of four sub-systems; resonator, stack, heat exchangers (hot and cold), and acoustic driver. Each of these sub-systems can be considered as a complete independent control volume system. The stack, however, is the heart of the system. It is where the desired cooling effects are designed for. The COP_s is given by,

$$COP_s = Q_c / W_s \tag{1}$$

The heat absorbed by the cold end of the stack is Q_c and the acoustic work done on the stack is W_s . They are a function of several parameters including the thermal boundary layer (δ_k), drive ratio (D), the stack length (L_s) and center position (x), the working gas Prandtl number (σ) and specific heat ratio (γ), drive ratio (D), and the blockage ratio (B). The dimensional parameters are normalized, the stack length and center position by the wave number, k ,

$$x_{sn} = (2\pi/\lambda)x_s = kx_s \tag{2}$$

$$L_{sn} = (2\pi/\lambda)L_s = kL_s \tag{3}$$

The thermal boundary layer is given by,

$$\delta_k = \sqrt{(2k/(\rho_m c_p w))} \tag{4}$$

where w is the circular frequency. Heat transfer between the oscillating gas particles occur only within this thickness. It is normalized by dividing it by the plate half spacing, y_0 . The cooling load and acoustic work are also normalized through division by the product of the sound speed in the working gas, the mean pressure and stack area, which upon simplifications gives [18],

$$Q_{cn} = - \frac{\delta_{kn} D^2 \sin(2x_n)}{8\gamma(1+\sigma)\Lambda} \left(\frac{\Delta T_{mn} \tan(x_n)}{(\gamma-1)BL_{sn}} \frac{1+\sqrt{\sigma}+\sigma}{1+\sqrt{\sigma}} - (1 + \sqrt{\sigma} - \sqrt{\sigma}\delta_{kn}) \right) \tag{5}$$

$$W_n = \frac{\delta_{kn} L_{sn} D^2 (\gamma - 1) B \cos(x_n)^2}{4\gamma} \left(\frac{\Delta T_{mn} \tan(x_n)}{B L_{sn} (\gamma - 1) (1 + \sqrt{\sigma}) A} - 1 \right) - \frac{\delta_{kn} L_{sn} D^2 \sqrt{\sigma} \sin(x_n)^2}{4\gamma B A} \tag{6}$$

The drive ratio is the ratio of the dynamic pressure to the mean design pressure,

$$D = p_0 / p_m \tag{7}$$

The blockage ratio is defined by,

$$B = y_0 / (y_0 + l_0) \tag{8}$$

The normalized temperature difference is given by,

$$\Delta T_{mn} = \Delta T_m / T_m \tag{9}$$

Where T_m is the design mean temperature and ΔT_m is the desired temperature difference across the stack. Figure 2 shows some of the most commonly used stack design to date with Mylar, a type of polyethylene material, being selected due to its low conductivity property and large specific heat capacity. At 200K, Mylar has the thermal conductivity, k , of 0.144 W/m.K, density, ρ_m , of 1365 kg/m³ and a heat capacity, c_p , of 740 K/kg.K. These characteristics encourage heat transfer between the oscillating gas parcels and the solid walls within the thermal boundary layer. Thus, for the plate spacing, $2y_0$, a minimum thickness of two times the thermal boundary layer is necessary for the desired thermoacoustic cooling to happen. Swift [19] recommended a plate spacing of two to four times the thermal boundary layers. For this study, a plate separation gap of three times the thermal boundary layer is chosen.

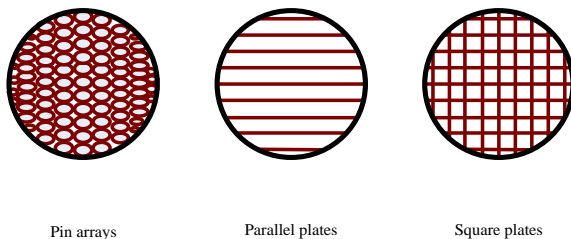


Figure 2 Commonly used stack design.

Although at first look, the pin arrays stack geometry seems to generate more thermal boundary layers, it is difficult to manufacture such a stack, so is the square stack geometry. A theoretical foundation for the selection of the geometry is obtained via the Rott's function [20], shown here in Figure 3.

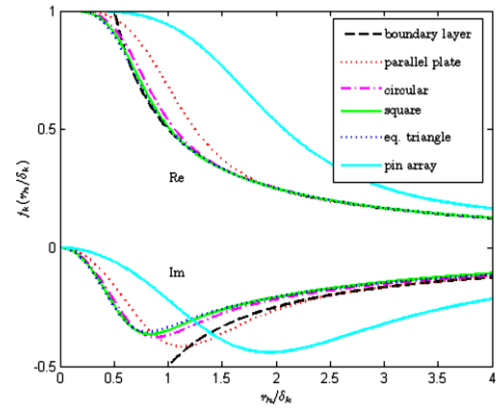


Figure 3 Rott's function for various stack geometry [21].

In addition to that, the parallel plate stack allows about 10 percent more energy flow compared with the other geometries under equal conditions [22].

2.1 Input Parameters

The required cooling load is considered as an input of the unit design which is taken to be 4 W for this particular study. The working gas is first determined since it is easier to design parameters according to the physical properties of a fluid rather than finding a fluid with prescribed physical properties. These properties are assumed constant at the selected design temperature which is 200K for the first case. The fluid is generally from the inert group due to its high Prandtl number – large thermal boundary layer where the thermoacoustic effects occur. In addition, the lighter gases in the inert group are preferred due to their higher sound velocities, the reason helium is often used and is used in this study. The design pressure is chosen to be 10bar with the drive ratio constrained by the maximum force at the acoustic driver and acoustic nonlinearities [23]. Here, a drive ratio of 3% is chosen [24]. Meanwhile, the stack thickness, $2l_0$, is twice its own thermal penetration depth; a larger thickness may induce a temperature gradient across its thickness which reduces the thermoacoustic effects and consequently the performance of the stack. The last of the input parameters for the stack design is its hydraulic radius which can be obtained from Figure 3. Further details of the arguments for the selection of the input parameters can be found in [25]. The design temperatures, which are being discretely varied to investigate the effects on the simultaneous optimization of the stack length and stack center position are 200, 250, 300 and 350K. Table 1 lists the input parameters for the optimization procedure.

Table 1 Selected design parameters for optimization

Mean pressure	p_m (bar)	10
Drive ratio	D	3%
Dynamic pressure amplitude	p_0 (bar)	0.3
Resonance frequency	F (Hz)	400
Cooling load	Q_c (W)	4

The frequency listed in Table 1 is the frequency of the acoustic driver that generates the oscillating fluid parcels.

2.2 Optimization Procedure

The maximization of the stack coefficient of performance (COP_s) under optimized conditions of the normalized stack length, $L_{sn} = kL_s$, and normalized stack center position, $x_n = kx$, is performed on the following Lagrange function, L ,

$$\frac{\partial L}{\partial \vec{X}} (COP_s(\vec{X}), \vec{Y}, \vec{\lambda}) = \frac{\partial COP_s}{\partial \vec{X}} (\vec{X}) + \sum_{j=1}^7 \lambda_j \frac{\partial G_j(\vec{X}, \vec{Y})}{\partial \vec{X}} = 0 \quad (10)$$

$$\frac{\partial L}{\partial \vec{\lambda}} (COP_s(\vec{X}), \vec{Y}, \vec{\lambda}) = \mathbf{G}_j(\vec{X}, \vec{Y}) = g_j(\vec{X}) + \vec{Y} = 0, j = 1, 2, \dots, 7 \quad (11)$$

$$\frac{\partial L}{\partial \vec{Y}} (\vec{X}, \vec{Y}, \vec{\lambda}) = 2\lambda_j y_j = 0 \quad (12)$$

with the vectors given by,

$$\vec{X} = \begin{Bmatrix} L_{sn} \\ x_n \end{Bmatrix}, \vec{\lambda} = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \end{Bmatrix} \text{ and } \vec{Y} = \begin{Bmatrix} y_1^2 \\ y_2^2 \\ y_3^2 \\ y_4^2 \\ y_5^2 \\ y_6^2 \\ y_7^2 \end{Bmatrix} \quad (13)$$

The vectors $\vec{\lambda}$ and \vec{Y} represent the Lagrange multipliers and the slack variables, respectively, while $\mathbf{G}_j(\vec{X}, \vec{Y})$ is the constraint function. These are listed in Table 2.

Table 2 The equality and inequality constraint functions

$G_1(L_{sn}, x_n)$	Stack plate spacing, $2y_0 = 3\delta k$
$G_2(L_{sn}, x_n)$	Gas velocity at driver, $u_0 = 0$
$G_3(L_{sn}, x_n)$	Modified Acoustic Mach no., $h_1 < 0.1$
$G_4(L_{sn}, x_n)$	Modified Reynolds no., $h_2 < 300$
$G_5(L_{sn}, x_n)$	Drive ratio, $h_3 < 3\%$
$G_6(L_{sn}, x_n)$	Modified wavelength, $h_4 > L_{sn}$
$G_7(L_{sn}, x_n)$	Modified wavelength, $h_5 > l_0$

The inequality constraints, g_3 to g_7 , are given by,

$$g_3 = h_1 + y_3^2 \quad (14)$$

$$g_4 = h_2 + y_4^2 \quad (15)$$

$$g_5 = h_3 + y_5^2 \quad (16)$$

$$g_6 = h_4 + y_6^2 \quad (17)$$

$$g_7 = h_5 + y_7^2 \quad (18)$$

The Lagrange multipliers and stack variables, Equations (10) through (18), are then solved with a MATLAB program. Further details on the solution approach can be found in [25].

3.0 RESULTS AND DISCUSSION

The relation between the stack coefficient of performance (COP) as a function of the optimized normalized stack length, L_{sn} , for different optimized normalized stack center position, x_{sn} , inside the resonator tube at the selected mean design temperature is shown in Figures 3 to 6.

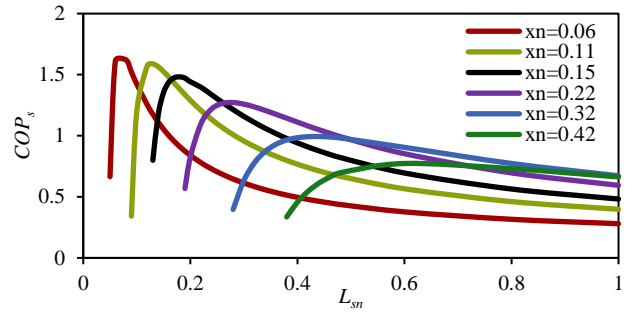


Figure 3 Stack COP as a function of L_{sn} , at different x_n at the mean design temperature 200K.

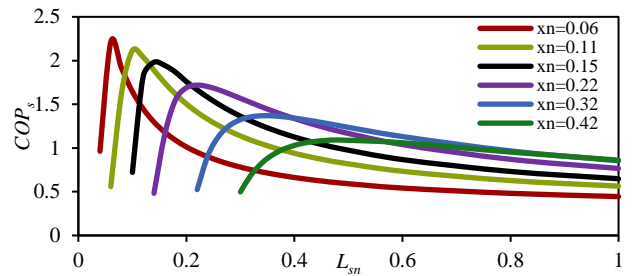


Figure 4 Stack COP as a function of L_{sn} , at different x_n at the mean design temperature 250K.

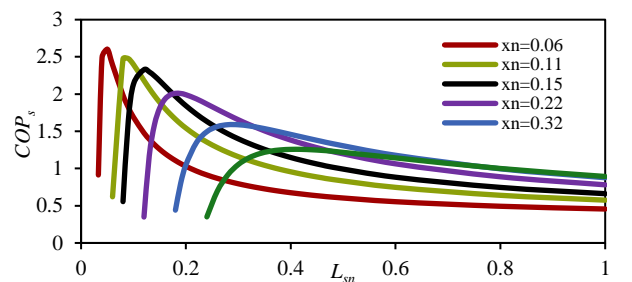


Figure 5 Stack COP as a function of L_{sn} at different x_n at the mean design temperature 300K.

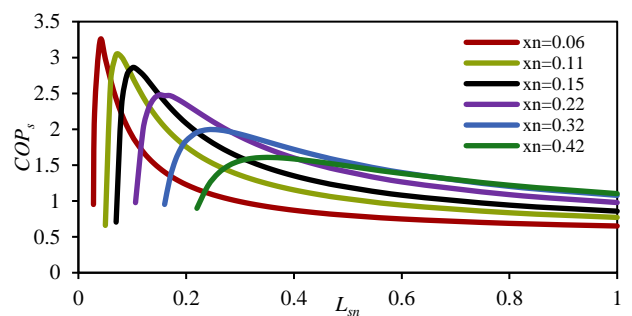


Figure 6 Stack COP as a function of L_{sn} at different x_n at the mean design temperature 350K.

The pattern and trend show similar behavior at each mean design temperature with a maximum value of the stack COP for each stack center position. These values correspond to the stack center position of $\lambda/20$ from the resonator closed end as recommended by Swift [23]. The results show that the value of the stack COP decreases as the stack center position increases or decreases from the value of $\lambda/20$. That means the optimal stack coefficient of performance is achieved when the stack center position is close to the pressure antinode (location of maximum pressure for the standing wave within the resonator). This is according to the fact that the stack should generally be located where the magnitude of the gas velocity amplitude is quite small i.e. pressure antinode, to reduce the viscous dissipation and therefore improve the performance of the stack and entire refrigerator as well [19].

As stated earlier, the stack length and center position are dependent on each other, the graphs show that for any particular maximum stack location, there is a specific stack length where the COP_s is a maximum. Any length shorter or longer would result in a lower COP_s . Furthermore, as the stack center position moves away from the pressure antinode, several stack lengths are available that gives the same COP_s .

Figure 7 compares the optimal value of the stack COP_s obtained with the optimization using the Lagrange Multiplier method against that of Tijani [20]. Both curves show similar behavior, increasing gradually with the mean design temperature with the optimized COP_s higher by 24.7% than that of Tijani's under the same operating parameters. This increase indicates the influence of the applied optimization technique which is promising considering that in this study only two parameters have been optimized.

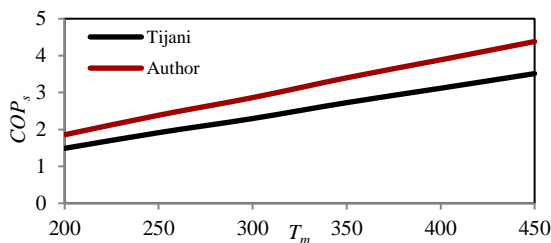


Figure 7 The optimized results and Tijani's results of the stack COP at 75 K temperature difference.

4.0 CONCLUSION

Optimization of a standing wave thermoacoustic refrigerator has been completed using a mathematical approach, the Lagrange Multiplier method. The approach which has never been utilized before for the optimization of a thermoacoustic refrigerator showed that optimization of the stack center position and stack length results in a 24.7%

increase in the stack coefficient of performance (COP_s) compared to a previous similar system. Optimization performed for various design temperatures showed similar pattern and trend. Among the conclusions that can be made are:

- The maximum COP_s is proportional to the design temperature and inversely proportional to the stack length. The increase in the maximum COP_s is observed as the stack position approaches the resonator closed end.
- The maximum COP_s achievable for several optimized stack length is the same when the position is further from the resonator closed end.

The outcomes of the study have indicated a potential in further increments of the stack COP_s with more design parameters being optimized.

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References

- [1] Hofer T.J. 1986. Thermoacoustic Refrigerator Design and Performance PhD. thesis, Physics Department, University of California; San Diego
- [2] Poesse M.E., Robert W.M., Garret S.L. 2004. Thermoacoustic Refrigeration for Ice Cream Sales. *J Acoust Soc Am.* 107(5): 2480-2486.
- [3] Minner B.L., Braun J.E., Mongeau L.G. 1996. Optimizing the Design of Thermoacoustic Refrigerator. *Proceedings of the International Refrigeration and Air Conditioning Conference.* Paper 343.
- [4] Reid R.S. and Swift G.W. 2000. Experiment with Flow-through Thermoacoustic Refrigerator. *J Acoust Soc Am.* 108: 2835 - 2842.
- [5] Tijani M.E.H., Zeegers J.C.H., De Waele A.T.A.M. 2002. Construction and Performance of a Thermoacoustic Refrigerator. *Cryogenics.* 42: 59-66.
- [6] Hariharan N.M., Sivashanmugam P., Kasthuriengan S. 2013. Experimental Investigation of a Thermoacoustic Refrigerator Driven by a Standing Wave Twin Thermoacoustic Prime Mover. *Int J Refrig.* 36: 2420-2425.
- [7] Reid R.S. and Swift G.W. 2000. Experiment with Flow-through Thermoacoustic Refrigerator. *J. Acoust Soc Am.* 108 (6).
- [8] Yougu L., Minner B.L., George T., Chiu C., Luc Mongeau, Braun J.E. 2002. Adaptive Tuning of an Electro-dynamically Driven Thermoacoustic Cooler. *J. Soc. Am.* 111(3).
- [9] Tijani M.E.H., Zeegers J.C.H., De Waele A.T.A.M. 2002. Prandtl Number and Thermoacoustic Refrigerators. *J. Acoust. Soc. Am.* 112(1).
- [10] Zoontjens L., Howard C.Q., Zander A.C., Cazzolato B.S. 2005. Feasibility Study of an Automotive Thermoacoustic Refrigerator. *Proceedings of Acoustics, Busselton, Western Australia.*
- [11] Zoontjens L., Howard C.Q., Zander A.C., Cazzolato B.S. 2006. Modelling and Optimization of Acoustic Inertance Segments for Thermoacoustic Devices. *Proceeding of Acoustic, Busselton, Western Australia.*
- [12] Paek I., Braun J.E., Mongeau L. 2007. Evaluation of Standing-wave Thermoacoustic Cycles for Cooling Applications. *Int. Journal of Refrigeration.* 30: 1059-1071.

- [13] Tasnim S.H., Mahmud S., Fraser R.A. 2011. Second Law Analysis of Porous Thermoacoustic Stack Systems. *Applied Thermal Engineering*. 31:2301-2311.
- [14] Dhuley R.C. and Atrey M.D. 2011. Investigation on a Standing Wave Thermoacoustic Refrigerator. *International Cryocooler Conference, Inc*, Boulder, CO, USA.
- [15] Tasnim SH., Mahmud S., Fraser R.A. 2011. Measurement of Thermal Field at Stack Extrimities of a Standing Wave Thermoacoustic Heat Pump. *Frontiers in Heat and Mass Transfer*. 2:013006.
- [16] Alahmer A., Omar M., Al-Zubi M. 2013. Demonstrating of Standing-wave – Thermoacoustic Refrigerator. *Int. J. of Thermal and Environmental Engineering*. 6(2): 75-81.
- [17] Kulcsar, T. and Timar, I. 2012. Mathematical Optimization Design - Overview and Application. *ACTA TechnicaCorviniensis - Bulletin of Engineering*. 21-26.
- [18] Tijani, M. E. H., Zeegers, J. C. H. and de Waele, A. T. A. M. 2002. Design of Thermoacoustic Refrigerators. *Cryogenics*. 42(1): 49-57.
- [19] Swift, G. W. 2002. *Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators*. New York, USA. Acoust. Soc. Am.
- [20] Tijani, M. E. H. 2001. *Loudspeaker-Driven Thermo-Acoustic Refrigeration*. PhD Dissertation, Physics Department, University of Eindhoven.
- [21] Ryan, T. S. 2009. *Design and Control of a Standing Wave Thermoacoustic Refrigerator*. MSc. Thesis, Department of Mechanical Engineering and Materials Science, University of Pittsburgh.
- [22] Arnoft W., Bass, H.E., Raspet, R.1991. General Formulation of Thermoacoustics for Stacks Having Arbitrarily-Shaped Pore Cross-Sections. *J. Acoust. Soc. Am*. 90: 3228-3237.
- [23] Swift, G. W. 1988. Thermoacoustic Engines. *J. Acoust. Soc. Am*. 84: 1145-1180.
- [24] Merkli, P. and Thomann, H. 1975. Transition to Turbulence in Oscillating Pipe Flow. *Journal of Fluid Mechanics*. 68: 567-576.
- [25] Mawahib H.A.E. 2015. *Thermoacoustic Refrigerator Design and Performance* PhD. thesis, Fac. Of Mech. Engineering, Universiti Teknologi Malaysia; Malaysia.